Pomeranchuk instabilities: Topic for Journal club discussion.

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Outline

- Introduction:- Fermi liquid theory and Pomeranchuk instabilities.
- Motivation.
- Ward Identities/conservation principles and continuity equations to determine the response function for generic order parameter.
- Implication on various conserved and non conserved order
- Conclusions.

Fermi liquid theory(brief idea)

- In fermi liquid theory, the energy change due to quasiparticle excitations is given as,

$$\delta E_{\text{qp}} = \sum_{\boldsymbol{k}\sigma} \varepsilon^*_{\boldsymbol{k}\sigma} \delta n_{\boldsymbol{k}\sigma}$$

with the single particle energy is,

$$\varepsilon^*_{\boldsymbol{k}\sigma} = v_F(|\boldsymbol{k}| - k_F) + \mu + \frac{1}{\rho_F} \sum_{\boldsymbol{k}',\sigma'} F^{\sigma,\sigma'}_{\boldsymbol{k},\boldsymbol{k}'} \delta n_{\boldsymbol{k}'\sigma'},$$

where the interaction function for the spin-isotropic fermi liquid is

$$F^{\sigma,\sigma'}_{\boldsymbol{k},\boldsymbol{k}'} = F^s_{\boldsymbol{k},\boldsymbol{k}'} + \sigma\sigma' F^a_{\boldsymbol{k},\boldsymbol{k}'}$$

Pomeranchuk instability.

- Pomeranchuk defined a condition of stability in FL as $1 + (2l+1)^{-1} F_l^{s,a} > 0.$
- Instabilities comes due to the deformation of the fermi surface as

$$k_F \to k_F + \delta k^s_{F,l}(\varphi_{\boldsymbol{k}_F}) + \sigma \delta k^a_{F,l}(\varphi_{\boldsymbol{k}_F})$$

when we approach this threshold value of

$$F_l^{s,a} \to -(2l+1)$$

- The energy change is given as,

$$\delta E_{\mathrm{qp}} = \frac{1}{\rho_F^0} \frac{m}{m^*} \sum_{l, r=\{s,a\}} |\delta n_l^r|^2 \left(1 + \frac{F_l^r}{2l+1}\right) + \mathcal{O}\left((\delta n_l^r)^4\right).$$

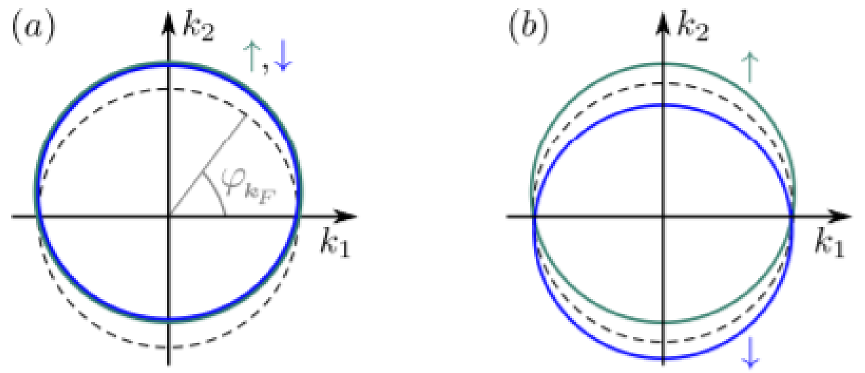


Figure 1: Distortion of the spin-up (green) and spin-down (blue) Fermi surface for the $l = 1$ Pomeranchuk instability in the (a) charge and (b) spin channel.

So therefore if $1 + (2l+1)^{-1} F_l^{s,a}$ becomes negative one can find a energetically favored state with finite order parameter, by breaking some symmetry in that particular l channel.

- Also the instabilities can be shown with quasiparticle susceptibility being divergent at the threshold value

$$\chi^r_{\mathrm{qp},l} = \frac{\rho_F}{1 + \frac{F_l^r}{2l+1}}.$$

- For l=0 channel we get the ferromagnetic instability.

Motivation.

The pomeranchuk criteria only incorporated the contributions to susceptibility from quasiparticles, but this is not the full susceptibility.

Main idea of my talk is to address the instabilities in l=1 channel for spin symmetric and spin antisymmetric sectors based on the conservation laws and continuity equation.

