6

STANDARD MODEL: One-Loop Structure

Although the fundamental laws of Nature obey quantum mechanics, microscopically challenged physicists build and use quantum field theories by starting from a classical Lagrangian. The classical approximation, which describes macroscopic objects from physics professors to dinosaurs, has in itself a physical reality, but since it emerges only at later times of cosmological evolution, it is not fundamental. We should therefore not be too surprised if unforeseen special problems and opportunities emerge in the analysis of quantum perturbations away from the classical Lagrangian.

The classical Lagrangian is used as input to the path integral, whose evaluation produces another Lagrangian, the effective Lagrangian, $\mathcal{L}_{eff}$, which encodes all the consequences of the quantum field theory. It contains an infinite series of polynomials in the fields associated with its degrees of freedom, and their derivatives. The classical Lagrangian is reproduced by this expansion in the lowest power of $\hbar$ and of momentum. With the notable exceptions of scale invariance, and of some (anomalous) chiral symmetries, we think that the symmetries of the classical Lagrangian survive the quantization process. Consequently, not all possible polynomials in the fields and their derivatives appear in $\mathcal{L}_{eff}$, only those which respect the symmetries.

The terms which are of higher order in $\hbar$ yield the quantum corrections to the theory. They are calculated according to a specific, but perilous path, which uses the classical Lagrangian as input. This procedure generates infinities, due to quantum effects at short distances. Fortunately, most fundamental interactions are described by theories where these infinities can be absorbed in a redefinition of the input parameters and fields, *i.e.* swept under the rug. These theories, which yield finite quantum corrections, are said to be *renormalizable*; the standard model is one of them. On the other hand, Quantum Gravity is not renormalizable, and its quantum corrections generate an intractable number of infinities. As a result, many physicists believe that gravity is an infrared approximation to a theory devoid of these ultraviolet infinities; likely candidates are the superstring theories.

In a renormalizable theory, quantum corrections generate *a priori* all possible terms consistent with the invariances of the input (classical) Lagrangian that survive quantization. These appear as terms in the effective Lagrangian, with coefficients to be calculated, either through perturbative or non-perturbative techniques.

By absorbing the ultraviolet divergences through a redefinition of the input fields and parameters, scale invariance is necessarily broken. In the process a new scale is introduced in the theory, and all the parameters become scale-dependent. Their runnings are determined by the first order differential equations of the renormalization group. Each has one integration constant which is roughly identified with the measured numerical value of the parameter at the scale determined by the experiment.

The rules for a renormalizable theory in four dimensions are rather easy to state. Start with *all* possible effective interactions of (mass) dimensions less than or equal to four, consistent with symmetries. All the ultraviolet infinities of the theory are then absorbed in its fields and parameters. Quantum corrections generate different types of terms. Some are of the same form as the input terms; they describe finite renormalizations of the basic interactions, and yield the scale-dependence of the input parameters. Some generate new interaction terms that were not in the classical description. The coefficients in front of effective interactions of mass dimension larger than four are finite and calculable in terms of the input parameters of the theory.

In general, the classical input Lagrangian must contain all terms of dimension four. Should one of the terms be absent, it is generated by the quantum corrections, with infinite strength. Thus it must be included as an input, so that its coupling strength can be used to absorb that infinity. There is one important exception to this rule: suppose that by deleting some terms of dimension four or less, the input Lagrangian acquires a larger symmetry. If that symmetry is of the type respected by quantization, (*i.e.* except scale invariance and anomalous symmetries), quantum corrections will not generate those terms and their associated infinities.

6.1 Quantum Electrodynamics

We begin with the quantum corrections of the mother of all renormalizable theories, Quantum Electrodynamics (QED). This section assumes prior knowledge of QED; it serves as an introduction to our method for analyzing the radiative structure of the electroweak theory.

QED is described by two fields, the electron field Ψ and the photon gauge field A_μ, one dimensionless gauge coupling constant e, the electric charge of the electron, and one mass parameter, M_e, the mass of the electron. The classical QED Lagrangian is Lorentz-invariant, gauge invariant, parity and charge conjugation invariant. The terms generated in its effective Lagrangian must then also be invariant under these same symmetries.

As a result, the gauge field appears in the covariant derivative combination, $\mathcal{D}_\mu = \partial_\mu + ieA_\mu$, acting on either itself or Ψ. The effective QED Lagrangian is just an infinite sum of polynomials in D_μ and Ψ, each of which is gauge invariant, Lorentz-invariant, even under parity and charge conjugation (*Furry's theorem*).

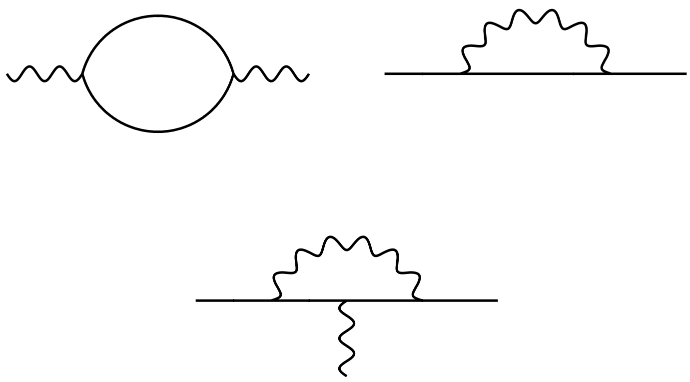
The most important tool for calculating the quantum corrections is the loop expansion with Feynman diagrams. In terms of diagrams, the effective Lagrangian is generated by one-particle irreducible or *proper* diagrams that cannot be disconnected by cutting one line. The effective Lagrangian is written as an expansion in $\hbar$,

$$\mathcal{L}_{eff} = \mathcal{L}_{cl.} + \hbar\mathcal{L}_1 + \hbar^2\mathcal{L}_2 + \cdots \ .$$

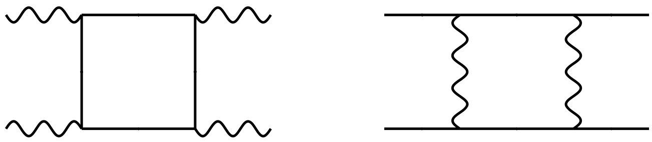
In the above, $\mathcal{L}_{cl.}$ is the same as the input Lagrangian: it contains only combinations of fields and derivatives of dimensions less than or equal to four. The higher order terms, $\mathcal{L}_n$ denote the terms generated by n-loop corrections; each includes infinite polynomials in the input fields and their derivatives.

Dimensional analysis is a potent tool in organizing the results. Since the action is dimensionless ($\hbar = 1$), the Lagrangian has (mass) dimension 4. The derivative has dimension 1, the electron field has dimension $3/2$, and the photon field has dimension 1. Terms of dimension less than or equal to 4 appear in the input Lagrangian. In a renormalizable theory, these are the only ultraviolet-divergent terms, and their divergences can all be absorbed in a redefinition of the input fields and parameters. Polynomials of higher dimensions either yield new interactions or finite corrections to the basic interactions; their strengths are in principle computable in terms of the input parameters.

We begin by organizing the expansion in terms of Feynman diagrams. Consider all possible one-loop Feynman diagrams of the same structure as the terms in the classical Lagrangian; these are the photon two-point function, the photon-electron vertex, and the electron two-point function. The relevant diagrams are



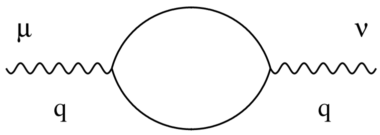
There are other one-loop diagrams which describe interactions absent from the classical Lagrangian; they are the box diagrams, which, in QED, generate four-fermion and multi-photon interactions.



By our rules, both of these must be finite, since QED is renormalizable. For example the four-fermion interaction has dimension 6, and the four photon has dimension 8 (using gauge invariance). However the four-fermion box diagram corrects a tree level process in which a four fermion interaction is generated by one-photon exchange. The four-photon interaction, on the other hand, is purely an effect of the quantum corrections, without classical analogue.

The corrections to the basic classical interactions contain the ultraviolet divergences, and the renormalization procedure results in a modification of the input parameters that makes them scale dependent. In QED, quantum effects modify the electron-photon vertex (vertex corrections), the photon propagator (vacuum polarization), the electron propagator (wave function correction) and the electron mass (mass correction). The separation of vertex and vacuum polarization is gauge invariant only in Abelian theories such as QED.

Consider the vacuum polarization diagram; it corrects the photon propagator in a simple way to give the coupling "constant" (electric charge) a momentum dependence.



At one loop, using dimensional regularization, it is calculated (details can be found in any primer on field theory) to be

$$\Pi_{\mu\nu}(q) = (\frac{q_\mu q_\nu}{q^2} - \delta_{\mu\nu})\Pi(q^2) \ , \tag{6.1}$$

where

$$\Pi(q^2) = \frac{e^2}{12\pi^2}q^2\left\{\Delta - \int_0^1 dxx(1-x)\ln\left(\frac{2m_e^2 + 2q^2x(1-x)}{\mu^2}\right)\right\} \ , \tag{6.2}$$

with

$$\Delta = \frac{2}{4-n} - \gamma + \ln 4\pi \ , \tag{6.3}$$

which contains the divergence; n is the dimension of space-time, and γ is the Euler-Mascheroni constant. The diagram diverges in the limit $n \to 4$, with the divergence absorbed in the input parameter.

The appearance of the arbitrary scale μ is a by-product of the regularization, in this case dimensional regularization. Physical quantities cannot depend on μ, which implies that the input parameters develop specific scale dependences of their own.

We note that this correction is purely transverse, and proportional to q^2. This feature continues to be true even at higher loops, enforced by the quantum BRST symmetry, a powerful remnant of the gauge invariance of the classical Lagrangian. This symmetry (through the Ward-Takahashi identities), reduces the degree of divergences of certain diagrams, and makes QED renormalizable.

In QED, it ensures that the photon remains massless even after quantum corrections. To see this, it suffices to examine the corrections to the photon propagator (two-point function). The diagrammatic expansion yields

$$\frac{e^2}{q^2} \quad \to \quad \frac{e^2}{q^2} + \frac{e^2}{q^2}\Pi(q^2)\frac{e^2}{q^2} + \cdots \equiv \frac{e^2(q^2)}{q^2} \ , \tag{6.4}$$

since Π is always proportional to q^2: the photon stays massless, and the only effect of the vacuum polarization diagrams is to make the coupling momentum-dependent.

