PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 9

Oct. 4, 2008

Due: Wednesday, Oct. 8, 2008
Reading: Boas sec. 5-4,10.8,10.9

1. Calculate $\vec{\nabla}\Phi$ and $\nabla^2\Phi$ if $\Phi =$

   (a) $\sin\phi$

   (b) $r^2\sin\theta$

   in spherical coordinates.

2. Consider the vector field in Cartesian coordinates

$$\vec{F} = (x^2y, 2yz, x+z) \tag{1}$$

   and calculate $\vec{\nabla}\cdot\vec{F}$ and $\vec{\nabla}\times\vec{F}$ in cylindrical coordinates, expressed in terms of cylindrical basis vectors $\hat{r}$, $\hat{\theta}$, and $\hat{z}$.

3. Show that the Laplace equation of electrostatics

$$\nabla^2\Phi = 0 \tag{2}$$

   is satisfied in cylindrical coordinates if $\Phi = r^n\sin n\theta$ for $r > 0$ and n an integer.

4. Calculate the volume integral

$$\int d\tau\sqrt{x_1^2 + x_2^2} \tag{3}$$

   over the region enclosed between the two surfaces

$$x_3 = x_1^2 + x_2^2 \quad \text{and} \quad x_3 = 9 - (x_1^2 + x_2^2). \tag{4}$$

   Hint: use cylindrical coordinates.

5. Calculate the arc length ds^2 for the elliptical cylindrical coordinate system, u, v, w such that

$$x_1 = a\cosh u\cos v \;\; ; \quad x_2 = a\sinh u\sin v \;\; ; \quad x_3 = w. \tag{5}$$