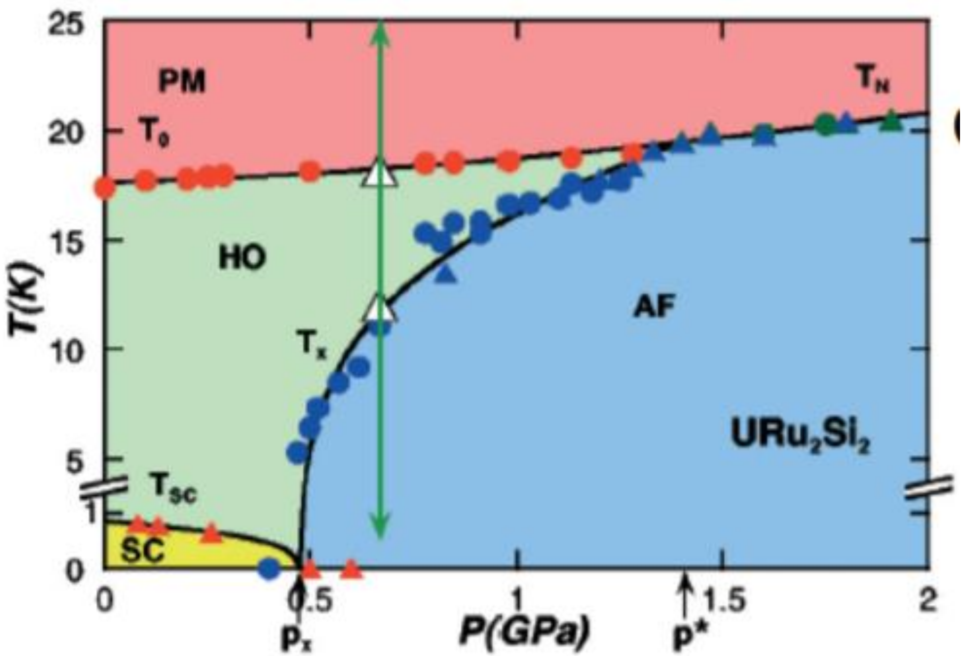
- l=1 spin-symmetric (charge) Pomeranchuk instability is forbidden more generally by gauge-invariance for Galilean invariant system.

1. But what about instability in the spin-antisymmetric channel for l=1 in a non-rel case?
2. If there is no instability in spin sector for l=1, how can this be implied from the fermi liquid picture?
3. How far this statement is true for l=1 channel in spin sector?
4. Are there any restrictions on the higher angular momentum channel as well?

Proposals for spin l=1 instabilities over the past many years…..(few examples)

