PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 5

September 17, 2008

Due: Friday, Sept. 19, 2008
Reading: Boas chapt. 4

1. Find
$$\frac{d}{dx}\int_1^{1/x}\frac{e^{xt}}{t}dt \tag{1}$$

2. The equation of state for a van der Waals gas is
$$\left(p+\frac{a}{v^2}\right)(v-b)=C. \tag{2}$$
Calculate dv/dp by the method of total differentials.

3. Expand
   (a) $f(x,y)=xy^2+2x^2+1$ around (1,2).
   (b) $g(x,y)=\frac{y^2}{x^3}$ around (1,0).

4. Minimize the function $f=x^2+y^2$ subject to the constraint $y=1-x^2$.

5. Problem 5 (Shankar) on next page

Problem 3.1.7. A Statistical Mechanics Interlude. *This example shows how Lagrange multipliers appear in Statistical Mechanics. Consider N particles in a box. According to quantum mechanics, the energies of the particles are quantized to some set of values or energy levels, $\varepsilon_1, \varepsilon_2, \ldots$. Let n_i be the number of particles in level i with energy ε_i. The multiplicity or number of distinct rearrangements of the particles consistent with any given distribution n_i, is given by*

$$W(n_1, n_2, \ldots) = \frac{N!}{n_1!n_2!\ldots} \tag{3.1.61}$$

For example if all the particles are in the lowest energy level, we have $n_1 = N$, rest of the $n_i = 0$ and $W = 1$. (Recall that $0! = 1$.) If one particle is in the second level and the rest are still in the lowest, $W = N!/(N-1)! = N$, the multiplicity reflecting the N ways to choose the one who goes up. The question is this: which distribution of particles, subject to the constraint that the total number equal N and the total energy equal E, *gives the biggest W? Proceed to find this as follows:*

a) • *Work with $S = \ln W$. Argue that*

$$\ln n! \simeq n \ln n - n$$

for large n by approximating the sum involved in $\ln n!$ by an integral.

b) • *Write the constraints on the n_i's due to total number N and energy E.*

c) • *Treat all n_i as continuous variables, introduce Lagrange multipliers α and β for N and E and maximize S.*

d) • *Derive the Boltzmann distribution $n_i = e^{-\alpha - \beta\varepsilon_i}$.*

The multipliers may then be found by setting the total number and energy coming from this distribution to N and E, respectively. But this is not our problem here.