Overview of Perturbative QCD

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Outline

- My Story of QCD.
- FF1: The Black Box Model.
- FF2: The QCD Improved Parton Model.
- Talk at ICHEP78 Tokyo and lectures at Boulder79.
- The QCD Improved Parton Model: Drell-Yan Lepton-Pair Production
- The QCD Improved Parton Model: Large Transverse Momentum Mesons in Hadron-Hadron Collisions.
- The QCD Improved Parton Model: Jet Production in Hadron-Hadron Collisions.
- From 7 GeV π^0’s to 2 TeV Jets.
My Story of QCD

The Quark Model
The Eightfold Way

Before we knew the partons were quarks and gluons!
Non-interacting partons!

The Parton Model
My Story of QCD

The Quark Model

The Eightfold Way

SU(3)$_{\text{flavor}}$ Triplet

Before we knew the partons were quarks and gluons!
Non-interacting partons!

The Parton Model

Partons = quarks, anti-quarks, and glue!
Still non-interacting partons!

The Naïve Parton Model

Proton momentum $P$ -> Partons

The QCD Improved Parton Model

Parton Model + Perturbative QCD!
Interacting quarks, anti-quarks, and gluons!

QCD Theory!
Toward and Understanding of Hadron-Hadron Collisions

Feynman-Field Phenomenology

From 7 GeV/c $\pi^0$’s to 2 TeV Jets.
The early days of trying to understand and simulate hadron-hadron collisions.
The Feynman-Field Days

1973-1983


The Feynman-Field Days

1973-1983


My 1st graduate student!
The Feynman-Field Days  

1973-1983


My 1st graduate student!
**The Naïve Parton Model**

**Electron-Positron Annihilations:**

\[ e^- + e^+ \rightarrow X \]

\[ e^- + e^+ \rightarrow \gamma^* \rightarrow q + \bar{q} \]

Born Term = \( \hat{t} \rightarrow \gamma^* \)

**Inelastic Electron Proton Scattering:**

\[ e^- + p \rightarrow e^- + X \]

\[ e^- + q \rightarrow e^- + q \]

Born Term = \( \hat{t} \rightarrow \gamma^* \)

**Quarks interacting with photons**

(electromagnetic interactions)

Virtual Photon
Naïve Parton Model
Electron-Positron Annihilations

Count the Number of Quark Flavors: Measure the ratio $R$ and verify that there are indeed three colors of quarks.

3 Flavors

$$3 \sum_{i=1}^{3} e_{q_i}^2 = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right)$$

$$= 3 \left( \frac{6}{9} \right) = 2$$
**Naïve Parton Model**

**Electron-Positron Annihilations**

**Count the Number of Quark Flavors:** Measure the ratio $R$ and verify that there are indeed three colors of quarks.

The data are from ORSAY, FEASCATI, NOVOSIBIRSK, SLAC-LBL, DASP, CLEO, DHHM, CELLO, JADE, MARK J, PLUTO, and TASSO. Compiled by B. Wiik (1978).

\[
5 \text{ Flavors: } 3 \sum_{i=1}^{5} e_{q_i}^2 = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = 3\left(\frac{11}{9}\right) = \frac{11}{3}
\]
Quark Fragmentation Functions:
Measure the probability of finding a hadron of type $h$ carrying the fraction $z$ of the parent quarks momentum.

$$e^+ + e^- \rightarrow h + X$$

$$D_{q \rightarrow h}^{(0)}(z)$$

Very unlikely that a parton will give all its momentum to a single hadron!
Naïve Parton Model
Electron-Positron Annihilations

Quark Fragmentation Functions:
Measure the probability of finding a hadron of type \( h \) carrying the fraction \( z \) of the parent quarks momentum.

\[
e^+ + e^- \rightarrow h + X
\]

Very unlikely that a parton will give all its momentum to a single hadron!