- hidden order parameter in URu_2Si_2

: Helicity order



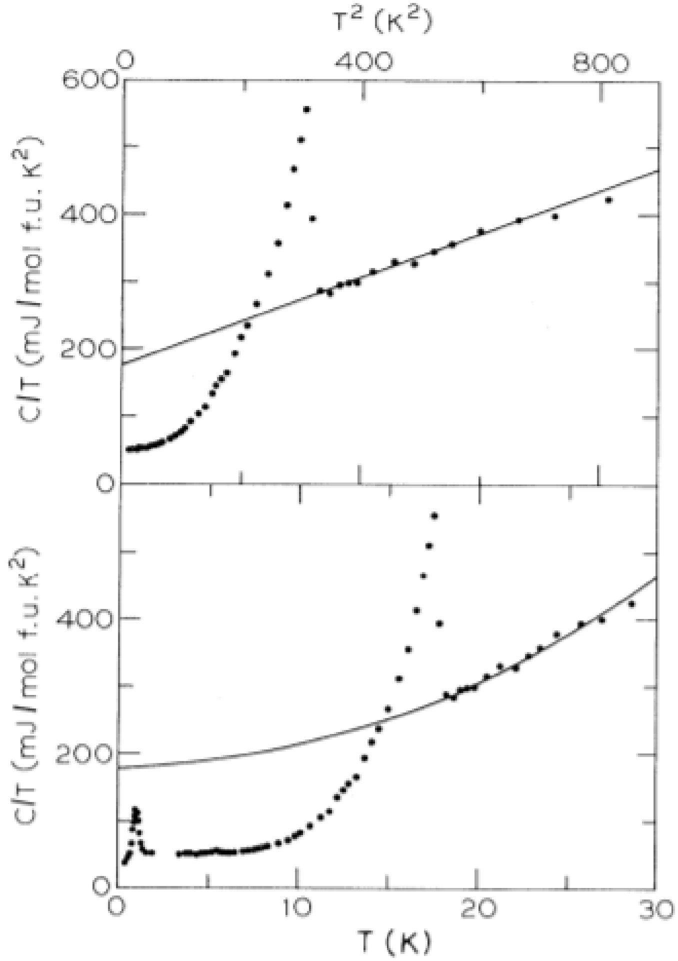


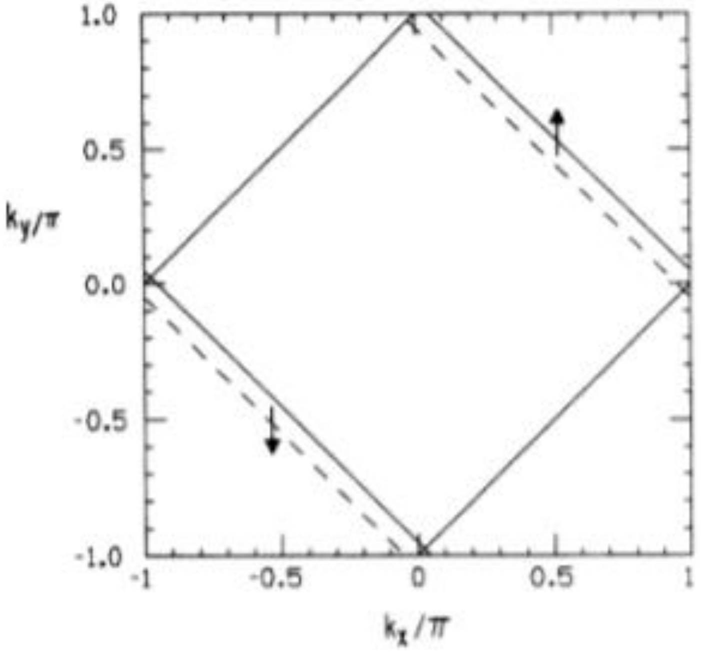
FIG. 1. Specific heat of URu_2Si_2 plotted as C/T vs T^2 (above) yielding γ and Θ_D, and as C/T vs T (below) showing the entropy balance.

C.M. Varma + L. Zhu., Phys. Rev. Lett. 96, 036405 (2006) :

phase diagram: A. Villaume, et al., Phys. Rev. B 78, 012504 (2008)

Specific heat: Palstra et al., Phys. Rev. B 55,1985

- **Spin-split states in metals, a proposal for Cr**



J. E. Hirsch, Phys. Rev. B 41, 6820 (1990)

J. E. Hirsch, Phys. Rev. B 41 6828 (1990)

^{3}He spin and charge response is very different
(near instability for l=1 in the spin channel?)

Pressure	0 bar	27 bar
F_1^s	5.39	12.79
F_1^a	-0.54	-1.00

G. Baym and Ch. Pethick. *Landau Fermi-liquid theory: concepts and applications* J. Wiley & Sons (2008)

- Exact expressions for a static susceptibility for a generic order parameter with any l, and any form factor $\lambda_l^{c(s)}(k)$.

$$\hat{\rho}_l^c(\mathbf{q}) = \sum_{\mathbf{k},\alpha} \lambda_l^c(k) c^\dagger_{\mathbf{k}-\mathbf{q}/2,\alpha} c_{\mathbf{k}+\mathbf{q}/2,\alpha},$$

$$\hat{\rho}_l^s(\mathbf{q}) = \sum_{\mathbf{k},\alpha\beta} \lambda_l^s(k) c^\dagger_{\mathbf{k}-\mathbf{q}/2,\alpha} \sigma^{\alpha\beta} c_{\mathbf{k}+\mathbf{q}/2,\beta},$$

The generic form of susceptibility is obtained by A. j Leggett in 1965 using the microscopic picture of FL theory where,

$$\chi_l^{c(s)} = \left(\Lambda_l^{c(s)} Z\right)^2 \chi_{l,qp}^{c(s)} + \chi_{l,inc}^{c(s)}.$$

$$G = \frac{Z}{\omega - \epsilon_{\mathbf{k}} + i0_\omega} + G^{\text{inc}}$$

Here

$$\chi_{l,qp}^{c(s)} = \frac{m^*}{m} \chi_{l,RPA}^{c(s)} \qquad \chi_{l,inc}^{c(s)} \equiv \lim_{\omega\to 0} \lim_{q\to 0} \chi_l^{c(s)}$$

Where Z is the quasiparticle residue and $\Lambda_l^{c(s)}$ accounts for the renormalization of the vertex containing the form factor.

