PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 2

Wednesday, Sept. 3, 2008

Due: Wednesday, Sept. 10, 2008
Reading: Boas chapt. 4

1. Show using L'Hospital's rule that
   (a) $x^n e^{-x} \to 0$ as $x \to \infty$ for any n,
   (b) $\ln x/x^p \to 0$ as $x \to \infty$ for any $p > 0$,

   i.e. an exponential "wins" over any power, and any power "wins" over a log.

2. Given $\int_0^\infty e^{-ax} \sin kx\, dx = \frac{k}{a^2+k^2}$, evaluate (using differentiation with respect to a parameter)
   (a) $\int_0^\infty xe^{-ax} \sin kx dx$
   (b) $\int_0^\infty xe^{-ax} \cos kx dx$

3. Calculate the total derivative dr/ds if $r = e^{-p^2-q^2}$, $p = e^s$, and $q = e^{-s}$.

4. For $u = e^y \sin x$, check that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}, \quad (1)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (2)$$

5. Calculate the total derivative dy/dx for

$$(a)\ xy^2 - 3x^2 = xy + 5\ ; \quad (b)\ x = \frac{3y-4}{y+2} \quad (3)$$

   using both implicit differentiation (Boas sec. 4-6) and explicitly solving for $y = y(x)$.