Observerved by the JADE detector at PETRA at \( E_{CM} = 30 \text{ GeV} \) (1978).
**Parton Model: DIS**

**Parton Model:** Virtual photon with 4-momentum \( q \) interacting with a parton of **type** \( i \) with mass \( m \) and electric charge \( e_i \) carrying fraction \( \xi \) of the proton’s 4-momentum \( P \). \( (M = \) proton mass) \[
m^2 = (\vec{p}')^2 = (\vec{p} + \vec{q})^2 = (\xi\vec{P} + \vec{q})^2 = \xi^2 \vec{P}^2 + \vec{q}^2 + 2\xi \vec{P} \cdot \vec{q} = \xi^2 M^2 - Q^2 + 2\xi MV \]

\[
\frac{2\xi MV}{Q^2} = 1 + \frac{(m^2 - \xi^2 M^2)}{Q^2} \quad \text{as } Q^2 \gg 1 \quad \xi = \frac{Q^2}{2MV} = x
\]

Provided \( Q^2 \) is large!

**Deep Inelastic Scattering (DIS)**

A fast moving proton is a collection of **partons** (constituents of the proton) each carrying a certain fraction \( \xi = x \) of the proton momentum, \( P \).

\[ G^{(0)}_{A \rightarrow i}(\xi) \]

is the **number of partons** of **type** \( i \) within a fast moving **hadron of type** \( A \) with fraction of momentum \( \xi \) (\( p_i = \xi P \)) between \( \xi \) and \( \xi + d\xi \).

**Scaling:** Predict that the DIS structure functions are not a function of both \( v \) and \( Q^2 \), but are simply a function of the parton scaling variable, \( x \).

\[
F^{ep}_{2}(x, Q^2) \rightarrow F^{ep}_{2}(x) = \sum_{i} e_i^2 x G^{(0)}_{p \rightarrow i}(x)
\]

Also predict: \( R^{DIS}(x, Q^2) \rightarrow 0 \) \( Q^2 \gg M^2 \)

**Momentum Sum Rule:** The sum of the momentum of all the constituents must equal one.

\[
\sum_{\text{All Partons}} \int_{0}^{1} x G^{(0)}_{A \rightarrow i}(x) dx = 1
\]
**Parton Model: DIS**

**Parton Model:** A virtual photon with momentum \( q \) interacting with a parton of type \( i \) with mass \( m \) and electric charge \( e_i \) carrying fraction \( \xi \) of the proton’s 4-momentum \( P \). \( M = \text{proton mass} \)

\[
\xi^2 M^2 - Q^2 + 2 \xi M \nu = \xi^2 M^2 - Q^2 + 2 \xi M \nu
\]

Provided \( Q^2 \) is large!

Deep Inelastic Scattering (DIS)

\[
\sum \rightarrow \nu + e^+ e^- 
\]

**In the early days of the Parton Model we did not know the partons were quarks, anti-quarks, and gluons!**

We learned this from DIS and \( e^+ e^- \) experiments!

**Scaling:** Note that the DIS structure functions are not a function of both \( \nu \) and \( Q^2 \), but are simply a function of the parton scaling variable, \( x \).

\[
F_2^{ep} (x, Q^2) \rightarrow F_2^{ep} (x, Q^2) \rightarrow F_2^{ep} (x, Q^2)
\]

**Momentum Sum Rule:** The sum of the momentum of all the constituents must equal one.

\[
\sum_{\text{All Partons}} \int x F_{2i} (x) dx = 1
\]
A fast moving proton is a collection of partons (quarks, anti-quarks, and gluons) each carrying a certain fraction $x$ of the proton momentum, $P$.

**Inelastic Electron-Proton Scattering:**

$$e^- + p \rightarrow e^- + X$$

$$F_2^{ep}(x) = \frac{4}{9} x \left( G_{p\rightarrow u}^{(0)}(x) + G_{p\rightarrow \bar{u}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p\rightarrow d}^{(0)}(x) + G_{p\rightarrow \bar{d}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p\rightarrow s}^{(0)}(x) + G_{p\rightarrow \bar{s}}^{(0)}(x) \right) + \ldots$$

