PHZ7427–Solid State II

Spring 2001

Final

April 25, 2001

Due: May 2, 2000. Do **3 out of 4** problems below, whichever you think are most interesting.

1. **Zero sound in Fermi liquid.** The transport equation for the distribution function $\delta f_{\mathbf{k}}(\mathbf{r}, t)$ in a Fermi liquid like ^3He looks quite similar to the Boltzmann equation:

$$\frac{\partial \delta f_{\mathbf{k}}}{\partial t} + \mathbf{v_k} \cdot \nabla_{\mathbf{r}} \left(\delta f_{\mathbf{k}} - \frac{\partial f^{(0)}_{\mathbf{k}}}{\partial \epsilon_{\mathbf{k}}} \delta \epsilon_{\mathbf{k}} \right) = \left(\frac{df}{dt} \right)_{coll},$$

where $\delta\epsilon_{\mathbf{k}}$ is the shift of the quasiparticle energy due to both external fields and the Landau interaction, and $f^{(0)}$ is the Fermi function.

   (a) Assume that the system is in the collisionless regime for frequencies of interest, i.e. neglect the right hand side of the equation. Fourier transform this equation with respect to time and space to obtain

$$(\omega - \mathbf{q} \cdot \mathbf{v_k})\delta f_{\mathbf{k}} - \delta(\epsilon_{\mathbf{k}} - \epsilon_F)\mathbf{q} \cdot \mathbf{v_k} \left(\sum_{\mathbf{k}'} f_{\mathbf{kk}'} \delta f'_{\mathbf{k}} \right) = 0$$

   where $f_{\mathbf{k}} = f_{\mathbf{k}}(\mathbf{q}, \omega)$ is now the F.T. distribution function, and $f_{\mathbf{kk}'}$ is the Landau interaction.

   (b) Assume that $f_{\mathbf{kk}'} = F^s_0/(N_0)$, i.e. the Landau interaction is isotropic and nonmagnetic. Show that the restriction of $\delta f_{\mathbf{k}}$ to the Fermi sphere $\delta f_{\hat{k}}$ obeys

$$\delta f_{\hat{k}} = \frac{\mathbf{q} \cdot \mathbf{v_k}}{\omega - \mathbf{q} \cdot \mathbf{v_k}} F^s_0 \int \frac{d\Omega'}{4\pi} \delta f_{\hat{k}'}.$$

   (c) Expand $\delta f_{\hat{k}}$ in Legendre polynomials on the Fermi sphere, and show there is a solution $\delta f_{\hat{k}}$ with isotropic symmetry only if

$$1 + F^s_0 \left(1 + \frac{1}{2} \frac{\omega}{q v_F} \log \left(\frac{\omega - q v_F}{\omega + q v_F} \right) \right) = 0$$

   (d) Plot the solution ω as a function of qv_F numerically, or sketch it at low qv_F and determine the behavior analytically. This is the dispersion for collisionless *zero sound*, a sound mode in a degenerate Fermi liquid which has no analog in the noninteracting system.

2. **Real materials.** Choose any real material currently being studied (or which has been studied recently) somewhere in the physics department of the University of Florida, and state its chemical formula.

(a) Why is it interesting?

(b) What property is being measured at U. Florida?

(c) Describe this material in terms of its basic electrical, thermodynamical, magnetic, and other properties.

(d) Find data on two experiments measuring properties of this material, at least one of which comes from a research group here, and at least one of which comes from another group elsewhere, and present it. (That is, turn in plots of data with your answer to this question, and discuss it as if you were explaining to a colleague what has been measured.) Explain how this data can be understood based on concepts you learned in Solid State II.

3. **Superconducting Sphere.** Consider a type-I superconductor with critical field H_c, placed in a uniform external field $\mathbf{H}$.

   (a) Show with a simple argument that the field at the surface of the sphere is parallel to the surface.

   (b) Use the previous fact to show that when the external field reaches a value of $2H_c/3$, some part of the sample must become normal.

   (c) At what external field does the entire sphere become normal?

4. **Localization in 1D.** Consider a tight-binding model in 1D with random chemical potentials:

$$\mathcal{H} = -t \sum_i (|i\rangle\langle i+1| + |i+1\rangle\langle i|) + \sum_i U_i |i\rangle\langle i|,$$

   where i runs over the 1D chain, and U_i is a random variable uniformly distributed between $-W/2$ and $W/2$.[1]

   (a) Show that a solution ψ_i to Schrödinger's equation with energy E satisfies

$$\begin{pmatrix} \psi_{i+1} \\ \psi_i \end{pmatrix} = \mathbf{T} \begin{pmatrix} \psi_i \\ \psi_{i-1} \end{pmatrix},$$

   where $\mathbf{T}$ is the 2×2 *transfer matrix*. Find $\mathbf{T}$.

   (b) Take boundary conditions $\psi_0 = 0$ and $\psi_1 = 1$. Find how $|\psi_i|^2$ behaves numerically for large i. (Hint: you will need to play around to figure out what "large" is for given W/t.) Take $E = 0$, and compare $W/t = 3$ and $W/t = 1$. Plot the your results for $|\psi_i|^2$ vs. i on a linear-log plot. (Further hint: do not forget to normalize the total wave function ψ over the entire finite chain!)

   (c) Can you say how the localization length ξ depends on W/t?

[1] equivalent to $\mathcal{H} = -t \sum_{\langle ij \rangle} c_i^\dagger c_j + \sum_i U_i c_i^\dagger c_i,$