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STATISTICAL MECHANICS FOR NATURAL FLOCKS OF BIRDS

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- **Flocking: Emergent collective behavior at macroscopic length scale.**
- **Microscopic theory using tools of statistical mechanics and physics, to understand directional ordering in a flock?**
- **Maximum entropy model for local, pairwise interactions among birds (motivated by Heisenberg's model for magnetism).**
- **Observed scale-invariance of long-range correlation length among the fluctuations in flight directions: a model where local correlations is able to describe correlations at all length scales (?)**

Model:

- Field data for flocks of European starlings, *Sturnus vulgaris*, across multiple flocking events for differently-sized flocks. Input data: position $\boldsymbol{r}_i$ and velocity $\boldsymbol{v}_i$ of individual birds in every flock set.
- Define normalized velocities $\boldsymbol{s}_i = \boldsymbol{v}_i / |\boldsymbol{v}_i|$. Hamiltonian and classical equation of motion (Langevin dynamics):

$$H(\{\vec{s}_i\}) = -(1/2)\sum_{i,j} J_{ij}\vec{s}_i \cdot \vec{s}_j$$

$$\frac{d\vec{s}_i}{dt} = -\frac{\partial H}{\partial \vec{s}_i} + \vec{\eta}_i(t) = \sum_{j=1}^{N} J_{ij}\vec{s}_j + \vec{\eta}_i(t)$$

- Trajectories controlled by "social forces".
- Define correlation element $\boldsymbol{C}_{ij} = <\boldsymbol{s}_i . \boldsymbol{s}_j>$, infinitely many possible probability distribution $P(\{\boldsymbol{s}_i\})$ consistent with measured correlations, but only one corresponds to maximum entropy distribution.

$$S[P] = -\sum_{\mathbf{x}} P(\mathbf{x}) \ln P(\mathbf{x})$$

$$P(\{\vec{s}_i\}) = \frac{1}{Z(\{J_{ij}\})} \exp\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\vec{s}_i \cdot \vec{s}_j\right]$$

Model:

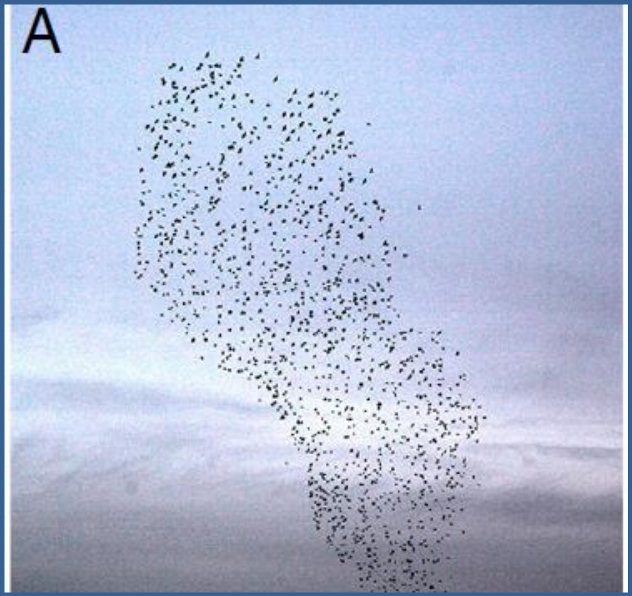
- Birds move and change neighbors. How to account for this intrinsic dynamics in the flock?
- Interaction J_{ij} cannot depend directly on individual identities labeled by i and j, but only on some function of their relative positions.