**Net Number of Quarks:** The net number of $u$ quarks in a proton is 2 and the net number of $d$ quarks is 1.

$$\int_0^1 \left( G_{p\rightarrow u}^{(0)}(x) - G_{p\rightarrow \bar{u}}^{(0)}(x) \right) dx = 2 \quad \int_0^1 \left( G_{p\rightarrow d}^{(0)}(x) - G_{p\rightarrow \bar{d}}^{(0)}(x) \right) dx = 1 \quad \int_0^1 \left( G_{p\rightarrow s}^{(0)}(x) - G_{p\rightarrow \bar{s}}^{(0)}(x) \right) dx = 0$$

**DIS Experiments (1972-1975):** Observe approximate scaling and measure quark distributions.

Find that only about one-half of the proton momentum is carried by the charged quarks:

$$\sum_{i=1}^{n_f} \int_0^1 x \left( G_{p\rightarrow q_i}^{(0)}(x) + G_{p\rightarrow \bar{q}_i}^{(0)}(x) \right) dx \approx 0.5$$

The remaining momentum must be carried by electrically neutral partons (i.e. gluons).
Quarks have fractional electric charge: By comparing deep inelastic electron-proton scattering with deep inelastic neutrino-proton scattering one can determine the electric charge of the quarks.

Inelastic Electron-Proton Scattering:  $e^- + p \rightarrow e^- + X$

$$F_2^{ep} (x) = \frac{4}{9} x \left( G_{p \rightarrow u}^{(0)} (x) + G_{p \rightarrow \bar{u}}^{(0)} (x) \right) + \frac{1}{9} x \left( G_{p \rightarrow d}^{(0)} (x) + G_{p \rightarrow \bar{d}}^{(0)} (x) \right)$$

$$+ \frac{1}{9} x \left( G_{p \rightarrow s}^{(0)} (x) + G_{p \rightarrow \bar{s}}^{(0)} (x) \right) + \ldots$$

Inelastic Neutrino-Proton Scattering:  $\nu_e + p \rightarrow e^- + X$

$$F_2^{vp} (x) = 2 x \left( G_{p \rightarrow u}^{(0)} (x) + G_{p \rightarrow \bar{u}}^{(0)} (x) \right) + 2 x \left( G_{p \rightarrow d}^{(0)} (x) + G_{p \rightarrow \bar{d}}^{(0)} (x) \right)$$

$$+ 2 x \left( G_{p \rightarrow s}^{(0)} (x) + G_{p \rightarrow \bar{s}}^{(0)} (x) \right) + \ldots$$
**Naïve Parton Model: DIS**

**Quarks have fractional electric charge:** By comparing deep inelastic electron-proton scattering with deep inelastic neutrino-proton scattering one can determine the electric charge of the quarks.

**Inelastic Electron-Proton Scattering:**

\[
F_2^{ep}(x) = \frac{4}{9} x G^{(0)}_{p\rightarrow s}(x) + \frac{1}{9} x G^{(0)}_{p\rightarrow s}(x) + 2 x G^{(0)}_{p\rightarrow s}(x)
\]

**Inelastic Neutrino-Proton Scattering:**

\[
F_2^{vp}(x) = 2 x G^{(0)}_{p\rightarrow s}(x) + 2 x G^{(0)}_{p\rightarrow s}(x) + 2 x G^{(0)}_{p\rightarrow s}(x)
\]
The Naïve Parton Model

**Inelastic Neutrino Proton Scattering:**
\[ \nu_e + p \to e^- + X \]

**Born Term:**
\[ \nu_e + q \to e^- + q' \]

**Quarks interacting with W & Z**
(weak interactions)

**Drell-Yan Muon-Pair Production:**
\[ p + p \to \mu^+ \mu^- + X \]

**Born Term:**
\[ q + \bar{q} \to \mu^+ + \mu^- \]

**Quarks interacting with photons**
(electromagnetic interactions)
Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Quark-Quark Elastic Scattering

\[ q + q \rightarrow q + q \]

2-to-2 Scattering

Born Term = \[ \hat{t} \rightarrow \]

Quarks interacting with quarks (strong interactions)

Field-Feynman Black-Box Model

What do I do now??
Hadron-Hadron Collisions

- What happens when two hadrons collide at high energy?