Implications

a) On susceptibility due to conserved order parameter (charge ,spin, momentum):- case of l=0 channel

- Conservation law requires that full susceptibility is only due to coherent term,

$$\chi_\rho(\boldsymbol{q} = \mathbf{0}, \omega \neq 0) = 0$$

$$\Downarrow$$

$$\Lambda Z = 1 \quad \chi_{inc} = 0$$

.

- For example in l=0 channel, both charge and spin order parameters with constant form factor are conserved quantity , hence the full static susceptibility is given by only quasiparticle response.

$$\chi^{c(s)}_{l=0} = \chi^{c(s)}_{l=0,qp} = \chi_{l=0,0}\frac{m^*/m}{1+F^{c(s)}_{l=0}}$$

b) On susceptibility of charge current (conserved) and spin current (not conserved) order parameter with $\lambda^{c(s)}_{l=1}(k) = \mathbf{k}$

- The continuity equation $(\Omega/q)^2\chi_\rho(q) = \chi^{\|}_J(q) - \chi^{\|}_J(\mathbf{q},0)$.

$$Z\Lambda^{c(s)}_{l=1} = \frac{m}{m^*}\left(1+F^{c(s)}_{l=1}\right) \longrightarrow (Z\Lambda^{c(s)}_{l=1})^2\chi^{c(s)}_{l,qp} \propto \left(1+F^{c(s)}_{l=1}\right)$$

and using the longitudinal sum rule we have,

$$\chi^{c(s)}_{l=1} = \chi_{1,0} \longrightarrow \chi^{c(s)}_{l=1,inc} = \chi_{l=1,0}\left(1-\frac{m}{m^*}\left(1+F^{c(s)}_{l=1}\right)\right)$$

- For Galilean invariant fermi liquid, the mass renormalization is given as

$$m^*/m = 1+F^c_1.$$

For charge current, it easily noticed that $\Lambda Z = 1$ and $\chi_{inc} = 0$ as it is a conserved quantity. [Charge current is simply momentum for a Galilean invariant system, hence above argument is understood from gauge invariance principle as well]
where as for spin current, $Z\Lambda^s_{l=1} = (1+F^s_1)/(1+F^c_1)$ and

$$\chi^s_{l=1,inc} = \chi_{l=1,0}(F^c_1 - F^s_1)/(1+F^c_1)$$

How generic is this result for l=1 channel?

- It has been argued that for certain form factor, $\lambda_{l=1}(k) = \mathbf{k} f_{l=1}^{c(s)}(|k|)$ with $f_{l=1}(|\mathbf{k}|) \neq 1$ there is no relation like $\Lambda_l^{c(s)} Z \propto (1+F_1^{c(s)})$, hence the instability can occur.
- Also statements have been made that there is no pomeranchuk instability in l=1 charge channel with lattice(which break Galilean invariance). Not completely convinced with this statement.

What about the higher angular momentum channel, l=2 etc?

- Recent calculations claims that one cannot put any such restrictions in l=2 channel or so on.

Conclusions.

- *For Galilean invariant system, charge current is conserved, hence, the incoherent part of susceptibility is zero. Therefore, no pomeranchuk instability in l=1 charge channel, also protected by gauge invariance.*
- *The spin current is not a conserved quantity. Hence if we reach the threshold value of PI's, the divergence is canceled by the vertex corrections, hence the entire response is coming from the incoherent part. Hence, no instability for spin channel as well.*
- *The instability can occur for different form factor with l =1 symmetry. As there is no such cancelation of the divergence.*
- *The above statement also supports the claim that pomeranchuk instability in higher l channel is also possible as opposed to what is proposed by some groups.*

References:- Chubukov, et al.Phys. Rev. B 97, 165101 (2018)
Kiselev, et al. Phys. Rev. B 95,125122(2017)
A. J. Leggett, Phys. Rev. A 140, 1869 (1965)