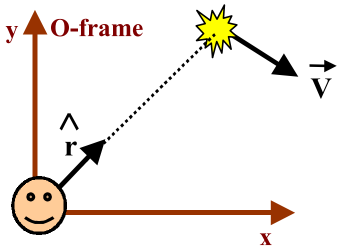
Relativistic Doppler Shift (general case)



Consider an observer at rest in the O-frame and a light source moving with velocity $\vec{\beta} = \vec{V}/c$. Then the observed period, wavelength, and frequency are given by

Relativistic Doppler Shift!

$$T = \gamma(1 + \vec{\beta}\cdot\hat{r})T_0$$
$$\lambda = \gamma(1 + \vec{\beta}\cdot\hat{r})\lambda_0$$
$$f = \frac{f_0}{\gamma(1 + \vec{\beta}\cdot\hat{r})}$$

Classical Doppler Shift!

$$T = (1 + \vec{\beta}\cdot\hat{r})T_0$$
$$\lambda = (1 + \vec{\beta}\cdot\hat{r})\lambda_0$$
$$f = \frac{f_0}{(1 + \vec{\beta}\cdot\hat{r})}$$

with $\vec{\beta} = \vec{V}/c \quad \gamma = 1/\sqrt{1-\beta^2}$ and where T_0, λ_0, and f_0 are the period, wavelength, and frequency of the light in the frame at rest with the source.

Case I "away" ($\vec{\beta}\cdot\hat{r} = 1$):

$$\lambda_{away} = \gamma(1+\beta)\lambda_0 \qquad f_{away} = \frac{f_0}{\gamma(1+\beta)}$$

Case II "toward" ($\vec{\beta}\cdot\hat{r} = -1$):

$$\lambda_{toward} = \gamma(1-\beta)\lambda_0 \qquad f_{toward} = \frac{f_0}{\gamma(1-\beta)}$$

Case III "transverse" ($\vec{\beta}\cdot\hat{r} = 0$):

$$\lambda_{transverse} = \gamma\lambda_0 \qquad f_{transverse} = \frac{f_0}{\gamma}$$

Transverse Doppler Shift!