- Most of the time the hadrons ooze through each other and fall apart (i.e. no hard scattering). The outgoing particles continue in roughly the same direction as initial proton and antiproton.

- Occasionally there will be a large transverse momentum meson. Question: Where did it come from?

- We assumed it came from quark-quark elastic scattering, but we did not know how to calculate it!
Hadron-Hadron Collisions

- What happens when two hadrons collide at high energy?

- Most of the time the hadrons ooze through each other (no hard scattering). The outgoing particles continue in roughly the same direction as initial proton and antiproton.

- Occasionally there will be a large transverse momentum meson.

**Question:** Where did it come from?

- We assumed it came from quark-quark elastic scattering, but we did not know how to calculate it!

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**FF1 1977**

**Feynman quote from FF1**

“The model we shall choose is not a popular one, so that we will not duplicate too much of the work of others who are similarly analyzing various models (e.g. constituent interchange model, multiperipheral models, etc.). We shall assume that the high $P_T$ particles arise from direct hard collisions between constituent quarks in the incoming particles, which fragment or cascade down into several hadrons.”

---

“Black-Box Model”
Quark Distribution Functions determined from deep-inelastic lepton-hadron collisions

Quark Fragmentation Functions determined from e^+e^- annihilations

Quark-Quark Cross-Section Unknown! Determined from hadron-hadron collisions.

FF1 1977

No gluons!
Quark Distribution Functions
determined from deep-inelastic
lepton-hadron collisions

Quark Fragmentation Functions
determined from e^+e^- annihilations

**Feynman quote from FF1**
“Because of the incomplete knowledge of
our functions some things can be predicted
with more certainty than others. Those
experimental results that are not well
predicted can be “used up” to determine
these functions in greater detail to permit
better predictions of further experiments.
Our papers will be a bit long because we
wish to discuss this interplay in detail.”
Quark-Quark Black-Box Model

**Predict**

- particle ratios

**FF1 1977**

- Predict increase with increasing CM energy $W$

- Predict overall event topology (FFF1 paper 1977)

- 7 GeV/c $\pi^0$'s!
Quark-Quark Black-Box Model

**Predict**
- Particle ratios

**FF1 1977**
- Predict increase with increasing CM energy W
- The beginning of the “underlying event”!
- 7 GeV/c π0’s!

**Predict**
- Overall event topology (FFF1 paper 1977)

“Beam-Beam Remnants”
QCD Approach: Quarks & Gluons

Parton Distribution Functions
Q^2 dependence predicted from QCD

Quark & Gluon Cross-Sections
Calculated from QCD
QCD Approach: Quarks & Gluons

Quark & Gluon Fragmentation Functions
Q^2 dependence predicted from QCD

Parton Distribution Functions
Q^2 dependence predicted from QCD

Quark & Gluon Cross-Sections
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QCD Approach: Quarks & Gluons

Parton Distribution Functions
Q² dependence predicted from QCD

Quark & Gluon Fragmentation Functions
Q² dependence predicted from QCD

Quark & Gluon Cross-Sections
Calculated from QCD

Feynman quote from FFF2
“We investigate whether the present experimental behavior of mesons with large transverse momentum in hadron-hadron collisions is consistent with the theory of quantum-chromodynamics (QCD) with asymptotic freedom, at least as the theory is now partially understood.”
Dynamics of High Energy Reactions

Rick Field
California Institute of Technology

(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)

The QCD Parton Model Approach - Outline of Talk

- The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale $\Lambda$.

- Quark and Gluon Distributions within Hadrons: Scale Breaking.
  - Analysis of ep and $\mu p$ Data.
  - Analysis of $F_2$ and $xF_3$ in Neutrino Processes.

- Muon-Pair Production in pp Collisions.
  - QCD Factorization – “Constant” Pieces.
  - Large $p_T$ Muon-Pair Production.
  - “Scaling” in $pp \rightarrow \mu^+ \mu^- + X$.

- Quark and Gluon Fragmentation Functions: Scale Breaking.
  - Gluon Jets.