The running coupling obeys the renormalization group differential equation

$$\frac{de}{dt} = \beta(e) \ , \tag{6.5}$$

where $t = \ln(q^2/\mu^2)$. The β function can be extracted from the coefficient of the divergence in Π. In dimensional regularization, it is natural to use the "minimal substraction" (MS) renormalization scheme where the finite parts of the counterterms are chosen to be zero. However since the divergence always occurs in combination with $\gamma - \ln 4\pi$, it is better to use the modified minimal subtraction ($\overline{MS}$), where this combination is the only momentum independent finite part of the counterterms. These renormalization prescriptions are chosen for their mathematical convenience, and the running parameters they produce must be carefully compared with measurable quantities.

In either scheme, the β function is zero below the electron threshold, when $t < t_e = \ln(4m_e^2/\mu^2)$; above the electron threshold, for $t > t_e$, it can be read off the coefficient of the divergent part of Π. At the one loop level, it is given by

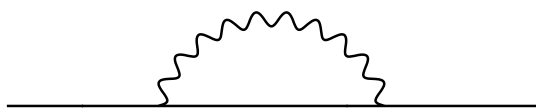
$$\beta(e) = \frac{e^3}{16\pi^2}\frac{4}{3} \ .$$

The arbitrary scale μ is fixed by measurement. In QED, it is traditional to use a different (on-shell) renormalization scheme, based on a direct comparison with the Thomson scattering cross-section; it yields the famous numerical value for the gauge coupling

$$\alpha = \frac{e^2}{4\pi}(q^2 = 2m_e^2) \approx \frac{1}{137} \ .$$

This numerical identification can be seen as setting the scale μ for QED. Fortunately, the QED coupling is smallest at large distances (that is after all the reason we recognize electrons as free particles). At the other end, the renormalization group equation implies that the effective gauge coupling increases in the deep ultraviolet, eventually reaching infinity at the *Landau pole* at an extraordinary large value of energy. This happens well beyond the domain of validity of this formula which assumes α to be small. It is not known if the singularity is really there; one can only relate that, when last seen, the gauge coupling was increasing with energy- what it does beyond that energy is unknown, beyond the reach of our puny perturbative methods. Inclusion of higher order effects does not alter this trend: there is a natural scale associated with QED, roughly speaking that of the Landau pole. Fortunately it is ridiculously small compared to those at which we operate, which offers some justification for ignoring it.

More QED infinities lurk in the remaining one-loop diagrams. The first



corrects the fermion propagator. It is calculated to be

$$\begin{aligned}\Sigma(p) = &\quad -i\Delta\tfrac{e^2}{16\pi^2}[\not{p}+4m_e] + i\tfrac{e^2}{16\pi^2}[\not{p}+2m_e] \\ + &\quad i\tfrac{e^2}{8\pi^2}\int_0^1 dx[\not{p}(1-x)+2m_e]\ln\left(\tfrac{p^2x(1-x)+m_e^2x}{\mu^2}\right) \ .\end{aligned} \tag{6.6}$$

Note that the divergence appears along both the kinetic term and the mass term. It is absorbed by a redefinition of the electron field and of the mass term. It follows that the mass, like any other parameter in the Lagrangian also becomes scale-dependent; its dependence on scale is dictated by the renormalization group equation

$$\frac{dm_e(t)}{dt} = m_e(t)\gamma_m(e) \ . \tag{6.7}$$

Its one loop expression is

$$\gamma_m = -6\frac{e^2}{(4\pi)^2} \ . \tag{6.8}$$

Finally, we note that the physical mass of the electron, M_e, is to be distinguished from this running mass. It is natural to make the identification

$$m_e(q^2 = M_e^2) = M_e[1 + \mathcal{O}(e^2)] \ .$$

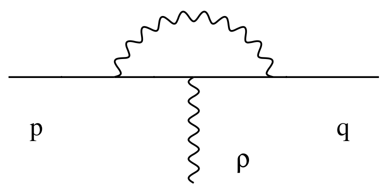
The scale dependence of the mass may also be interpreted in terms of the dimension of the two-fermion operator, and thus as the anomalous dimension of the fermion field.

This renormalization group equation tells us that if the running mass is zero at any scale, it will remain so at all other scales. This is a reflection of the added symmetry gained by setting the mass equal to zero. Indeed if $m_e = 0$, the QED Lagrangian becomes invariant under the chiral transformation

$$\Psi \to e^{i\alpha\gamma_5}\Psi \ , \tag{6.9}$$

which forbids a mass term for the electron to all orders of perturbation theory.

The one-loop correction to the interaction vertex is described by the diagram,



It is written in the form

$$\Gamma_\rho(p,q) = \Gamma_\rho^{(1)}(p,q) + \Gamma_\rho^{(2)}(p,q) \ ,$$

where the first part contains the divergence and the second part is finite. The first term has exactly the same matrix structure as the interaction term, specifically

$$\begin{aligned}\Gamma_\rho^{(1)}(p,q) \quad &= \quad -ie\mu^{2-\frac{n}{2}}\gamma_\rho \frac{e^2}{16\pi^2}\Bigg\{\Delta - 1 - \int_0^1 dx \int_0^{1-x} dy \\ &\qquad ln\left[\frac{m_e^2(x+y) + p^2x(1-x) + q^2y(1-y) - 2p\cdot qxy}{\mu^2}\right]\Bigg\}\end{aligned} \tag{6.10}$$

The second term is more complicated. It yields contributions along both

γ_ρ and $\sigma_{\rho\mu}k_\mu$, with $k_\mu = q_\mu - p_\mu$. They are both ultraviolet finite, but the γ_ρ term diverges in the infrared when the external particles are on their mass-shells.

$$\begin{aligned}\Gamma^{(2)}_\rho(p,q) &= \frac{e^2}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \ \times \\ & \left\{ \frac{2m_e^2\gamma_\rho[(x+y)^2 - 2(1-x-y) - 2k^2(1\ -x)(1-y)]}{m_e^2(x+y) + p^2x(1-x) + q^2y(1-y) - 2p\cdot qxy} \right. + \\ &+ \left. \frac{8im_e\sigma_{\rho\sigma}k_\sigma[x - y(x+y)]}{m_e^2(x+y) + p^2x(1-x) + q^2y(1-y) - 2p\cdot qxy} \right\} . \end{aligned} \tag{6.11}$$

The infrared divergence in the first term results from integration over the Feynman parameters. It can be shown, on the grounds of relativistic invariance that the vertex corrections can be cast in the form

$$\Gamma_\rho(p,q) = F_1(k^2)\gamma_\rho + F_2(k^2)\sigma_{\rho\nu}k_\nu \ . \tag{6.12}$$

The one-loop diagram contributes to both $F_{1,2}$. The first term is the form factor which corrects the basic electron-photon vertex, while the second contributes to the electron's magnetic moment.

We have said that the effect of the quantum corrections is to generate in the effective Lagrangian terms of higher dimensions that respect all the symmetries of the classical Lagrangian which survive quantization. The finite quantum corrections generate in $\mathcal{L}_{eff}$ interaction terms with dimension higher than four. Divided by the appropriate power of the only dimensionful parameter, in this case the electron mass, they decouple in the limit of large electron mass. The reader is cautioned that it does not mean that the theory is trivial at energies below that scale: the electron can still contribute as a virtual particle, say in the scattering of light by light.

We now turn to the uses of dimensional analysis and symmetries, which are powerful tools in drawing a catalog of the finite quantum corrections of QED.

• *Dimension Five Interactions.* We begin the catalog by enumerating all possible interactions of dimension five, made out solely of the covariant derivative and the electron field. There are no Lorentz-invariant term with five D_μ alone. This leaves only Lorentz-invariant combinations of two fermion fields and two D_μ's:

$$\mathcal{O}_5^{(1)} = \overline{\Psi}\sigma^{\mu\nu}\Psi F^{\mu\nu} \ ; \qquad \mathcal{O}_5^{(2)} = \overline{\Psi}D_\mu D_\mu\Psi \ , \tag{6.13}$$

as well as

$$\mathcal{O}_5^{(1)\prime} = \overline{\Psi}\gamma_5\sigma^{\mu\nu}\Psi F^{\mu\nu} \; ; \qquad \mathcal{O}_5^{(2)\prime} = \overline{\Psi}\gamma_5 D_\mu D_\mu \Psi \; . \tag{6.14}$$

The last two terms cannot be generated by QED alone (see problem), leaving only the first two terms. Both **must** appear in $\mathcal{L}_{eff}$ divided by the electron mass, and with a dimensionless prefactor which is **finite** and **computable** in perturbation theory. The first term describes the famous correction to the gyromagnetic ratio of the electron. It is generated by the one-loop vertex diagram. Eventually, comparison with Eq. (6.11) yields the following effective interaction

$$\mathcal{L}_1 = (\frac{\alpha}{\pi})\frac{1}{m_e}\overline{\Psi}\sigma_{\mu\nu}\Psi F_{\mu\nu} \; , \tag{6.15}$$

where α is the fine structure constant. Contributions to the magnetic dipole of the electron are also generated in $\mathcal{L}_n$, with a coefficient of n^{th} order in the fine structure constant.

The second term in (6.13) breaks fermion chirality, and thus cannot contribute to the kinetic term. By expanding the covariant derivative, we see that it contains three different terms. The first just provides a correction proportional to the momentum squared to the electron mass. The second generates a momentum-dependent chirality-breaking correction to the electron photon vertex, and the third yields a two-electron-two-photon vertex.

• *Dimension Six Interactions.* There are three types of terms of dimension six, containing four Ψ's, two Ψ's and three D_μ's, and six D_μ's; they will be divided by the square of the electron mass, with a finite prefactor calculated in perturbation theory. It is easy to see that the only possible terms with two fermion fields are

$$\mathcal{O}_6^{\prime} = \epsilon_{\rho\sigma\mu\lambda}\overline{\Psi}\gamma_\rho F_{\sigma\mu} D_\lambda \Psi \; , \tag{6.16}$$

which is odd under parity, and does not appear in pure QED, and the parity-invariant interactions

$$\mathcal{O}_6^{\pm} = \overline{\Psi}(\gamma^\rho D^\sigma \pm \gamma^\sigma D^\rho) D^\rho D^\sigma \Psi \; . \tag{6.17}$$

The antisymmetric combination corrects the basic vertex. After integration by parts, it yields the interaction

$$\mathcal{O}_6^{(1)} = \overline{\Psi}\gamma_\rho \Psi \partial_\sigma F_{\rho\sigma} \; . \tag{6.18}$$

In momentum space, it is the coefficient of the term linear in momentum squared of the form factor for the electron-photon vertex; it is called the charge radius of the electron. It appears in the expansion of the form factor F_1.

