PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 3

Sept. 11, 2008

Due: Friday. Sept. 12, 2008
Reading: Boas chapt. 4

1. If $u = f(x - ct) + g(x + ct)$, show that
$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}. \tag{1}$$
What can you say about the possible physical interpretation of u?

2. Given $s(v, T)$ and $v(p, T)$, define $c_p \equiv T(\partial S/\partial T)_p$, $c_v \equiv T(ds/dT)_v$. Show that
$$c_p - c_v = T\left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \tag{2}$$
[Hint: you need $s(p, T)$ to calculate c_p, i.e. find $dS = (\dots)dp + (\dots)dT$.]

3. Find the point on the curve $x^2 - 2\sqrt{3}xy - y^2 = 2$ which is closest to the origin $0, 0$.

4. The temperature of a rectangle bounded by the lines $x = \pm 1$, $y = \pm 2$ is given by $T = x^2 - 4y^2 + y - 5$. Find the hottest and coldest point.

5. Transform the differential equation
$$x^2\left(\frac{d^2y}{dx^2}\right) + 2x\left(\frac{dy}{dx}\right) - 5y = 0 \tag{3}$$
with the help of the substitution $x = e^z$ into another differential equation in d^2y/dz^2, dy/dz and y which has only *constant* coefficients of the derivative terms.