- Large $p_T$ Production of Mesons and Jets in pp Collisions.
  - Scale Breaking Effects: $Ed\sigma/d^3p$.
  - Correlations – Evidence for Gluons.
  - The Jet Cross Section.

- A Look to the Future.
  - $pp \rightarrow \pi^0 + X$ at $W = 500$ GeV.
  - Large $p_T$ Charm Production.
  - $\gamma \gamma \rightarrow$ Jet + Jet and $e^+e^- \rightarrow e^+e^- +$ Jet + Jet.
  - Three Jets: $e^+e^- \rightarrow q + q\bar{q} +$ Jet and $pp \rightarrow$ Jet + Jet + Jet + X.
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  - $pp \rightarrow \pi^0 + X$ at $W = 500$ GeV.
  - Large $p_T$ Charm Production.
  - $\gamma\gamma \rightarrow$ Jet + Jet and $e^+e^- \rightarrow e^+e^- +$ Jet + Jet.
  - Three Jets: $e^+e^- \rightarrow q+q\bar{q}+\text{Jet and pp} \rightarrow \text{Jet+Jet+Jet}+X$.
The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale $\Lambda$.

Quark and Gluon Distributions within Hadrons: Scale Breaking.
- Analysis of $ep$ and $\mu p$ Data.
- Analysis of $F_2$ and $xF_3$ in Neutrino Processes.

Muon-Pair Production in $pp$ Collisions:
- QCD Factorization – "Constant" Pieces.
- Large $p_T$ Muon-Pair Production.
- "Scaling" in $pp \rightarrow \mu^+\mu^- + X$.

The QCD Parton Model Approach
1. The effective strong interaction coupling $\alpha_s(Q^2)$ and the mass scale $\Lambda$.
2. Quark and Gluon Distributions within Hadrons: Scale Breaking.
   - Analysis of $ep$, $pp$ data.
   - Analysis of $F_2$, $xF_3$ in neutrino processes.
   - QCD factoring in "constant" pieces.
4. Large $p_T$ muon pair production.
5. Scaling in $pp \rightarrow \mu^+\mu^- + X$.
6. Quark and gluon fragmentation functions – scale breaking.
   - Gluon jets.
   - Scale breaking effects.
   - Correlations – evidence for gluons.
   - The jet cross section.
8. A Look to the Future.
   - $pp \rightarrow X$, extra jets.
   - Large $p_T$ charm production.
   - $\gamma \gamma \rightarrow$ Jet + Jet and $e^+e^- \rightarrow e^+e^- + \text{Jet} + \text{Jet}$.
   - Three Jets: $e^+e^- \rightarrow q\bar{q} + \text{Jet}$ and $pp \rightarrow \text{Jet} + \text{Jet} + \text{Jet} + X$.

Final Remarks.
QCD Improved Parton Model

Drell-Yan Muon-Pair Production

\[ p + p \rightarrow \mu^+ \mu^- + X \]

**Born Term**

\[ q + \bar{q} \rightarrow \mu^+ + \mu^- \]

**Born Amplitude**

\[ A_0 = \gamma^* \]

Include Leading Order QCD Corrections (Order \( \alpha_s \))

**Annihilation**

\[ q + \bar{q} \rightarrow \gamma^* + g \]

**Virtual Corrections**

\[ A_V = \gamma^* + g_\gamma + g_\gamma + g_\gamma \]

**Compton**

\[ q + g \rightarrow \gamma^* + q \]

**Compton**

\[ C_R = \gamma^* + g_\gamma + g_\gamma + g_\gamma \]
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative predictions. The “Compton” and “annihilation” contributions are given by the dashed and dotted curves, respectively.
QCD Improved Parton Model

Real Gluon Emissions Order $\alpha_s$

$$\sigma^D_Y (s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \frac{d\sigma^{A+B\rightarrow\gamma^*+X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2} (s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) + \frac{d\sigma^{A+B\rightarrow\gamma^*+X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2} (s, M_{\mu\mu}^2, y_{\mu\mu}, p_T)$$