The symmetric combination yields a finite correction to the fermion kinetic term

$$\mathcal{O}_6^{(2)} = \overline{\Psi}\gamma_\rho D_\rho D_\mu D_\mu \Psi \ , \tag{6.19}$$

that is linear in the momentum squared, as well as more complicated corrections to the interaction of one electron with one, two, and three photons.

The combination with six covariant derivatives can appear in many different ways. A possible term with three field strengths vanishes identically because one cannot make a Lorentz invariant out of three field strengths. Another can be made up of two field strengths and two covariant derivatives, such as

$$\mathcal{O}_6^{(3)} = \partial_\mu F_{\mu\nu} \partial_\rho F_{\rho\nu} \ , \tag{6.20}$$

which gives a finite correction to the photon propagator.

The analysis of the four fermion combinations is more complicated. Using Lorentz invariance, we arrange the 16 fermion bilinears in their Lorentz-covariant form (S,P,V,A,T). The allowed four-fermion interactions are their Lorentz-invariant, charge conjugation-even combinations. They are of the form SS, PP, VV, AA, and TT, but they are not all independent. The *Fierz* identities yield the following relations

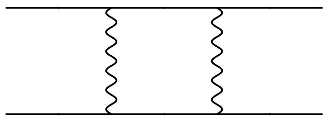
$$SS - PP = -\frac{1}{2}(VV - AA) \ , \tag{6.21}$$

$$SS + PP = -\frac{1}{3}TT \ , \tag{6.22}$$

which leave only the three independent interactions

$$\mathcal{O}_6^{(4)} = \overline{\Psi}\Psi\overline{\Psi}\Psi \ , \qquad \mathcal{O}_6^{(5)} = \overline{\Psi}\gamma_5\Psi\overline{\Psi}\gamma_5\Psi \ , \qquad \mathcal{O}_6^{(6)} = \overline{\Psi}\gamma_\mu\Psi\overline{\Psi}\gamma_\mu\Psi \ . \tag{6.23}$$

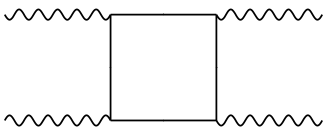
The finite contributions to these interactions are generated by the box diagram



When weak interactions are included, we expect other interactions to be generated, such as SP and VA, because of parity violation.

It is important to emphasize that there already are one-particle-reducible contributions which yield four-fermion interactions, generated by one-photon exchange. In QED, the dimension six terms in the effective Lagrangian yield finite corrections to these processes, but do not generate new types of interaction. As we shall see later, in the standard model, some new four-fermion interactions, forbidden at tree level, do appear at one-loop in the effective Lagrangian. In QED, we have to go to terms of dimension eight to see a similar effect.

• *Dimension Eight.* We leave it to the reader to list all possible terms of dimension eight. Rather we focus on the one term that does not correspond to any interaction of lower dimensions. We can indeed form a term of dimension eight by putting together eight covariant derivatives, which is the same as four field strengths. It produces a direct interaction between photons; it is the famous scattering of light by light, or Delbrück scattering. It is generated at the lowest order by a box diagram



where the internal lines are electrons, and the external lines are photons. Naive power counting implies this diagram to be logarithmically divergent in the ultraviolet, which would spoil the renormalizability of the theory. However, in gauge theories, there are magic cancellations, and the diagram is finite. In the static approximation, it yields the finite interaction

$$\mathcal{L}_1 = \frac{\alpha^2}{90 m_e^4}[(F_{\mu\nu}F_{\mu\nu})^2 + \frac{7}{12}(\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma})^2] \ .$$

This important example shows that the effective Lagrangian can contain totally new interactions. In particular, imagine a world where the electron mass is so large that it has not yet been produced in the laboratory. At low energy, its presence would still be felt indirectly through the observation of photon-photon scattering! However, in the limit of very large electron mass, these effects become negligible. This is an example of the *decoupling theorem* (See T. Appelquist and J. Carazzone, *Phys. Rev.* **D11**, 2856(1975)), which says that all the quantum effects of massive particles become insignificant as their masses become infinite. The important exception, is for particles that get their masses through vacuum expectation values.

6.1.1 PROBLEMS

A. Identify the symmetries that forbid QED from generating the terms in Eq. (6.14).

B. Show that the box diagram that describes Delbrück scattering is ultraviolet finite.

C. Find the Lorentz structure of the QED four-fermion one-loop box diagram, to determine which of the interactions in the text it generates. Then compare these with interactions generated by one photon exchange.

D. Enumerate all the possible operators of dimension seven, and interpret their contributions physically.

E. Identify the lowest order Feynman diagrams which contribute to $\mathcal{O}_5^{(2)}$ and to $\mathcal{O}_6^{(3)}$.

6.2 One-Loop Standard Model

The organization of the one-loop corrections of the standard model is much more challenging. Their detailed analysis is complicated by its non-Abelian gauge symmetries, and their spontaneous breaking. In spite of these technical difficulties, the corrections have the same structure as those encountered in our study of QED: they cause the parameters of the theory to run with scale as prescribed by the renormalization group, and they also generate interactions; some appear as corrections to those already present in the classical Lagrangian, others generate entirely new interactions.

To distinguish between these two types of corrections, it is useful to analyze in detail the symmetries of the standard model. We start with the electroweak part of the theory, leaving QCD aside for the moment.

6.2.1 Partial Symmetries of the Standard Model

From Chapter 2, we recall the symmetries of the standard model classical Lagrangian: besides the gauged symmetries, $SU(2) \times U(1) \times SU(3)$, it contains four global continuous symmetries, the three lepton numbers, $L_e,\ L_\mu,\ L_\tau$, and the total quark number, $B/3$, and no discrete symmetries. Non-perturbative quantum effects, associated with the anomaly of the non-Abelian weak $SU(2)$, break these down to two relative lepton numbers, $L_e - L_\mu,\ L_\mu - L_\tau$, and $B - L$, where L is the total lepton number. These effects are negligible and they have no practical effect for our present purposes, although they are important in the study of standard model cosmology.

When we consider a subset of all its interactions, the standard model displays a much richer structure than implied by these symmetries alone. One reason is that particular subset of the interactions may display a larger symmetry than in the whole Lagrangian. We call these partial symmetries, or *accidental* symmetries. For example, QED, which is part of the standard model, preserves P and C, while the weak interactions do not. Of course the rest of the interactions break such symmetries, but by terms which have specific covariance properties with respect to the *accidental symmetries.* As with the Wigner-Eckhardt theorem, this generally implies selection rules amenable to experimental tests.

For instance, tree-level processes which involve interactions with the larger symmetry, will reflect that symmetry either by producing (tree-level) relations among parameters, or by the absence of some processes. To these must be added quantum corrections (one-loop or beyond), which use interactions from other parts of the Lagrangian that break the partial symmetry. It may even happen that one-loop corrections break the accidental symmetry only to a lesser accidental symmetry, and so on. The radiative corrections may generate new interactions forbidden by the accidental tree level symmetries, or correct the tree level relations implied by the partial symmetries. An example is the electric dipole of the electron, which is forbidden by the symmetries of QED; in the standard model, it is no longer protected, and we expect it to be generated by weak quantum corrections.

Clearly, the study of accidental symmetries will prove very useful in the understanding of the radiative structure of the standard model. Let us now apply this analysis for the whole electroweak model. Its Lagrangian can be split into different parts,

$$\mathcal{L}_{SM} = \mathcal{L}_{YM} + \mathcal{L}_{WD} + \mathcal{L}_{Y} + \mathcal{L}_{H} \ ,$$

each part characterized by different global symmetries that are generally larger than those of $\mathcal{L}_{SM}$. We have already encountered simple examples: the classical Yang-Mills part, $\mathcal{L}_{YM}$ is not only invariant under the gauge groups, but also under the discrete space-time symmetries P and C; the anomaly associated with QCD generates non-perturbatively interactions that break C and CP; the QED part which emerges from spontaneous breaking is invariant under parity.

Lepton Symmetries

The leptonic Weyl-Dirac Lagrangian $\mathcal{L}_{WD}$ displays a much larger global invariance. With three chiral families, it is invariant under $U(3)_L \times U(3)_R$, where $U(3)_L$ acts on the three lepton doublets, and $U(3)_R$ acts on the lepton singlets.

This leptonic global chiral symmetry is explicitly broken by Yukawa interactions, leaving only the three lepton number symmetries. The leptonic bilinears which appear in the Yukawa couplings are of the form $L_i\bar{e}_j$ and transform as $(\mathbf{3}, \bar{\mathbf{3}})$ under this global symmetry. If these bilinears were coupled to a Higgs matrix $\mathbf{H}_e$, itself transforming as a $(\bar{\mathbf{3}}, \mathbf{3})$, invariance could be preserved, but this would require nine Higgs doublets. The standard model contains only one doublet, with the Higgs matrix that couples to the lepton bilinear given by

$$\mathbf{H}_e(x) = \mathbf{Y}_e \tau_2 H^*(x) \ , \tag{6.24}$$

where $\mathbf{Y}_e$ is the lepton Yukawa matrix which breaks the symmetry. If all three leptons had equal mass, the remaining symmetry would be the diagonal $U(3)_{L+R}$, but since the lepton masses are different, this vectorial $U(3)$ is broken to its three diagonal generators, yielding the three lepton numbers. At high energies, where we can neglect the electron and muon masses, the remaining global symmetry is $U(2)_L \times U(2)_R \times U(1)_{L+R}$.

Quark Symmetries

The same analysis applied to the quark Weyl-Dirac Lagrangian is more complicated. The quark gauged kinetic terms are invariant under the global chiral symmetry generated by $U(3)_L \times U(3)_R \times U(3)_R$, the first acting on the three quark left-handed weak doublets, the other two on the charge 2/3 and -1/3 right-handed singlets, respectively. The Yukawa terms involve two types of weak doublets quark bilinears, $\widehat{\mathbf{Q}}_i \bar{\mathbf{u}}_j$ and $\widehat{\mathbf{Q}}_i \bar{\mathbf{d}}_j$, transforming as $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$ and $(\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$ under the global chiral symmetry, respectively. With only one Higgs doublet, these bilinears couple to the Higgs matrices

$$\mathbf{H}_u(x) = \mathbf{Y}_u H(x) \; ; \qquad \mathbf{H}_d(x) = \mathbf{Y}_d \tau_2 H^*(x) \; . \tag{6.25}$$

Since the Yukawa matrices are different for the up and down quark sectors, there remains only one unbroken symmetry, the (vectorial) quark number. It is nevertheless very instructive to keep track of the individual quark numbers.

