$PHY3113-Introduction\ to\ Theoretical\ Physics$

Fall 2007

Test 3 - 55 minutes

Oct. 26, 2007

No other materials allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer-define clearly. Do 4 of 6 problems, clearly indicating which you want graded! You may attempt the remaining two as extra credit as well. All regular parts are worth 10 pts., each extra credit 5 each, for maximum of 50 points.

Useful formulae:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$
 (1)

$$\oint f(z)dz = 0$$
(2)

$$\frac{1}{2\pi i} \oint_C \frac{f(z)}{z - a} dz = f(a) \tag{3}$$

$$f^{n}(a) = \frac{n!}{2\pi i} \oint \frac{f(z)dz}{(z-a)^{n+1}} \tag{4}$$

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}$$
 (5)

$$a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{(z - z_0)^{n+1}} \quad ; \quad b_n = \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{(z - z_0)^{-n+1}}$$
 (6)

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1.	Two	blocks of	$\max n$	n are	connected	to	springs.	The	equations	ot	motion	can	be	written	as

$$m\ddot{x}_1 = -kx_1 + 2k(x_2 - x_1) \tag{7}$$

$$m\ddot{x}_2 = -kx_2 - 2k(x_2 - x_1) \tag{8}$$

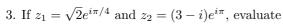
(a) Write the two coupled equations as a matrix equation of the form $|\ddot{X}\rangle = A|X\rangle$, where $|X\rangle$ is a two-component column vector and A is a 2×2 matrix.

(b) Find the eigenvalues and normalized eigenvectors of A.

(c) Draw a picture indicating the motion of the blocks represented by each of the eigenvectors.

- 2. Consider the 2×2 matrix B with elements $B_{11} = B_{22} = 0$, $B_{12} = 1$ and $B_{21} = -1$.
 - (a) Show that B rotates any two dimensional vector by 90° . (Hint: two vectors are at 90° with respect to each other if the dot product vanishes.)

(b) Show that $e^{\theta B} = R$, where R is the two dimensional rotation matrix $R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. (Hint: note that $B^2 = -I$, and that $R = \cos \theta I + \sin \theta B$).



(a) Re
$$z_1z_2$$

(b)
$$\sqrt[3]{z_1 + z_2}$$
 (give all roots!)

(c)
$$(z_1 + z_2)^{1+i}$$
 (hint: use properties of ln)

4. (a) Find the analytic function
$$w(z) = u(x,y) + iv(x,y)$$
 if $v(x,y) = 2xy$.

(b) Show explicitly that the function f(z) = |z| is not analytic at z = 0 by calculating the derivative along 2 different paths.

5.	Use	contour	integration	to	evaluate	the	integral	S

(a)

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{i+x} dx \tag{9}$$

$$\int_{-\infty}^{\infty} \frac{x \sin \pi x}{1 + x^2} dx \tag{10}$$

6. (a) Find the first two a_n terms and first two b_n terms of the Laurent series for $f(z) = (z + 2)^{-2}z^{-1}$ around z = -2.

(b) Evaluate the residues of $f(z) = (z+2)^{-2}z^{-1}$ at z = -2 and z = 0.