PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 6

September 19, 2008

Due: Wednesday, Sept. 24, 2008
Reading: Boas chapt. 5, secs. 2-4; chapt. 3 sec. 4.

1. Evaluate

$$\int\int_A (2x - 3y)\, dx\, dy, \tag{1}$$

where A is the triangle with vertices (0,0), (2,1) and (2,0). Do the integral in both orders!

2. Find the moment of inertia

$$I_x = \int\int\int (y^2 + z^2)\rho(x,y) dV, \tag{2}$$

of the solid cone with surface $x^2 + y^2 = z^2$, with variable density $\rho(x,y) = (x^2 + y^2)b$, with b a constant.

3. Consider a thin plate whose form is given by the boundaries $x = 0$, $x = 1$, $y = 0$ and $y = x^3$. Calculate the coordinates of the center of mass of the plate, if the density is given by $\rho(x,y) = cxy^2$, with c a constant.

4. Calculate $\nabla\phi$ and $(\nabla \cdot \nabla)\phi$, for $\phi(x_1, x_2, x_3) = \sin x_1 + x_1^2 x_2 x_3$.

5. Show that (use ϵ_{ijk})

   (a) $\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \nabla^2 \vec{v}$

   (b) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0$

   for any smooth vector field $\vec{v}$.