Born Term Plus Virtual Gluon Emissions Order $\alpha_s$

$$\sigma^V_Y (s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{d\sigma^{A+B\rightarrow\gamma^*+X}}{dM_{\mu\mu}^2 dy_{\mu\mu}} (s, M_{\mu\mu}^2, y_{\mu\mu})$$

Total Cross-Section Order $\alpha_s$

$$\sigma^{DY}_{tot} (s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{d\sigma^{A+B\rightarrow\gamma^*+X}_{B+V}}{dM_{\mu\mu}^2 dy_{\mu\mu}} (s, M_{\mu\mu}^2, y_{\mu\mu}) + \int \sigma^D_Y (s, M_{\mu\mu}^2, y_{\mu\mu}, p_T^2) dp_T^2$$

Finite = $\alpha_s$ + Infinite

Total Cross-Section Leading Order

$$\sigma^{DY}_{tot-LO} (s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{d\sigma^{DY}_{tot-LO}}{dM_{\mu\mu}^2 dy_{\mu\mu}} (s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{4\pi\alpha^2_{em}}{9M_{\mu\mu}^2s} \sigma^{DIS}_{q\bar{q}} (x_a, x_b, M_{\mu\mu}^2)$$

$$\sigma^{DIS}_{q\bar{q}} (x_a, x_b, M_{\mu\mu}^2) = \sum_{i=1}^{nf} e_i^2 \left[ G^{DIS}_{A\rightarrow q_i} (x_a, M_{\mu\mu}^2) G^{DIS}_{B\rightarrow \bar{q}_i} (x_b, M_{\mu\mu}^2) + G^{DIS}_{A\rightarrow \bar{q}_i} (x_a, M_{\mu\mu}^2) G^{DIS}_{B\rightarrow q_i} (x_b, M_{\mu\mu}^2) \right]$$
Parton Intrinsic (Primordial) Transverse Momentum:

From the uncertainty principle one expects that the partons will have some intrinsic (primordial) transverse momentum.

\[ c\Delta p_x \approx \frac{\hbar c}{\Delta x} = \frac{197.3 \text{ MeV} \cdot \text{fm}}{1 \text{ fm}} \approx 200 \text{ MeV} \]

Expect intrinsic \( k_T(P \rightarrow q) \approx 200 – 300 \text{ MeV/c}! \)

Jet Size: Also expect that the hadrons within a jet will have some intrinsic transverse momentom.

Two Jet Final State: \( e^+e^- \) to Jet + Jet

Expect intrinsic \( k_T(q \rightarrow h) \approx 200 – 300 \text{ MeV/c}! \)
Primordial $k_T$ Smearing

\[
f(k_T^2) = \frac{1}{4\pi\sigma^2_{\text{primordial}}} e^{-\frac{k_T^2}{4\sigma^2_{\text{primordial}}}}
\]

\[
<k_T>_{\text{primordial}} = 2\sigma^2_{\text{primordial}}
\]

\[
\sigma^{DY}_{\text{smear}}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma_{\text{primordial}}) = \int f(k_T^2) [\sigma^{DY}_R (s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) + \sigma^{DY}_V (s, M^2_{\mu\mu}, y_{\mu\mu})\delta(q_T^2)] d^2k_T
\]

\[
\sigma^{DY}_{\text{smear}}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma_{\text{primordial}}) = \int \sigma^{DY}_R (s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) \left[ f(k_T^2) - f(p_T^2) \right] d^2k_T + f(p_T^2) \int \left[ \sigma^{DY}_R (s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) + \sigma^{DY}_V (s, M^2_{\mu\mu}, y_{\mu\mu})\delta(q_T^2) \right] d^2q_T
\]

\[
\sigma^{DY}_{\text{smear}}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma) = \int \sigma^{DY}_R (s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) \left[ f(k_T^2) - f(p_T^2) \right] d^2k_T + f(p_T^2) \sigma^{DY}_{\text{tot}} (s, M^2_{\mu\mu}, y_{\mu\mu})
\]

Finite

Finite
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle = 600$ MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle = 600$ MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.
Energy dependence of the large $p_T$ tail expected for $pp \rightarrow \mu^+\mu^- + X$ from the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $<k_T> = 600$ MeV.

