PHY3113–Introduction to Theoretical Physics

Fall 2007

Test 1 – 55 minutes

Sept. 28, 2007

No other materials allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer–define clearly. **Do 5 of first 6 problems, clearly indicating which you want graded!** *You may attempt extra credit problems as well. All regular parts are worth 10 pts., extra credit 5 each, for maximum of 60 points. Good luck!*

Phz3113 Fall '07
Test 1

1. Expand $x/(e^x - 1)$ to order x^2 for $x \ll 1$.

2. The equation of state for a van der Waals gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT, \tag{1}$$

where a, b and R are constants. Consider two experiments on such a gas confined to a cylinder where you may control p, V and/or T.

(a) Hold T constant and find dV/dp.

(b) Hold p constant and find dV/dT.

3. Change variables $x = u + v$, $y = u - v$, to rewrite the differential equation

$$\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} = 1 \tag{2}$$

in terms of u and v (no need to solve the equation).

4. Evaluate the integral

$$\int_{y=0}^{\pi} dy \int_{x=y}^{\pi} dx \frac{\sin x}{x}. \tag{3}$$

5. If $\vec{\nabla} \cdot \vec{A} = 0$ and $\vec{\nabla} \cdot \vec{B} = 0$, show that

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B}. \tag{4}$$

[Hint: $\epsilon_{ijk}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}$]

6. Look for a minimum of the function $1/x+4/y+9/z$ for $x, y, z > 0$ and $x+y+z = 12$ by the method of Lagrange multipliers.

7. (Extra credit.) Consider the vector $\vec{V} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ and the scalar field $\psi(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2}$.
   (a) show $\vec{\nabla} \times \vec{V} = -3\hat{k}$

   (b) evaluate $\int \vec{V} \cdot d\vec{r}$ from the origin (0,0,0) to (1,1,1) along the line $x = t$, $y = t^2$, $z = t^3$.

   (c) evaluate $\vec{\nabla}\psi$ and $\vec{\nabla} \times \vec{\nabla}\psi$).

8. (Extra credit.) Calculate the radii of convergence of the following series:
   (a)

$$\sum_{n=1}^{\infty} \frac{(nx)^n}{n!} \tag{5}$$

   (b)

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2 + 1} \tag{6}$$