# PHZ7427–Solid State II

**Spring 2023**

**Midterm (take-home)**

**Mar. 13, 2023**

Due Monday, Mar. 27. Attempt all problems below, by yourself. Use of books and other course materials is allowed.

1. **"Short" answer (please write enough to convince me you understand what is going on). Note in some cases I did not explicitly cover the material in class, but information is contained in the class notes, and elsewhere.**

   (a) Explain how neutron scattering can be used to *detect* and *characterize* a magnetic state in a solid. Discuss both diffraction and inelastic scattering.

   (b) What is a spontaneously broken symmetry of a statistical system? *Explain* two different examples of systems which exhibit such symmetry breaking.

   (c) Give a definition of the plasma frequency of a simple metal, and describe how you would go about determining it with an optics experiment.

   (d) In the elementary theory of solids, systems with one electron per atomic site are metals. Under what circumstances can such a system be an insulator?

   (e) A charged impurity placed in a semiconductor will influence electrons moving 10 nm away, whereas the same impurity in a metal (assume same charge on the impurity) will only influence electrons over a distance of 0.1 nm or less. Explain.

   (f) Write down a valid single quantized wave function for 3 electrons at positions $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ in the three distinct single-particle states $\phi_a(\mathbf{r})$, $\phi_b(\mathbf{r})$, $\phi_c(\mathbf{r})$. Note the $a, b, c$ label *includes* spin. Now express the same wave function in 2nd quantized notation.

   (g) Name one property of a typical solid which cannot be explained using the quantum theory of a harmonic lattice. Explain briefly.

2. **Friedel oscillations.** Note the *real* part of the Lindhard function $\mathrm{Re}L(q, 0)$ is given in A& M 17.58. Make a plot of this function vs. $qv_F/\varepsilon_F$ using Mathematica, etc. for $T = 0$. Now make a simple model; your sketch should look a bit like

$$L(q,0) \simeq \begin{cases} 1 & q < 2k_F \\ 0 & q > 2k_F \end{cases} \tag{1}$$

   when suitably normalized. Find the asymptotic ($r \to \infty$) behavior of the electrostatic potential $\phi(\mathbf{r})$ around the impurity using this model form. Compare to the result 17.59 in Ashcroft & Mermin. What is the difference?

3. **Spin-boson transformations.**

   (a) Show that the Holstein-Primakoff representation of spin operators in terms of bosons $a^\dagger$, $a$, satisfies the usual spin algebra (commutation relations):

$$\begin{aligned} S^- &= (2S)^{1/2} a^\dagger \sqrt{1 - \frac{a^\dagger a}{2S}} \\ S^+ &= (2S)^{1/2} \sqrt{1 - \frac{a^\dagger a}{2S}}\; a \\ s^z &= (S - a^\dagger a) \end{aligned} \tag{2}$$

   (b) Show that the Dyson-Maleev representation of spin operators in terms of bosons $a^\dagger$, $a$, satisfies the usual spin algebra (commutation relations):

$$\begin{aligned} S^- &= (2S)^{1/2} a^\dagger \left(1 - \frac{a^\dagger a}{2S}\right) \\ S^+ &= (2S)^{1/2} a \\ s^z &= (S - a^\dagger a) \end{aligned} \tag{3}$$

4. **Specific heat due to spin waves.** Calculate the low temperature specific heat of a ferromagnet described by the spin-1/2 Heisenberg model within linear spin wave theory. Show that the leading asymptotic temperature dependence is $C \sim AT^\alpha$, find $A$ and $\alpha$ in terms of J and the lattice constant $a$.

5. **1D Anisotropic Exchange.** Take a Heisenberg-type model in one dimension with nearest-neighbor only interactions:

$$H = -\sum_i \left[ J_z S_i^z S_{i+1}^z + \frac{1}{2} J_\perp \left( S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ \right) \right] \tag{4}$$

   (a) Show that for $J_z > J_\perp$ the magnon dispersion $E(\mathbf{q})$ is similar to the isotropic case, but with nonzero energy as $q \to 0$.

   (b) Show for the opposite case $J_z < J_\perp$ the ferromagnetically ordered state with magnetization along $z$ is not the ground state!