Data on the energy dependence of the $<p_T>$ of the $\mu^+\mu^-$ pair in proton-nucleon collisions which indicate that the $p_T$ distribution is becoming broader as the energy increases.

Naïve Parton Model

A+B→h+X

External Variables (neglect the mass of A, B, and h)

\[ s = (\vec{P}_A + \vec{P}_B)^2 = 2\vec{P}_A \cdot \vec{P}_B \]
\[ t = (\vec{P}_h - \vec{P}_A)^2 = -2\vec{P}_h \cdot \vec{P}_A \]
\[ u = (\vec{P}_h - \vec{P}_B)^2 = -2\vec{P}_h \cdot \vec{P}_B \]
\[ x_1 = -u/s \]
\[ x_2 = -t/s \]
\[ x_T = 2p_T/W \]

Internal-External Connection

\[ \vec{p}_a = x_a \vec{P}_A \]
\[ \vec{p}_b = x_b \vec{P}_B \]
\[ \vec{p}_h = z_c \vec{p}_c \]

Internal Variables (neglect masses)

\[ \hat{s} = (\vec{p}_a + \vec{p}_b)^2 = 2\vec{p}_a \cdot \vec{p}_b = 2x_a x_b (\vec{P}_A \cdot \vec{P}_B) = x_a x_b s \]
\[ \hat{t} = (\vec{p}_c - \vec{p}_a)^2 = -2\vec{p}_c \cdot \vec{p}_a = -2x_a (\vec{P}_h \cdot \vec{P}_A) / z_c = x_a t / z_c \]
\[ \hat{u} = (\vec{p}_c - \vec{p}_b)^2 = -2\vec{p}_c \cdot \vec{p}_b = -2x_b (\vec{P}_h \cdot \vec{P}_B) / z_c = x_b u / z_c \]
\[ \hat{s} + \hat{t} + \hat{u} = 0 \rightarrow x_a x_b - \frac{x_a x_2}{z_c} - \frac{x_b x_1}{z_c} = 0 \]

\[ x_a = \frac{x_b x_1}{z_c x_a - x_2} \]
\[ x_b = \frac{x_a x_2}{z_c x_a - x_1} \]
\[ z_c = \frac{x_1 + x_2}{x_a x_b} \]
Naïve Parton Model

A+B→h+X

\[
W = E_{cm} = \sqrt{s} \quad x_T = \frac{2 p_T}{W}
\]

\[
t = -x_2 s = -\frac{1}{2} s x_T e^{-y}
\]

\[
u = -x_1 s = -\frac{1}{2} s x_T e^{+y}
\]

\[
x_1 \equiv -u/s = \frac{1}{2}(x_E + x_L) = \frac{1}{2} x_T e^{+y}
\]

\[
x_2 \equiv -t/s = \frac{1}{2}(x_E - x_L) = \frac{1}{2} x_T e^{-y}
\]

\[
d\sigma^{A+B\rightarrow h+X}(s,t) = G^{(0)}_{A \rightarrow a}(x_a) dx_a G^{(0)}_{B \rightarrow b}(x_b) dx_b \left( \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{dt} (\hat{s}, \hat{t}) \right) d\hat{t} D^{(0)}_{c\rightarrow h}(z_c) dz_c
\]

\[
d\hat{t} d\hat{z}_c = \frac{\partial (\hat{s}, \hat{t})}{\partial (x_1, x_2)} dx_1 dx_2 = s/z_c dx_1 dx_2 \quad dx_1 dx_2 = \frac{\partial (x_1, x_2)}{\partial (y, x_T)} dy dx_T = \frac{1}{2} x_T dy dx_T
\]

\[
E \frac{d\sigma^{A+B\rightarrow h+X}}{d^3 p}(s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} x_{a \min} x_{b \min}^{1.0} \int dx_a^{1.0} \int dx_b^{1.0} G^{(0)}_{A \rightarrow a}(x_a) G^{(0)}_{B \rightarrow b}(x_b) D^{(0)}_{c\rightarrow h}(z_c) \frac{1}{z_c} \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}} (\hat{s}, \hat{t})
\]