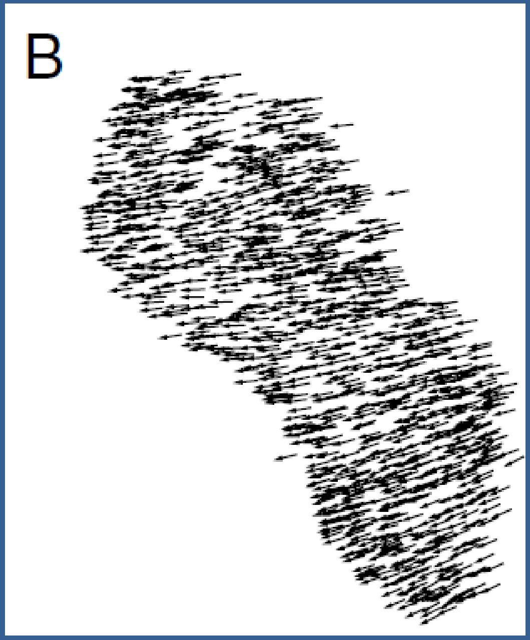
$$P(\{\vec{s}_i\}) = \frac{1}{Z(J, n_c)} \exp\left[\frac{J}{2}\sum_{i=1}^{N}\sum_{j\in n_c^i} \vec{s}_i \cdot \vec{s}_j\right]$$

$$C_{\text{int}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{1}{n_c}\sum_{j\in n_c^i} \vec{s}_i \cdot \vec{s}_j.$$

- Problem reduces to finding the values of J and n_c that reproduce the observed correlation C_{int}. Fundamental correlations are between birds and their directly interacting neighbors; all more distant correlations should be derivable from these local correlations.
- If that is correct, a model that appropriately reproduces the fundamental correlations up to the scale n_c must be able to describe correlations on all length scales (?)

Input data:





Individual 3D coordinates and velocities were measured using stereometric photography and innovative computer vision techniques.

- 21 distinct flocking events
- Sizes ranging from 122 to 4,268 individuals
- Linear extensions from 9.1 m to 85.7 m
- Each event consists of up to 40 consecutive 3D configurations at time intervals of 1⁄10 s.
- Boundary birds, belonging to the exterior border, are treated differently due to asymmetric neighborhood.

Evaluations:

- Global order parameter, or polarization: $\vec{S} = (1/N)\sum_i \vec{s}_i = S\vec{n}$
- Individual orientations:

  $\vec{s}_i = s_i^L \vec{n} + \vec{\pi}_i$ $\quad S \sim 1,\ |\vec{\pi}_i| \ll 1,$ $\quad s_i^L \sim 1 - |\vec{\pi}_i|^2/2$

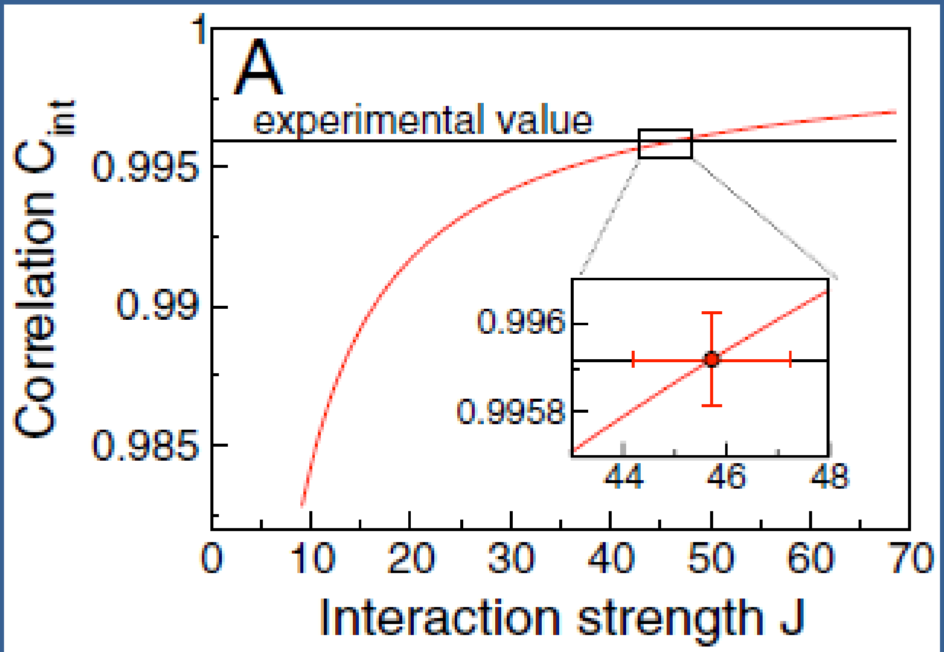
- For highly polarized flocks $S \sim 1$:

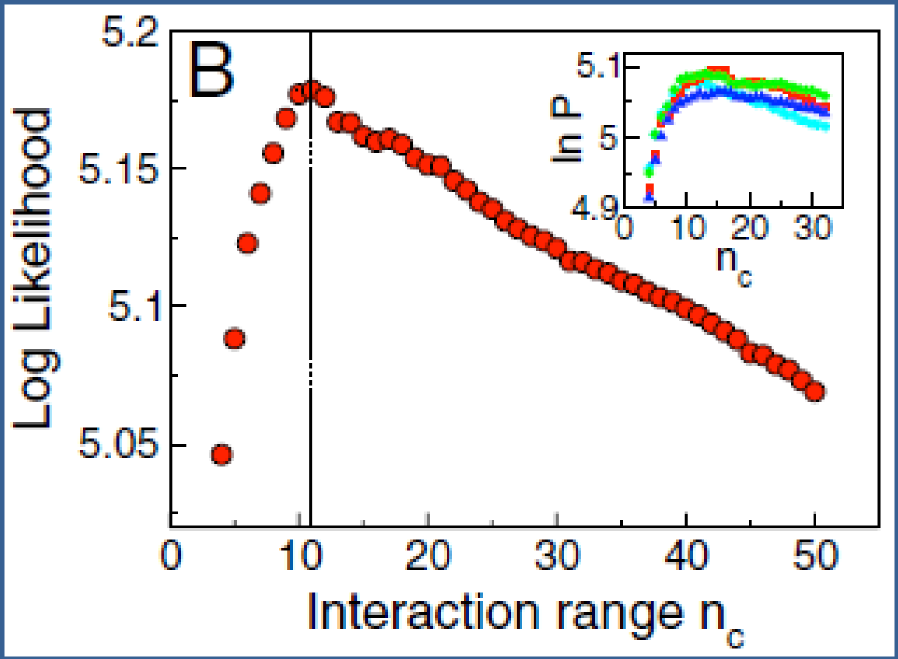
$$Z(\{J_{ij}\};\mathscr{B}) = \int d^{\mathscr{I}}\vec{\pi} \exp\left[-\frac{1}{2}\sum_{i,j\in\mathscr{I}} A_{ij}\vec{\pi}_i\cdot\vec{\pi}_j + \sum_{i\in\mathscr{I}}\vec{\pi}_i\cdot\vec{h}_i + \frac{1}{2}\sum_{i,j\in\mathscr{I}} J_{ij} + \frac{1}{2}\sum_{i\in\mathscr{I}} h_i^L + \frac{1}{2}\sum_{i,j\in\mathscr{B}} J_{i,j}\vec{s}_i\cdot\vec{s}_j\right].$$

$$A_{ij} = \delta_{ij}\left(\sum_{k\in\mathscr{I}} J_{ik} + \sum_{l\in\mathscr{B}} J_{il}s_l^L\right) \qquad \vec{h}_i = \sum_{l\in\mathscr{B}} J_{il}\vec{s}_l$$

- Whether both individuals, just one, or none, belong to the local n_c-neighborhood of the other: $J_{ij} = Jn_{ij}$, with $n_{ij} = 1,\ 1/2$, or 0
- Maximum log-likelihood of data: $\langle \log P(\{\vec{s}_i\})\rangle_{\exp} = -\log Z(J, n_c;\mathscr{B}) + \frac{1}{2}Jn_cNC_{\text{int}}^{\exp}$
- Maximizing w.r.t. J corresponds to equating expected and experimental correlations (leads to analytic solution relating optimal J and n_c).
- Maximizing w.r.t. n_c is performed numerically.