In the spontaneously broken vacuum of the standard model, the different quark mass eigenstates, each of which carries its own quark number, mix with one another at tree level *only by emitting a charged W-boson.* These interactions change both quark number and electric charge at the same time. It is traditional to attribute the different quarks with a *flavor* F of their own, usually named after the quark itself; thus the strange quark has *strangeness* (for strange historical reasons, the strange quark has strangeness -1); the charmed quark has *charm*, the bottom quark has *beauty*, and the top quark carries *truth.* The up and down quarks are not assigned special flavor names. W-exchange imply the important **tree-level** selection rules

$$|\Delta F| = 1 \; , \; |\Delta Q| = 1 \; ; \qquad |\Delta F| = 0 \; , \; |\Delta Q| = 0 \; , \tag{6.26}$$

where F is any of the quark flavors. In words, there are no flavor-changing interactions among quarks of the same electric charge at tree level. As these selection rules do not come from the symmetries of the full Lagrangian, we expect radiative corrections to break them. It follows that the standard model flavor-changing neutral interactions among quarks arise purely from quantum effects; they are strictly **predicted** in terms of the input parameters, and thus provide important experimental checks of the standard model. This situation is similar to that in QED where scattering of light by light occurs only through quantum effects, and is predicted in terms of QED's input parameters, the electron charge and mass.

Higgs Symmetry

The Higgs sector of the standard model has a global $SU(2)_R$ symmetry of its own, which we have already discussed. It is convenient for the analysis that follows to write the complex Higgs doublet in terms of four real fields

$$H = \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix} \ . \tag{6.27}$$

The same doublet with opposite hypercharge is just

$$\overline{H} \equiv -i\tau_2 H^* = \begin{pmatrix} -h_3 + ih_4 \\ h_1 - ih_2 \end{pmatrix} \ . \tag{6.28}$$

We form the matrix

$$\mathcal{H} = (H, \overline{H}) = \begin{pmatrix} h_1 + ih_2 & -h_3 + ih_4 \\ h_3 + ih_4 & h_1 - ih_2 \end{pmatrix} \ . \tag{6.29}$$

The weak gauged $SU(2)_L$ acts on the two rows, the global $SU(2)_R$ on the two columns; $SU(2)_R$ clearly violates hypercharge, since the first and second rows have opposite hypercharges. The Higgs matrix transforms as $(\mathbf{2}, \mathbf{2})$ under this symmetry

$$\mathcal{H} \to \mathcal{H}' = U_L \mathcal{H} U_R \ , \tag{6.30}$$

where the unitary matrices $U_{L,R}$ represent $SU(2)_{L,R}$, respectively. These two groups combine to an $SO(4)$ acting on the four real components of the Higgs field. It is easy to check that the Higgs potential involves only the $SO(4)$-invariant combination

$$H^\dagger H = \frac{1}{2}\mathrm{Tr}(\mathcal{H}^\dagger \mathcal{H}) = \det \mathcal{H} = h_1^2 + h_2^2 + h_3^2 + h_4^2 \ . \tag{6.31}$$

The gauged weak $SU(2)_L$ is clearly preserved, but what happens to $SU(2)_R$ in the rest of the Lagrangian? The Higgs kinetic term clearly preserves the full symmetry, but since the two columns of the matrix have opposite unit values of hypercharge, hypercharge interactions violate $SU(2)_R$ by two units.

The quark Yukawa interactions violate $SU(2)_R$ as well, but in an interesting way, best seen by rewriting the quark Yukawa coupling in the suggestive form

$$\mathcal{L}_Y = \widehat{\mathbf{Q}}\left[\left(\frac{\mathbf{Y}_u + \mathbf{Y}_d}{2}\right)(\overline{\mathbf{u}}H + \overline{\mathbf{d}}\overline{H}) + \left(\frac{\mathbf{Y}_u - \mathbf{Y}_d}{2}\right)(\overline{\mathbf{u}}H - \overline{\mathbf{d}}\overline{H})\right] \ . \quad (6.32)$$

The first term is obviously invariant if the combination $(\overline{\mathbf{u}}, \overline{\mathbf{d}})$ transforms as a doublet under $SU(2)_R$ (this is the reason for the subscript R). The second term violates $SU(2)_R$, with the quantum numbers of a triplet. In analogy with the Wigner-Eckhardt theorem, this results in additional sum rules. The contributions from this term are significant because of the sizeable value of the mass *difference* between the top and bottom quarks, which dwarfs the contributions from the lighter families. The gauged kinetic term of the $(\overline{\mathbf{u}}, \overline{\mathbf{d}})$ doublet is not $SU(2)_R$ invariant, since its components have different hypercharges.

A similar reasoning applied to the lepton Yukawa couplings shows that they are not symmetrical in any limit, because the standard model contains no electroweak-singlet leptons to serve as the $SU(2)_R$ partner of $\overline{e}_i$. However these partners do appear in many extensions of the standard model.

The standard model is therefore symmetric under a global $SU(2)_R$ only in the limits

$$\mathbf{Y}_e = \mathbf{Y}_u - \mathbf{Y}_d = 0 \ ; \qquad g_1 = 0 \ . \quad (6.33)$$

Our analysis has not taken into account the spontaneous breakdown of the standard model symmetry. As the Higgs gets its vacuum value, the $SO(4)$ symmetry is spontaneously broken to an $SO(3)$ subgroup. The three broken symmetries yield Nambu-Goldstone bosons, which are eaten by the three gauge vector bosons. The surviving symmetry preserves the trace of the Higgs matrix; it is the vectorial diagonal subgroup $SU(2)_{L+R}$, the so-called custodial $SO(3)$.

In the limit $g_1 = 0$, $\cos\theta_W = 1$, and electromagnetism decouples. The three massive gauge bosons W^+_μ, W^-_μ, Z_μ transform as a custodial triplet, and have the same mass. The three currents to which they couple also transform as a custodial triplet. One can see this directly by noting that both left- and right-handed quarks transform as custodial doublets.

Does this symmetry manifest itself at tree level? In the static limit, the exchange of the gauge bosons produces with a current-current four-fermi interaction among these currents. The custodial symmetry simply requires that neutral and charged current interactions appear with the same strength at this level of approximation.

When hypercharge interactions are restored ($g_1 \neq 0$), this situation does not change at tree-level, since the strengths of the charged and neutral current-current interactions are still equal

$$\frac{g_2^2}{m_W^2} = \frac{g_1^2}{m_Z^2} = \frac{g_2^2}{m_Z^2 \cos^2\theta_W} , \tag{6.34}$$

although the form of the neutral current changes by acquiring the electromagnetic current multiplied by $\sin^2\theta_W$. The above relation is often expressed by introducing a parameter ρ ρ *parameter*which is the ratio of these two interaction strengths

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} . \tag{6.35}$$

It trivially satisfies the tree-level relation

$$\rho - 1 = 0 . \tag{6.36}$$

Since the rest of the Lagrangian violates the custodial symmetry, one expects quantum corrections to this relation. This situation is analogous to $g-2$ in QED, which is zero at tree-level, but is calculably corrected by quantum effects. The custodial symmetry is most badly broken by the large mass difference between the top and bottom quark masses divided by the W-mass (to make it dimensionless). There will also be smaller contributions involving the $W-Z$ mass difference, and electromagnetic corrections, with strengths proportional to $\sin^2\theta_W$.

6.2.2 Running the Standard Model Parameters

As a result of quantization, all the parameters of the standard model become scale dependent. In this section we only state the resulting equations for its parameters, and refer the reader interested in the calculational details to standard texts on quantum field theory.

Gauge Couplings

All three gauge couplings are scale-dependent. In the one-loop approximation, their evolution is governed by the equations

$$\frac{d\alpha_l^{-1}}{dt} = \frac{1}{2\pi} b_l , \tag{6.37}$$

where $\alpha_l = g_l^2/4\pi$, $t = \ln\mu$ and $l = 1, 2, 3$, corresponding to the gauge groups $U(1) \times SU(2) \times SU(3)$. For any gauge group, the coefficients are given by

$$b = \frac{11}{3}C_{\rm adj} - \frac{2}{3}\sum_f C_f - \frac{1}{6}\sum_h C_h \ , \tag{6.38}$$

where $C_{\rm adj}$ is the *Dynkin index* of the adjoint representation of the gauge group, C_f is the Dynkin index of the representation of the left-handed Weyl fermions, and C_h is that of the representation of the (real) Higgs field. Applying this formula for one Higgs doublet and $n_{\rm fam}$ chiral families, we find

$$\begin{aligned} b_1 &= -\frac{4}{3}n_{\rm fam} - \frac{1}{10} \ , \\ b_2 &= \frac{22}{3} - \frac{4}{3}n_{\rm fam} - \frac{1}{6} \ , \\ b_3 &= 11 - \frac{4}{3}n_{\rm fam} \ . \end{aligned} \tag{6.39}$$

The coefficient of the hypercharge has been normalized in such a way that

$$b_i = -\frac{3}{20}\left\{\frac{2}{3}\sum_L (Y^2)_L + \frac{1}{3}\sum_H (Y^2)_H\right\} . \tag{6.40}$$

In the standard model, with $n_{\rm fam} = 3$, the numerical values of these coefficients are just

$$(b_1, b_2, b_3) = (-\frac{41}{10}, \ \frac{19}{6}, \ 7) \ .$$

These equations are modified by higher loop effects, but as long as the couplings are reasonably small, they should suffice. All three gauge couplings are perturbative over an enormous range of energies. The QCD coupling becomes strong in the infrared, where we need to include higher order effects, and the hypercharge coupling tends towards a Landau pole in the deep ultraviolet.

