

PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 15

Nov. 15, 2008

Due: Wednesday, Nov. 19, 2008

Reading: Boas Ch. 14

1. Show that

$$\oint_C (z - z_0)^n dz = \begin{cases} 2\pi i & n = -1 \\ 0 & n \neq -1 \end{cases}, \quad (1)$$

where the contour encircles the point $z = z_0$ in a positive (counterclockwise) sense. The exponent n is an integer.

2. Show that

$$\frac{1}{2\pi i} \oint z^{m-n-1} dz = \delta_{mn}, \quad m \text{ and } n \text{ integers} \quad (2)$$

3. Evaluate

$$\oint \frac{dz}{z^2 - 1} \quad (3)$$

where C is the circle $|z| = 2$.

4. Find the first three nonzero terms of the Laurent series of

$$f(z) = (e^z - 1)^{-1} \quad (4)$$

5. Determine the nature of the singularities of each of the following functions and evaluate the residues (take $a > 0$).

(a)

$$\frac{1}{z^2 + a^2} \quad (5)$$

(b)

$$\frac{z^2}{(z^2 + a^2)^2} \quad (6)$$

(c)

$$\frac{\sin 1/z}{z^2 + a^2} \quad (7)$$

(d)

$$\frac{ze^{iz}}{z^2 + a^2} \quad (8)$$