PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 17

Nov. 21, 2008

Due: Wednesday, Dec. 3, 2008
Reading: Boas Ch. 7

1. A sawtooth wave is given by the periodic extension of

$$f(x) = x, \quad -\pi < x < \pi. \tag{1}$$

Show that

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx. \tag{2}$$

2. Verify that

$$\delta(\phi_1 - \phi_2) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\phi_1 - \phi_2)}, \tag{3}$$

is the Dirac δ-function, by showing that it satisfies the fundamental definition

$$\int_{-\pi}^{\pi} f(\phi_1) \left(\frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\phi_1 - \phi_2)} \right) d\phi_1 = f(\phi_2). \tag{4}$$

3. (a) Find the Fourier series of

$$f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ x, & 0 \leq x < \pi \end{cases} \tag{5}$$

(b) From the result of a), show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \tag{6}$$

4. Find the Fourier transform $f(x)$ of the function

$$a(k) = \begin{cases} \frac{1}{\sqrt{\epsilon}} & -\epsilon/2 \leq k \leq \epsilon/2 \\ 0 & |k| > \epsilon/2 \end{cases} \tag{7}$$

5. Find the first 3 terms in the expansion of

$$f(x) = \begin{cases} 0, & -1 < x \leq 0 \\ 1, & 0 \leq x < 1 \end{cases} \tag{8}$$

in terms of Legendre polynomials.