PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 7

September 24, 2008

Due: Friday, Sept. 26, 2008
Reading: Boas chapt. 6

1. Given a vector field $\vec{F} = (2x - 3y)\hat{\imath} - (3x - 2y)\hat{\jmath}$, find $\int \vec{F} \cdot d\vec{r}$ along the circle $x^2 + y^2 = 2$ from (1,1) to (1,-1) in 2 different ways:
   (a) by changing to polar coordinates $x = \sqrt{2}\cos\theta$ and $y = \sqrt{2}\sin\theta$
   (b) by showing that $\vec{\nabla} \times \vec{F} = 0$, arguing that this means the integral is independent of the path, and choosing a simpler path.
2. The gravitational force on a mass m a displacement $\vec{r}$ from another mass M is $\vec{F}(\vec{r}) = -GMm\vec{r}/r^3$. Show that it's a conservative force.
3. A vector field $\vec{a}$ has the form $\vec{a}(x_1, x_2, x_3) = (x_1 + \alpha x_2, x_2 + \beta x_1, x_3)$. Determine α and β such that $\vec{\nabla} \times \vec{a} = 0$, and find the corresponding scalar potential.
4. If $f(r)$ is a differentiable function of $r = |\vec{r}|$, calculate $\vec{\nabla} \times (\vec{r} f(r))$.
5. If $\vec{V}$ is any smooth vector field, S a closed surface with measure $d\vec{\sigma} = \hat{n} d\sigma$, where $\hat{n}$ is the unit normal to the surface, prove that

$$\int_S \vec{\nabla} \times \vec{V} \cdot d\vec{\sigma} = 0 \tag{1}$$

   Explain in a sentence or two why this is true.