Results:





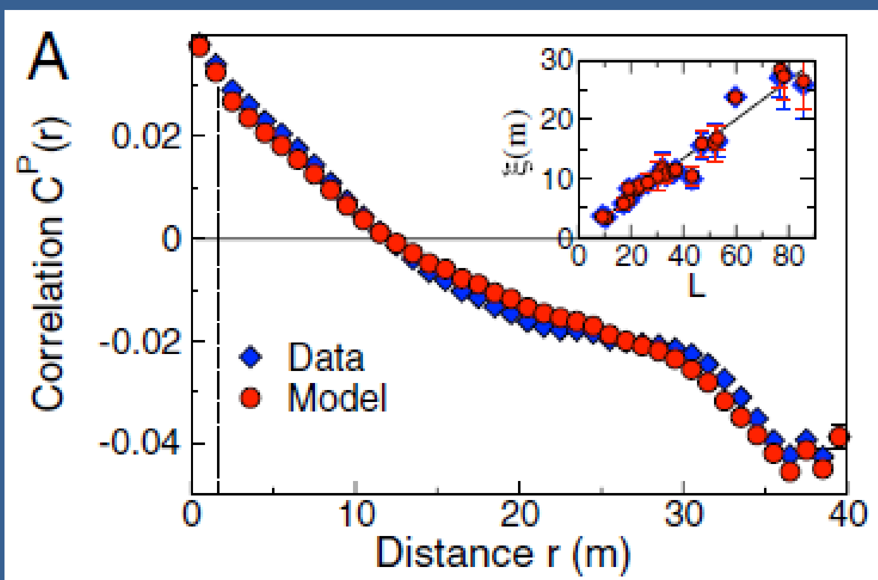
For $J = 45.73$ matches with experimental $C_{int} = 0.99592$

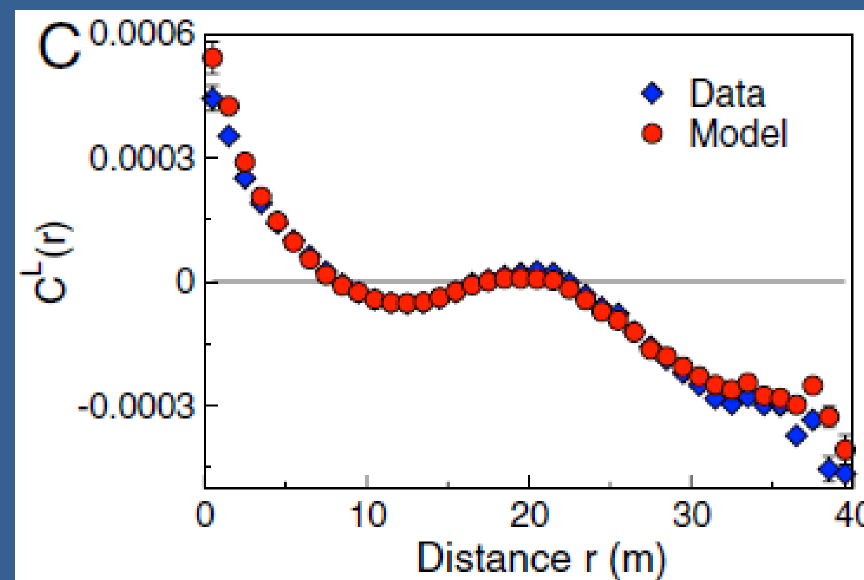
$(< lnP(\{s_i\}) >_{exp}/N)$ vs n_C for optimized J values

Results:

Scale-invariance of correlation length at all length-scales:

Maximum entropy model finds local interactions ($n_C \sim 20$ for flocks of up to 1000 birds). Scalar correlation obtained from integral of $C(r) \sim \sum_{ij} \boldsymbol{s}_i . \boldsymbol{s}_j \delta(r - r_{ij})$ over small interval close to $r = 0$ gives C_{int}: long range contributions does not count in. Local correlations are able to describe correlations on all length scales.