Yukawa Couplings

The one-loop renormalization group equations of the Yukawa couplings are of the form

$$\frac{d\mathbf{Y}_{u,d,e}}{dt} = \frac{1}{16\pi^2}\mathbf{Y}_{u,d,e}\boldsymbol{\beta}_{u,d,e} \ . \tag{6.41}$$

The matrix coefficients are given by

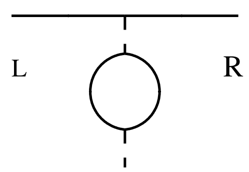
$$\begin{aligned}\boldsymbol{\beta}_u &= \frac{3}{2}(\mathbf{Y}_u{}^\dagger\mathbf{Y}_u - \mathbf{Y}_d{}^\dagger\mathbf{Y}_d) + T - (\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\ ,\\ \boldsymbol{\beta}_d &= \frac{3}{2}(\mathbf{Y}_d{}^\dagger\mathbf{Y}_d - \mathbf{Y}_u{}^\dagger\mathbf{Y}_u) + T - (\frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2)\ ,\\ \boldsymbol{\beta}_e &= \frac{3}{2}\mathbf{Y}_e{}^\dagger\mathbf{Y}_e + T - \frac{9}{4}(g_1^2 + g_2^2)\ ,\end{aligned} \tag{6.42}$$

with

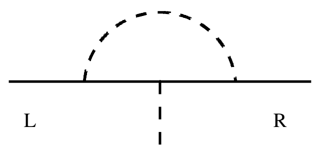
$$T = \mathrm{Tr}\{3\mathbf{Y}_u{}^\dagger\mathbf{Y}_u + 3\mathbf{Y}_d{}^\dagger\mathbf{Y}_d + \mathbf{Y}_e^\dagger\mathbf{Y}_e\}\ . \tag{6.43}$$

The structure of these equations reflects the fact that chiral symmetry that appears in the limit where these couplings are zero, is not broken by (perturbative) radiative corrections. The origin of the different terms can be understood in terms of Feynman diagrams. In the following, we show only diagrams with physical particles (unitary gauge), but in any calculationally-friendly gauge, these diagrams must be supplemented by those involving the longitudinal gauge bosons and various associated ghosts.

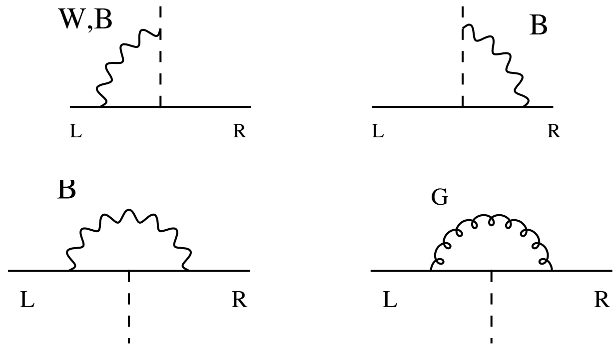
The universal factor T comes from the fermion-loop renormalization of the Higgs line



The pure Yukawa coupling contributions come from diagrams of the form



The gauge couplings contributions come from corrections to the fermion and Higgs lines (not shown here) as well as from the one-particle irreducible diagrams



The last diagrams applies only when the external fermions are quarks. We observe that the Yukawa couplings contributions tend to make the couplings blow up in the ultraviolet, while those of the gauge couplings tend towards asymptotic freedom. In order to see which of these two effects prevails, let us look at the coefficients in the (not unrealistic) limit where we keep only the top quark Yukawa, and neglect g_1 and g_2. Then the lepton Yukawa couplings evolve as

$$\frac{dy_\tau}{dt} \approx \frac{3}{16\pi^2} y_\tau y_t^2 \ , \tag{6.44}$$

resulting in a *Landau pole* in the ultraviolet. The mass of the τ lepton, and thus its Yukawa coupling, is sufficiently small that y_τ blows up beyond the Planck energy. The *bottom* quark Yukawa obeys

$$\frac{dy_b}{dt} \approx \frac{1}{16\pi^2} y_b (\frac{3}{2} y_t^2 - 8 g_3^2) \ . \tag{6.45}$$

As the larger QCD coupling dominates, y_b is asymptotically free. Similarly, the *top* Yukawa coupling, which varies according to,

$$\frac{dy_t}{dt} \approx \frac{1}{16\pi^2} y_t (\frac{9}{2} y_t^2 - 8 g_3^2) \ , \tag{6.46}$$

is still asymptotically free, although less so than that of the *bottom* quark. For representative values $y_t \approx .7$ and $g_3^2 \approx 1.5$, in the neighborhood of M_Z, we both quark Yukawa couplings decrease at short distances.

Higgs self-coupling

The Higgs quartic coupling has a complicated scale dependence. It evolves according to

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\beta_\lambda \ , \tag{6.47}$$

where the one loop contribution is given by

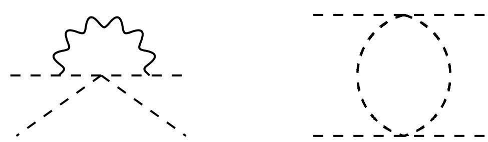
$$\beta_\lambda = 12\lambda^2 - (\frac{9}{5}g_1^2 + 9g_2^2)\lambda + \frac{9}{4}(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4) + 4T\lambda - 4H \ , \tag{6.48}$$

in which

$$H = \mathrm{Tr}\{3(\mathbf{Y}_u{}^\dagger \mathbf{Y}_u)^2 + 3(\mathbf{Y}_d{}^\dagger \mathbf{Y}_d)^2 + (\mathbf{Y}_e{}^\dagger \mathbf{Y}_e)^2\} \ . \tag{6.49}$$

We note that since λ is not protected by symmetry, β_λ is not proportional to λ. Hence setting to zero the Higgs self coupling does not enhance symmetry. This is to be contrasted with the Yukawa couplings whose absence generates chiral symmetries.

The first two terms involving λ come from diagrams of the form



while the pure gauge terms, and H are generated by the gauge and fermion loop corrections, respectively

Finally, the renormalization of the Higgs line are all proportional to λ give contributions to the second and fourth groups of terms.

The value of λ at low energies is related the physical value of the Higgs mass according to the tree level formula

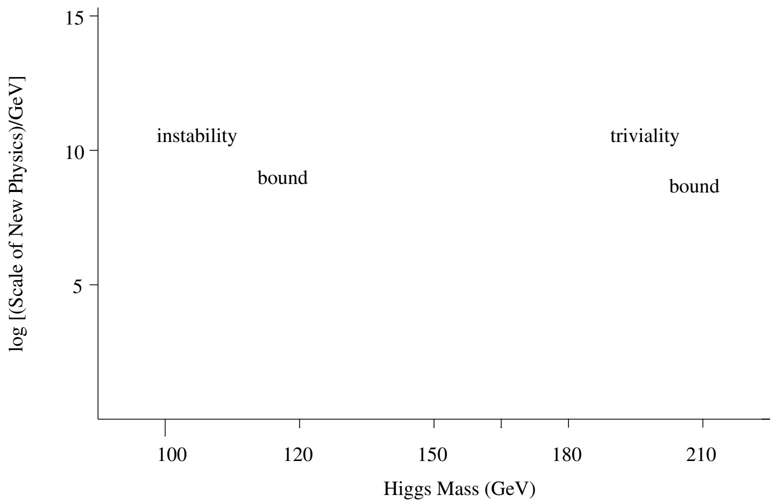
$$m_H = v\sqrt{2\lambda} \ , \tag{6.50}$$

while the vacuum value is determined by the Fermi constant G_F of β decay. Since the Higgs mass is not yet known, we do not have a physical boundary condition for Eq. (?). Still we can discuss the evolution of λ as a function of the Higgs mass.

We discuss below the qualitative features of its running, leaving datails to the problems. First, for a fixed vacuum value v, let us assume that the Higgs mass, and therefore λ is large. In that case, β_λ is dominated by the λ^2 term, which drives the coupling towards its Landau pole at higher energies. Hence the higher the Higgs mass, the higher λ is and the closest the Landau pole to experimentally accessible regions. This means that for a given (large) Higgs mass, we expect the standard model to enter a strong coupling regime at relatively low energies, losing in the process our ability to calculate. This does not necessarily mean that the theory is incomplete, only that we can no longer handle it. In analogy with the chiral model description of pion physics, it is natural to think that this effect is caused by new strong interactions, and that the Higgs actually is a composite of some hitherto unknown constituents. An example of such a theory is a generalization of the standard model called *technicolor*. The resulting bound on λ is sometimes called the triviality bound. The reason for this unfortunate name (the theory is anything but trivial) stems from lattice studies where the coupling is assumed to be finite everywhere; in that case the coupling is driven to zero, yielding in fact a trivial theory. In the standard model λ is certainly not zero.

In the opposite limit of a small *Higgs mass*, another strange behavior sets in, leading to another interesting constraint. In this regime, λ is small and its β function is dominated by the term coming from fermion loops. This term, proportional to the fourth power of the Yukawa couplings, can becomes dominant for a heavy top quark. Its effect is to decrease the value of λ in the ultraviolet. Since the change is not proportional to λ, it can in fact drive λ to negative values beyond a certain energy. Naively, this implies a negative contribution to the potential, which destabilizes the theory: large field configurations become energetically favored, and the theory tumbles out of control. The standard model description becomes inconsistent above a certain scale. This yields the instability bound. Should the Higgs particle prove to be light, this bound means that something must happen to the standard model at that scale, perhaps in the form of new contributions to the renormalization group evolution appear, from particles not in the standard model. In the supersymmetric generalization of the standard model, for instance, new particles appear and λ is not a fundamental coupling constant, but rather the square of gauge coupling constants.

We can summarize these two bounds in one graph showing the scale at which new physics is expected as a function of the Higgs mass for a given value of the top quark mass, which we take to be 180 GeV.



We see that with a (low) Higgs mass of 100 GeV, the instability sets in around 1 TeV; on the other hand, a (large) Higgs mass of 300 GeV implies, through the triviality bound new physics around 10 TeV. This discussion makes it clear that knowledge of the Higgs mass is an important figure of merit for the scale at which new physics will appear. OF course, if the Higgs mass is between 130 and 200 GeV, this analysis does not require new physics below the Planck scale!

