**PHZ3113–Introduction to Theoretical Physics**

**Fall 2008**

**Problem Set 8**

**September 26, 2008**

Due: Friday, Oct. 3, 2008
Reading: Boas chapt. 6

1. Verify the divergence theorem for the vector field $\vec{v} = (x_2, -2, x_1)$ and the volume bounded by $0 \le x_1 \le 1,\ 0 \le x_2 \le 3,\ 0 \le x_3 \le 2$. In other words, calculate the volume integral of the divergence, and the surface integral over its boundary, and show they are equal.

2. The magnetic field is related to the vector potential by $\vec{B} = \vec{\nabla} \times \vec{A}$.

   (a) Show that the vector field $\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$ corresponds to a constant $\vec{B}$.

   (b) Vector $\vec{B}$ is formed by the product of two gradients, $\vec{B} = (\vec{\nabla} u) \times (\vec{\nabla} v)$, where $u$ and $v$ are scalar functions. Show that $\vec{B}$ is solenoidal, i.e. divergenceless.

   (c) Show that $\vec{A} = \frac{1}{2}(u\vec{\nabla} v - v\vec{\nabla} u)$ is a vector potential for $\vec{B}$ of the previous part.

   (d) Since $\vec{B} = \vec{\nabla} \times \vec{A}$, one may write Stokes' theorem as

   $$\int \vec{B} \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{r}. \tag{1}$$

   Show that this equation is invariant under the transformation $\vec{A} \to \vec{A} + \vec{\nabla}\psi$, where $\psi$ is any scalar.

3. Show

   $$\delta(g(x)) = \sum_{a_n} \frac{\delta(x - a_n)}{|g'(a_n)|}, \tag{2}$$

   where $a_n$ are the zeros of $g(x) = 0$.

4. Evaluate $\int_0^\infty (3x^2 + 5)\delta(2x^2 + 3x - 2)$

5. Write expressions for the charge density (charge/unit volume) in terms of the appropriate $\delta$-functions for:

   (a) a point charge located at the point $\vec{R}$.

   (b) an infinitesimal shell of charge of radius $a$.