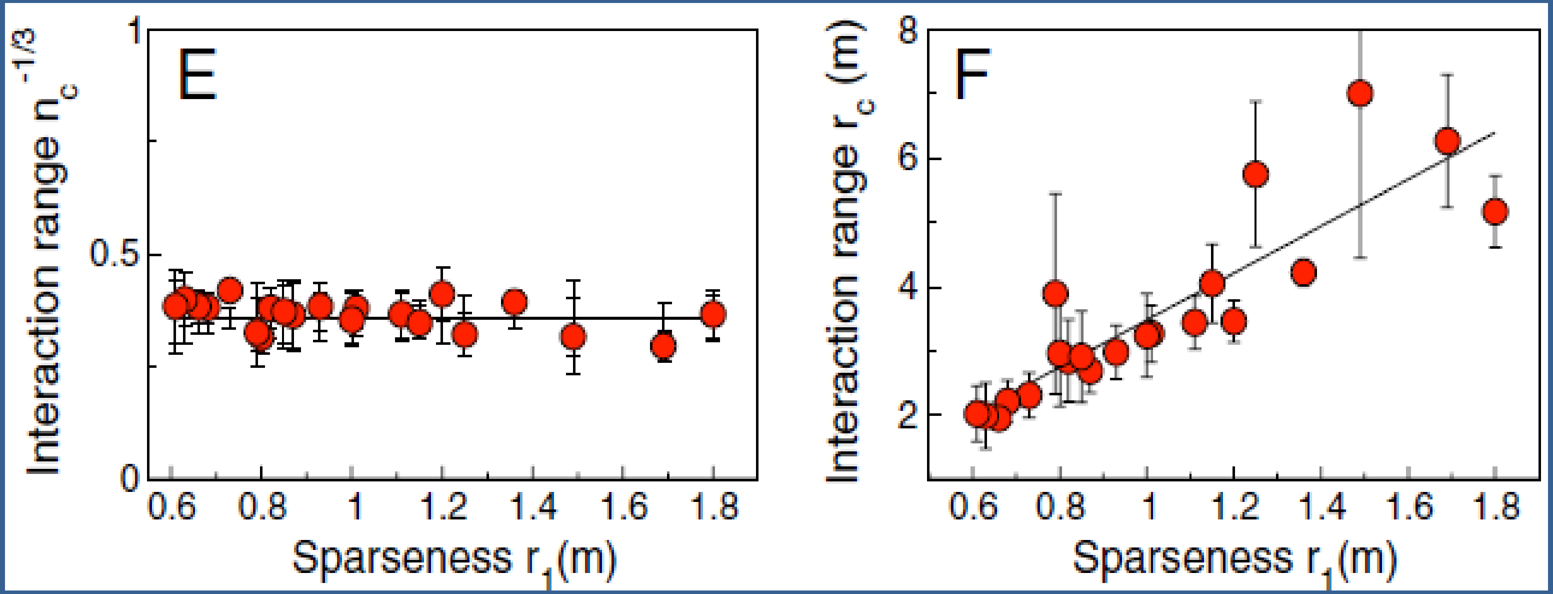
(A) $C^P(r) = \langle \vec{\pi}_i \cdot \vec{\pi}_j \rangle$

(C) $C^L(r) = \langle s_i^L s_j^L \rangle - S^2$

Results:

Signature of interactions with a fixed number of topological neighbors:

If interactions extend over some fixed metric distance r_0, then we expect $n_C^{-\frac{1}{3}} \propto r_1/r_0$ and $r_C =$ constant. The plots display opposite patterns.