At low energy, the effective theory becomes $SU(3) \times U(1)_{EM}$, with only two gauge coupling constants. Their evolution is given by

$$\frac{dg_3}{dt} = \frac{g_3^3}{(4\pi)^2}[\frac{2}{3}(n_u + n_d) - 11] \ , \tag{6.51}$$

and

$$\frac{de}{dt} = \frac{e^3}{(4\pi)^2}[\frac{16}{9}n_u + \frac{4}{9}n_d + \frac{4}{3}n_l] \ , \tag{6.52}$$

where n_u, n_d, and n_l are the number of light fermions, up-like and down-like quarks, and leptons. The remaining parameters of the low energy theory run as well. In the Landau gauge, the vacuum expectation value of the Higgs field runs like its anomalous dimension, that is

$$\frac{d\ln v}{dt} = \frac{1}{16\pi^2}\left(\frac{9}{4}(\frac{1}{5}g_1^2 + g_2^2) - T\right) \ . \tag{6.53}$$

Finally, the fermion masses in the low energy theory evolve as

$$\frac{dm_i}{dt} = m_i\gamma_{(i)} \ , \qquad i = l, q \ , \tag{6.54}$$

where the l and q refer to a particular lepton or quark. At one-loop

$$\gamma_{(i)} = \gamma_{(i)}^{[QED]}\frac{e^2}{(4\pi)^2} + \gamma_{(i)}^{[QCD]}\frac{g_3^2}{(4\pi)^2} \ , \tag{6.55}$$

with one-loop values for fermions of electric charge $Q_{(i)}$

$$\gamma_{(i)}^{[QED]} = -6Q_i^2 \ , \qquad \gamma_{(l)}^{[QCD]} = 0 \ ; \qquad \gamma_{(q)}^{[QCD]} = -8 \ . \tag{6.56}$$

6.2.3 PROBLEMS

A. 1-) Assume that the standard model symmetry is broken by a Higgs field that transforms not as a doublet of the weak $SU(2)$, but according to some arbitrary represntation of weak isospin j. Derive the general formula for the tree-level value of the ρ parameter.

2-) Now assume that you have two Higgs, one is the standard weak iso doublet, and the other is an isotriplet. What constraints does the experimental value of the ρ parameter put on the ratio of their vacuum values? Neglect the effect of quantum corrections.

B. Show that when the standard Higgs doublet gets its vacuum value, the global $SO(4)$ symmetry is broken to an $SO(3)$ subgroup. Identify the surviving symmetry as the diagonal subgroup $SU(2)_{L+R}$. Show that in the limit $g_1 = 0$, the three gauge bosons form a degenerate triplet under that symmetry.

C. 1-) Keeping only the top quark Yukawa and the QCD coupling in the one-loop renormalization equations, find the location of the Landau pole for the τ lepton Yukawa coupling.

2-) Suppose a heavy τ were found. Derive an upper bound on its mass based on Landau pole arguments.

D. Assume one family of quarks and leptons, and neglect g_1 and g_2. Using the RG equations for both the top quark Yukawa coupling and the strong gauge coupling, show that the top Yukawa coupling has an *infrared fixed point*. Estimate its value, and discuss its significance. For reference, see B. Pendleton and G. G. Ross *Phys.Lett.* **98B** 291(1981), as well as C. T. Hill, *Phys. Rev.* **D24**, 691(1981).

E. Using one-loop expressions, plot the instability and triviality bounds for the measured value of the top quark mass. Suppose the Higgs particle weighs in at 110 GeV. At what energy scale does the standard model cease to be valid?

6.3 Higher Dimension Electroweak Operators

We now apply to the standard model what we have learned in organizing the QED quantum corrections. Our procedure was to find all possible invariant field combinations of a given dimension. In perturbation theory, all these combinations are generated in the effective quantum Lagrangian, suppressed by inverse powers of the electron mass, M_e^{d-4}, where d is the engineering dimension of the invariant. The coefficients in front of each combination are determined in perturbation theory, by calculating the appropriate Feynman diagrams.

The same technique also provides an elegant and efficient method to describe and organize the radiative structure of the standard model. It is of course more complicated simply because there are more symmetries to keep track of, and there is an essential complication we have not encountered in QED: the electroweak symmetry is spontaneously broken. In spite of these, the basic ideas we have introduced in our study of QED still apply, with some caveats we proceed to discuss.

6.3.1 Higgs Polynomials

The spontaneous breaking of the electroweak symmetry introduces some subtelty, as we can perform the analysis either in the broken or in the unbroken formulation of the theory. It is more economical to list invariants under the full electroweak symmetry, rather than under its broken remnants. However, invariants under the full symmetry will in general contain polynomials in the Higgs field, which must be evaluated in the electroweak vacuum. As a result, an infinite number of operators with arbitrarily high dimensions in the unbroken formulation can be expected to contribute to one operator of much lower dimension in the broken theory. Nevertheless, as long as we stick to identifying operators by their quantum numbers, this method provides a powerful way to identify interactions in the broken theory. This technique cannot be used for the perturbative calculation of the coefficients in front of the operators, since the false and true vacua are not perturbatively related. Calculations make sense only in the true electroweak vacuum. Since spontaneous breaking brings in another scale, v, masses of the particles will not necessarily appear in the effective Lagrangian as inverse powers (as in QED), but also as positive powers, logarithms, etc... .

To summarize, invariant interactions in the broken theory formulation with a given dimension can be generated either from electroweak invariants of the same dimension, and/or from invariants of higher dimensions that contain polynomial combinations of the Higgs doublet that do not vanish in the electroweak vacuum. The difference in dimension is the order of the Higgs polynomial. Fortunately, these polynomials have a limited set of electroweak quantum numbers, which keeps our method practical.

It is not very hard to list all possible Higgs polynomials which do not vanish in the electroweak vacuum. The first is of course the Higgs doublet itself (or its conjugate) which can be set equal to its vacuum value, v. Any combination of fields of dimension d that transforms with the conjugate quantum numbers of the Higgs doublet stems from a full invariant of dimension $d+1$.

There are several Higgs polynomials of second order. Of the two Higgs binomials with $Y=2$, the isoscalar combination $H^t\tau_2 H$ vanishes identically, since there is only one Higgs doublet. The second is the weak isovector

$$H^t\tau_2\vec{\tau}H \sim (\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1}^c)_2 \ , \tag{6.57}$$

where the first two entries refer to the Lorentz group, $SU(2)\times SU(2)$, the third to the weak isospin, the fourth to color, and the subscript is the hypercharge. The same combination with H replaced by $\overline{H}$ is the conjugate isovector with hypercharge -2. Full invariants of dimension $d+2$ that contain this polynomial yield interactions of dimension d along the electrically neutral component of this weak isotriplet. There are two $Y=0$ combinations,

$$H^\dagger H \sim (\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1}^c)_0 \; ; \qquad H^\dagger \vec{\tau} H \sim (\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1}^c)_0 \; , \tag{6.58}$$

The first, with no quantum numbers, plays no role in the listing of invariants, as it can appear in any power. The second is another isovector.

There are of course other polynomials quadratic in the Higgs doublet, but they vanish in the electroweak vacuum. For instance, by adding the covariant derivative acting on the Higgs doublet, we obtain a Lorentz vector polynomial

$$H^T \tau_2 \vec{\tau} \mathcal{D}_\mu H \sim (\mathbf{2},\mathbf{2};\mathbf{1}\oplus\mathbf{3},\mathbf{1}^c)_2 \; . \tag{6.59}$$

It vanishes in the Lorentz invariant electroweak vacuum. This vector polynomial does appear in higher dimension polynomials, coupled to another with conjugate quantum numbers. Evaluated in the electroweak theory, it gives rise to interactions that involve Higgs scalars.

The reader is encouraged to show that, with one Higgs doublet, there is only one new cubic Higgs polynomial with isospin $3/2$ and $Y = \pm 3$, and that there are no new quartic polynomials. Hence all higher order Higgs polynomials with electroweak vacuum values are made up of the combinations we have already listed, leaving polynomials with four possible quantum numbers

$$(\mathbf{1},\mathbf{1};\mathbf{2},\mathbf{1}^c)_{\pm 1} \; ; \; (\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1}^c)_{\pm 2} \; ; \; (\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1}^c)_0 \; ; \; (\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1}^c)_{\pm 3} \; , \tag{6.60}$$

together of course with their Kronecker products. This enables us to proceed with our main task: building electroweak-invariant polynomials of a given dimension, using the basic building blocks of the standard model: the left-handed Weyl fermions $f(d = \frac{3}{2})$, the Lorentz vector covariant derivatives $\mathcal{D}(\lceil = \infty)$, and the scalar Higgs doublet $H(d = 1)$.

6.3.2 Dimension-Five Interactions

We proceed to list those operators which are invariant under the full symmetry of the standard model, *as well as* those which have the quantum numbers of the Higgs polynomials with electroweak vacuum values. We start by discussing the invariants.

It is not difficult to enumerate all invariants with $d = 5$. Dimension-five invariants must necessarily contain one fermion bilinear: without fermions, the weak doublet Higgs must appear in pairs to conserve weak isospin, so that $d = 5$ combinations must contain either one or three covariant derivatives, which are not possible without losing Lorentz invariance. Hence the possible dimension-five invariants are restricted to the forms $ffHH$, $ff\mathcal{D}H$, and $ff\mathcal{D}\mathcal{D}$, where the covariant derivatives can act on any of the fields including themselves. Here f denote fermions of either chirality (f or $\bar{f}$).

It is useful to recall the Lorentz properties of Weyl fermion bilinears. The products of two left- or right-handed Weyl fermions transform as a linear combination of scalar and tensor, and the products of left and right fermions transforms as a vector and/or axial vector

$$f\ f \sim \bar{f}\ \bar{f} \sim (\mathbf{1}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{1})\ ; \qquad f\ \bar{f} \sim (\mathbf{2}, \mathbf{2})\ . \tag{6.61}$$

• Consider terms of the form $ffHH$. Lorentz invariance requires the ff combination to be a Lorentz scalar, with zero color triality; thus both f must be leptons of the same chirality. Also, the two Higgs combination must be an isotriplet, restricting each lepton to be isodoublet. The quantum numbers of the antisymmetrized product of two lepton doublets are

$$L_{(i}L_{j)} \sim (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{1}^c)^{(ij)}_{-2} \oplus (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1}^c)^{(ij)}_{-2}\ , \tag{6.62}$$

when symmetrizing over the family indices, and

$$L_{[i}L_{j]} \sim (\mathbf{3}, \mathbf{1}; \mathbf{3}, \mathbf{1}^c)^{[ij]}_{-2} \oplus (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}^c)^{[ij]}_{-2}\ , \tag{6.63}$$

when antisymmetrizing over the family indices. We have a match for the family-symmetric combination, and the dimension-five operator

$$L^T_{(i}\sigma_2\tau_2\vec{\tau}L_{j)} \cdot H^T\tau_2\vec{\tau}H\ , \tag{6.64}$$

is invariant under the gauge groups of the standard model. Unfortunately, it is not invariant under its global symmetries: it violates total and relative lepton numbers by two units. It is not generated in perturbation theory.

Any interaction forbidden only by global symmetries deserves further analysis. Evaluate in the electroweak vacuum, it yields $v^2\widehat{\nu}_{(i}\nu_{j)}$, which we recognize as *Majorana mass* terms for the neutrinos. This is an example of a dimension-five operator which produces a dimension-three operator in the electroweak vacuum. This analysis shows with hardly any calculation that the standard model neutrinos stay massless to all orders of perturbation theory, not because of its gauge symmetries, but only because of the global lepton number symmetries.

• There are no invariants of the form $ff\overline{H}H$ in the absence of standard model fermion bilinears with zero hypercharge.

• Consider terms of the form $ff\mathcal{D}H$. Simple invariance considerations restrict the fermion bilinear to be a color singlet or octet, a weak doublet, and a Lorentz vector. For quarks, the only possible combinations necessarily have color triality two. For leptons, the only Lorentz vector isodoublet combination has hypercharge ± 3. We conclude that there are no dimension-five invariants of this form.

• The last combination to consider is of the form $ff\mathcal{D}\mathcal{D}$. Each covariant derivative has the following quantum numbers

$$\mathcal{D}_\mu \sim (\mathbf{2}, \mathbf{2}; \mathbf{1} \oplus \mathbf{3}, \mathbf{1}^c \oplus \mathbf{8}^c)_0 \ . \tag{6.65}$$

Since the standard model fermion bilinears with zero hypercharge are Lorentz vectors, no dimension-five invariants of this form can exist.

We conclude that the *unbroken* standard model generates *no invariant dimension-five interactions*. How then did the dimension-five operators of QED come about? They are generated solely from higher dimension operators evaluated in the electroweak vacuum.

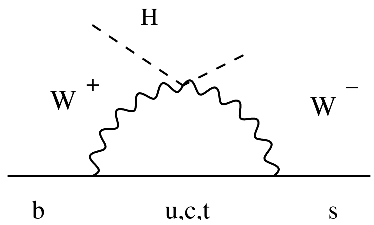
This shows that our classification is not complete, and we need to take into account the $d = 5$ combinations with the quantum numbers of Higgs polynomials that take electroweak vacuum values. Coupled with these Higgs polynomials, these produce standard model invariants of higher dimensions that reduce to $d = 5$ interactions in the electroweak vacuum. The dimension-five Higgs polynomial covariants can be several types:

• There are combinations without fermions, of the form $\mathcal{D}^2H^3$ and $\mathcal{D}^4H$. We leave it as an exercise to list the Higgs polynomial-covariants of that dimension.

• All combinations of the form $\bar{f}_i f_j \mathcal{D}H$, where $f_i = \mathbf{Q}_i$, $\overline{\mathbf{u}}_i$, $\overline{\mathbf{d}}_i$, L_i, or $\overline{e}_i$, where i, j are the family indices, can have the quantum numbers of the Higgs doublet. This follows because the combinations $\bar{f}_i f_j \mathcal{D}$ always contain electroweak singlets, since they can transform like (singlet) kinetic terms. They yield dimension-six invariants in the unbroken formulation when coupled to the conjugate Higgs. Lepton number conservation requires that $i = j$ for leptons; not so for quarks, leading to flavor-changing decays of the scalar Higgs. These interactions describe chirality-preserving emission and absorption of Higgs scalars from fermions. The flavor-changing processes (unlikely to ever be observed!) such as

$$\mathbf{s}^\dagger\sigma_\mu\overline{\mathbf{b}}\mathcal{D}_\mu H\ ; \qquad \overline{\mathbf{d}}^\dagger\sigma_\mu\overline{\mathbf{s}}\mathcal{D}_\mu H\ , \tag{6.66}$$

do not appear at tree level where the Higgs decay is flavor-diagonal. In the electroweak vacuum, they are generated by a diagram of the form,



where the cross on the Higgs line means that it is evaluated in the vacuum.

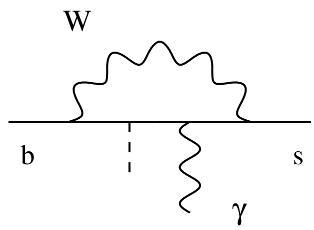
• Terms with one fermion pair and two covariant derivatives, $ff\mathcal{D}\mathcal{D}$, for which Lorentz invariance requires both fermions to be left- or right-handed, yield the operators

$$\mathcal{D}\mathcal{D}\mathbf{Q}_i\overline{\mathbf{u}}_j\ ; \qquad \mathcal{D}\mathcal{D}\mathbf{Q}_i\overline{\mathbf{d}}_j\ ; \qquad \mathcal{D}\mathcal{D}L_i\overline{e}_j\ , \tag{6.67}$$

all with the quantum numbers of the Higgs doublet: they are generated by at least $d = 6$ invariants in the unbroken formulation. Of special interest are the quark magnetic moment interactions

$$\mathbf{Q}_i\sigma_{\mu\nu}\lambda^A\overline{\mathbf{d}}_j\mathbf{G}^A_{\mu\nu}\ ; \qquad \mathbf{Q}_i\sigma_{\mu\nu}\tau^a\overline{\mathbf{d}}_jW^a_{\mu\nu}\ ; \qquad \mathbf{Q}_i\sigma_{\mu\nu}\overline{\mathbf{d}}_jB_{\mu\nu}\ , \tag{6.68}$$

and similar interactions with $\overline{\mathbf{d}}$ replaced by $\overline{\mathbf{u}}$. If $i \neq j$, they generate flavor-changing but charge-preserving interactions. Forbidden at tree-level, these processes provide a direct glimpse into the radiative corrections to the standard model. Evaluated in the electroweak vacuum, they describe rare interactions of the type gluon $\rightarrow \mathbf{s}\overline{\mathbf{d}}$, $Z \rightarrow \mathbf{s}\overline{\mathbf{d}}$, $\mathbf{b} \rightarrow \mathbf{s}\gamma$, or $\gamma \rightarrow \mathbf{s}\overline{\mathbf{d}}$, etc... . These processes occur at the one-loop level through diagrams like



Similar invariants with lepton pairs are allowed only if $i = j$ because of lepton number conservation. There are more subtle covariant combinations involving leptons, but they are forbidden by lepton number conservation. For example,

$$L_i^\dagger \sigma_\mu \overline{e}_j \mathcal{D}_\mu \vec{\tau} \overline{H} \ , \tag{6.69}$$

which transforms as an isovector with $Y = 2$, can be upgraded to a $d = 7$ invariant by adding the isovector Higgs binomial with $Y = -2$. The same remark applies to the weak isovector with $Y = -2$, generically of the form $\mathcal{D}\mathcal{D}L_i L_j$. Neither term is generated in perturbation theory.

6.3.3 Dimension-Six Interactions

They come in many different combinations, $\mathcal{D}^6$, $\mathcal{D}^4H^2$, $\mathcal{D}^2H^4$, H^6, ffH^3, $ff\mathcal{D}H^2$, $ff\mathcal{D}^2H$, $ff\mathcal{D}^3$, and $ffff$, not including operators linear in H, which we have already discussed.

• Invariants of the form $\mathcal{D}^6$ contain either three field strengths or two covariant derivatives of field strengths. In QED, it was not possible to form a symmetric invariant product of three Maxwell field strengths: the symmetric product of two field strengths is a symmetric second rank Lorentz tensor. In the standard model we can escape this restriction by antisymmetrizing on the group indices. This always produces the adjoint representation, leading to invariants of the form

$$\epsilon^{abc} W^a_{\mu\rho} W^b_{\rho\nu} W^c_{\nu\mu} \ ; \qquad f^{ABC} \mathbf{G}^A_{\mu\rho} \mathbf{G}^B_{\rho\nu} \mathbf{G}^C_{\nu\mu} \ , \tag{6.70}$$

involving $SU(2)$ and $SU(3)$ field strengths, respectively. The P and CP violating weak interactions can generate similar interactions with the one field strength replaced by its dual

$$\epsilon^{abc} W^a_{\mu\rho} W^b_{\rho\nu} \widetilde{W}^c_{\nu\mu} \ ; \qquad f^{ABC} \mathbf{G}^A_{\mu\rho} \mathbf{G}^B_{\rho\nu} \widetilde{\mathbf{G}}^C_{\nu\mu} \ , \tag{6.71}$$

There are other operators containing two field strengths

$$\mathcal{D}_\mu W^a_{\nu\rho} \mathcal{D}_\mu W^a_{\nu\rho} \ ; \qquad \partial_\mu B_{\nu\rho} \partial_\mu B_{\nu\rho} \ ; \qquad \mathcal{D}_\mu \mathbf{G}^A_{\nu\rho} \mathcal{D}_\mu \mathbf{G}^A_{\nu\rho} \ , \tag{6.72}$$

which, together with a permutation of the Lorentz indices, provide finite renormalizations to the kinetic terms, and to the higher order vertices. By taking dual field strengths, we generate new interactions

$$\mathcal{D}_\mu W^a_{\nu\rho}\mathcal{D}_\mu \widetilde{W}^a_{\nu\rho}\ ; \qquad \partial_\mu B_{\nu\rho}\partial_\mu \widetilde{B}_{\nu\rho}\ ; \qquad \mathcal{D}_\mu \mathbf{G}^A_{\nu\rho}\mathcal{D}_\mu \widetilde{\mathbf{G}}^A_{\nu\rho}\ . \tag{6.73}$$

• Invariants of the form $\mathcal{D}^4H^2$. Except for operators which are products of invariants, such as $H^\dagger H B_{\mu\nu}B_{\mu\nu}$, ..., we have the interesting interaction

$$H^\dagger\tau^a H W^a_{\mu\nu}B_{\mu\nu}\ . \tag{6.74}$$

As before there is a similar term with one dual field strength. Finally there are several types with one field strength and two Higgs derivatives

$$(\mathcal{D}_\mu H)^\dagger\mathcal{D}_\nu H B_{\mu\nu}\ ; \qquad (\mathcal{D}_\mu H)^\dagger\tau^a\mathcal{D}_\nu H W^a_{\mu\nu}\ , \tag{6.75}$$

and

$$(\mathcal{D}_\nu\mathcal{D}_\mu H)^\dagger\mathcal{D}_\nu\mathcal{D}_\mu H\ , \tag{6.76}$$

which describe interactions of gauge and Higgs fields.