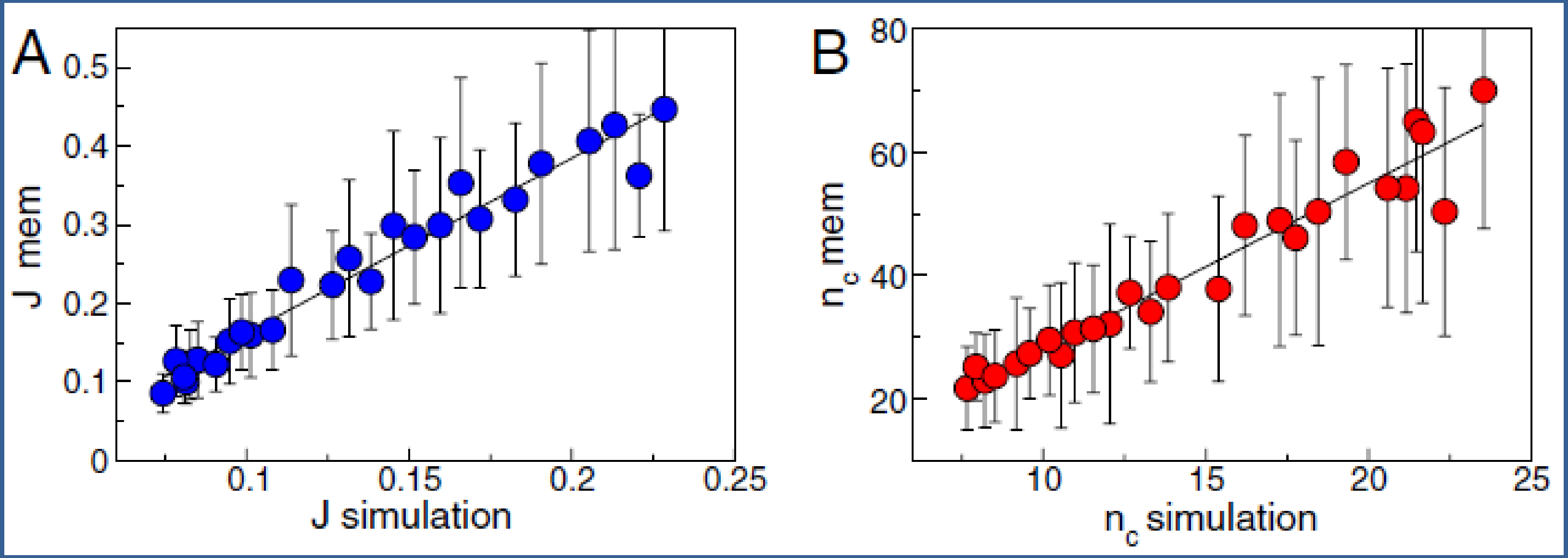


(E) Inferred value of the topological range $n_C^{-\frac{1}{3}}$ as function of mean interbird distance in the flock, for all flocks.

(F) Inferred value for the metric range r_C as function of mean interbird distance in the flock, for all flocks.

Testing the Mechanistic Interpretation:

Simulated Population of self-propelled particles in 3D moving according to social forces that tend to align each particle with the average direction of its neighbors. Plots show comparison of simulation parameters (J^{sim}, n_C^{sim}) to the values (J^{mem}, n_C^{mem}) obtained by applying the maximum entropy method to snapshots drawn from the simulation.



Conclusions:

- Construction of a minimal model that is consistent with a single number C_{int} characterizing the interactions among birds in a flock, the average correlation between the flight directions of immediate neighbors.
- Qualitative effects such as the presence of long range, scale-free correlations among pairs of birds can be successfully accounted for by the discussed model.
- The structure of the model corresponds to pairwise interactions with a fixed number of (topological) neighbors n_C, rather than with all neighbors that fall within a certain (metric) distance r_C.
- A full theory must connect the velocities of the birds to their evolving positions, which requires more accurate measurements of trajectories over time, and we must consider the fluctuations in the speed as well as direction of flight.

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