• Invariants of the form $\mathcal{D}^2H^4$ are

$$H^\dagger H(\mathcal{D}_\mu H)^\dagger\mathcal{D}_\mu H\ ; \qquad H^\dagger\tau^a H(\mathcal{D}_\mu H)^\dagger\tau^a\mathcal{D}_\mu H\ . \tag{6.77}$$

Other interactions of this type with a different distribution of the weak isospin indices can be obtained from these through $SU(2)$ Fierz transformations.

• Fermion bilinears in invariants of the form $ff\mathcal{D}HH$ are zero triality Lorentz vectors and/or axial vectors. We have already analyzed terms of the form $f_i\overline{f}_j\mathcal{D}H\overline{H}$. An interesting invariant is

$$\overline{\mathbf{d}}^\dagger_i\sigma_\mu\overline{\mathbf{u}}_j H^T\tau_2\mathcal{D}_\mu H\ . \tag{6.78}$$

A similar operator

$$\overline{e}^\dagger_j\sigma_\mu L_i H^T\tau_2\mathcal{D}_\mu H\ , \tag{6.79}$$

with the quantum numbers of the Higgs doublet, will be generated in a term of dimension-seven.

• Invariants of the form $ff\mathcal{D}\mathcal{D}H$ contain only fermions of the same chirality, one being a weak doublet. These are the terms we encountered in constructing dimension-five invariants. They are

$$\mathbf{Q}_i\overline{\mathbf{u}}_j\mathcal{D}\mathcal{D}H\ ; \qquad \mathbf{Q}_i\overline{\mathbf{d}}_j\mathcal{D}\mathcal{D}\overline{H}\ ; \qquad L_i\overline{e}_j\mathcal{D}\mathcal{D}\overline{H}\ . \tag{6.80}$$

They look like Yukawa terms but can also couple to field strengths. They are especially relevant in the quark sector when those with different family indices break the tree-level flavor symmetry of the gauge interactions.

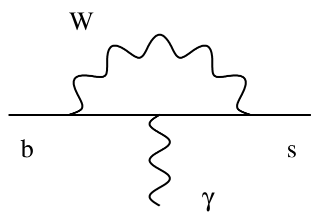
• Invariants of the form $ff\mathcal{DDD}$ exist when the fermion bilinear transforms as Lorentz vector and/or axial vector. They must also have zero triality and no hypercharge. The bilinears must therefore be of the form $\overline{f}_i f_j$. The quark family index can be different, leading to flavor-changing charge-preserving interactions. Some interesting examples where two covariant derivatives form into one field strength are

$$\mathbf{Q}_i^\dagger \sigma_\mu \mathbf{Q}_j \mathcal{D}_\rho B_{\rho\mu} \ ; \ \mathbf{Q}_i^\dagger \sigma_\mu \tau^a \mathbf{Q}_j (\mathcal{D}_\rho \cdot W_{\rho\mu})^a \ ; \ \mathbf{Q}_i^\dagger \sigma_\mu \lambda^A \mathbf{Q}_j (\mathcal{D}_\rho \cdot \mathbf{G}_{\rho\mu})^A \ , \quad (6.81)$$

which involve only the quark doublets. We also have

$$\overline{\mathbf{d}}_i^\dagger \sigma_\mu \overline{\mathbf{d}}_j \mathcal{D}_\rho B_{\rho\mu} \ ; \qquad \overline{\mathbf{d}}_i \sigma_\mu \lambda^A \overline{\mathbf{d}}_j (\mathcal{D}_\rho \cdot \mathbf{G}_{\rho\mu})^A \ , \quad (6.82)$$

and others with $\overline{\mathbf{d}}$ replaced by $\overline{\mathbf{u}}$. At the one-loop level, these are generated through chirality-preserving diagrams like



They are called penguin diagrams. With different quark flavors, they describe flavor-changing emission of gluons, photons, and decays of the Z boson. Note that we have already encountered flavor-changing interactions of this type, but they were chirality-changing of the magnetic moment variety, induced by the Higgs vacuum value.

• As expected, gauge invariants of the form $ffff$ have a much richer structure than in QED. Some, which do not violate the tree-level partial symmetries provide finite corrections to tree-level processes. Others, which do not respect the same symmetries, lead to new processes, and allow direct measurements of radiative corrections. Fortunately, many of these operators violate lepton and baryon numbers, and do not appear in the effective Lagrangian.

We can build Lorentz-invariants of this type in two ways. One is $(f\ f)\cdot(f\ f)$, and its conjugate, with bilinears forming scalar and tensor combinations. The other is of the form $(f\ f)\cdot(\overline{f}\ \overline{f})$, with each fermion bilinear in a scalar combination. Other possible invariants, for instance with each bilinear transforming as vector and axial vectors, is Fierz-equivalent to the above. It follows that all invariants can be assembled by first forming the product of any two of the (left-handed) fermion fields of the standard model, $\mathbf{Q}_i$, $\overline{\mathbf{u}}_i$, $\overline{\mathbf{d}}_i$, L_i, or $\overline{e}_i$, and then by contracting them with either themselves or their conjugates. This construction is simplified by assembling the fermion pairs in terms of their triality, hypercharge, baryon and lepton numbers.

– There are 15 different types of fermion bilinears. Multiplied with their conjugates, they yield fifteen types of four-fermion interactions. Some examples are

$$(\mathbf{Q}_i\mathbf{Q}_j)\cdot(\mathbf{Q}_k^\dagger\mathbf{Q}_l^\dagger)\ , \qquad (\overline{\mathbf{u}}_i\overline{\mathbf{u}}_j)\cdot(\overline{\mathbf{u}}_k^\dagger\overline{\mathbf{u}}_l^\dagger)\ \ldots . \tag{6.83}$$

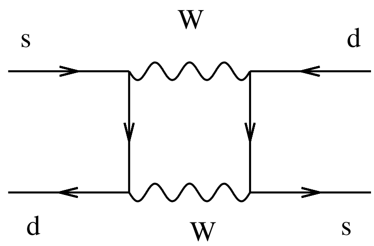
– There are only three types of bilinears with $\Delta B = \Delta L = 0$

$$\mathbf{Q}_i\overline{\mathbf{u}}_j\ , \qquad \mathbf{Q}_i\overline{\mathbf{d}}_j\ , \qquad L_i\overline{e}_i\ . \tag{6.84}$$

Multiplied together, they generate four-fermion interactions that preserve lepton and baryon numbers, but can cause flavor-changing interactions. They enter the standard model effective Lagrangian as

$$\mathbf{Q}_i\mathbf{Q}_j\overline{\mathbf{d}}_k\overline{\mathbf{u}}_l\ ; \qquad \mathbf{Q}_i\overline{\mathbf{u}}_jL_k\overline{e}_l\ ; \qquad \mathbf{Q}_i\overline{\mathbf{d}}_j\overline{L}_ke_l\ . \tag{6.85}$$

They are generated in one-loop order through box diagrams like



which can violate flavor. This diagram cause charge-preserving interactions that violate strangeness by two units.

Although not generated in the standard model, invariants that violate B

and L, but preserve $B-L$ are of interest in many of its extensions. Some of these are, $\mathbf{Q}_i\mathbf{Q}_j\mathbf{Q}_k L_l$, $\mathbf{Q}_i\mathbf{Q}_j\overline{\mathbf{u}}_k^\dagger\overline{e}_l^\dagger$, $\mathbf{Q}_i L_j\overline{\mathbf{u}}_k^\dagger\overline{\mathbf{d}}_l^\dagger$, and $\mathbf{u}_i\mathbf{u}_j\overline{\mathbf{d}}_k^\dagger\overline{e}_l^\dagger$.

Finally, we must also consider four-fermion covariants that contain Higgs polynomials with electroweak vacuum values. To that effect, we list the $Y = 1, 2, 3$ combinations that yield invariants when multiplied by the appropriate Higgs polynomial.

– We do not show the $Y = 1$ combinations since all can be shown to violate either lepton and/or baryon number.

– There are six $Y = \pm 2$ combinations which do not violate any global symmetries:

$$\begin{aligned} &\mathbf{Q}_i\overline{\mathbf{u}}_j L_k^\dagger\overline{e}_l^\dagger \ , \qquad L_i L_j\overline{e}_k\overline{e}_l \ , \qquad \mathbf{Q}_i\overline{\mathbf{d}}_j L_k\overline{e}_l \ , \\ &\overline{\mathbf{u}}_i\overline{\mathbf{u}}_j\overline{\mathbf{u}}_k^\dagger\overline{\mathbf{d}}_l^\dagger \ , \quad \overline{\mathbf{d}}_i\overline{\mathbf{d}}_j\overline{\mathbf{u}}_k^\dagger\overline{\mathbf{d}}_l^\dagger \ , \quad \mathbf{Q}_i\mathbf{Q}_j\overline{\mathbf{u}}_k\overline{\mathbf{u}}_l \ , \quad \mathbf{Q}_i\mathbf{Q}_j\overline{\mathbf{d}}_k\overline{\mathbf{d}}_l \ . \end{aligned} \tag{6.86}$$

There is in addition one combination with $\Delta B = \Delta L = 1$, of the form $\mathbf{Q}_i\mathbf{Q}_j\overline{\mathbf{d}}_k\overline{\mathbf{d}}_l$. Finally, all the $Y = 3$ four-fermion combinations violate the global symmetries.

The effective Lagrangian contains interactions of arbitrarily high dimensions. Fortunately, we need not list invariants of higher dimensions, as those already listed are sufficient for a thorough discussion of the lowest order radiative structure of the standard model.

6.3.4 PROBLEMS

A. Find the representations contained in the product of two antisymmetric second rank tensors, each transforming as $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ of the Lorentz group. Use the result to verify the list of all possible invariants built out of three field strengths presented in the text.

B. Show that with one Higgs doublet, there is only one new cubic Higgs polynomial with $Y = \pm 3$, and no new quartic polynomial. Deduce that all higher order Higgs polynomials that do not vanish in the vacuum are made up of the combinations already listed.

C. Evaluate the operators with two Higgs and one field strength in the electroweak vacuum, and interpret the results.

D. Find the quantum numbers of the symmetric and antisymmetric product of two covariant derivatives in the standard model. Interpret the results physically.

E. Evaluate the mixed interaction (6.74) in the electroweak vacuum. Show that it apparently mixes the photon and the Z boson. What do you conclude?

F. Show that the list of operators with four Higgs and two covariant derivatives is complete, up to integrations by parts and Fierz transformations.