\[
x_a = \frac{x_b x_1}{z_c x_b - x_2} \quad x_b = 1 \quad x_a^{\min} = \frac{x_1}{1-x_2} \quad x_b^{\min} = \frac{x_a x_2}{x_a - x_1}
\]
QCD Parton-Parton Scattering
Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Include All The QCD 2-to-2 Parton-Parton Processes (Order \( \alpha_s^2 \))
Parton = quark, anti-quark, & gluon

\[
\frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2(Q^2)}{\hat{s}^2} |A^{a+b\rightarrow c+d}(\hat{s},\hat{t})|^2
\]

(1) \( q_i + q_j \rightarrow q_i + q_j \)
(2) \( q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j \)
(3) \( q_i + q_i \rightarrow q_i + q_i \)
(4) \( q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i \)
(5) \( q_i + \bar{q}_i \rightarrow g + g \)
(6) \( gg \rightarrow q_i + \bar{q}_i \)
(7) \( q_i + g \rightarrow q_i + g \)
(8) \( \bar{q}_i + g \rightarrow \bar{q}_i + g \)
(9) \( g + g \rightarrow g + g \)

TABLE I. Cross sections for the various constituent quark-quark, quark-gluon, and gluon-gluon subprocesses. The differential cross section is given by \( d\hat{\sigma}/d\hat{t} = \pi\alpha_s^2(Q^2) |A|^2/\hat{s}^2 \), where \( \alpha_s(Q^2) \) is the effective coupling given by Eq. (3.1).
Parton Model Scaling

\[ \hat{s} + \hat{t} + \hat{u} = 0 \quad \hat{p}_T^2 = \frac{\hat{t}u}{\hat{s}} \quad \hat{x}_T = 2 \hat{p}_T / \sqrt{\hat{s}} \quad \hat{s} = 4 \hat{p}_T^2 / \hat{x}_T^2 \]

\[ \hat{t} = -\frac{1}{2} \hat{s} \left( 1 - \sqrt{1 - \hat{x}_T^2} \right) \quad \hat{u} = -\frac{1}{2} \hat{s} \left( 1 + \sqrt{1 - \hat{x}_T^2} \right) \]

\[ \frac{d\hat{\sigma}}{d\hat{t}} (\hat{s}, \hat{t}) = \frac{4}{9} \pi \alpha_s^2 \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \frac{\hat{x}_T^2}{\hat{p}_T^4} \left( \frac{\hat{t}u}{\hat{s}} \right) \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \]

\[ \left( \frac{\hat{x}_T^2}{9} \right) \left( \frac{\hat{t}u}{\hat{s}} \right) \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \left( \frac{\hat{u}}{\hat{s}} \right) = \left( \frac{\hat{x}_T^2}{9} \right) \left( \frac{\hat{u}}{\hat{s}} \right) \left( 1 + \left( \frac{\hat{u}}{\hat{s}} \right)^2 \right) = f_i (\hat{x}_T) \]

\[ P_1^{(0)} (x_a, x_b, z_c) = \sum_{i \neq j} G_{A \rightarrow q_i}^{(0)} (x_a) G_{B \rightarrow q_j}^{(0)} (x_b) D_{q_i \rightarrow h}^{(0)} (z_c) \]

\[ p_T^4 E \frac{d\sigma_{A+B \rightarrow h+X}}{d^3 p} (s, p_T, \theta_{cm}) = \int_{x_a}^{x_{a_{\text{min}}}} dx_a \int_{x_b}^{x_{b_{\text{min}}}} dx_b P_1^{(0)} (x_a, x_b, z_c) \frac{\alpha_s^2}{z_c} f_i (x_a, x_b, x_T) = F_1 (x_T, \theta_{cm}) \]

\[ x_1 = \frac{1}{2} x_T / \tan(\theta_{cm} / 2) \quad x_a^{\text{min}} = \frac{x_1}{1 - x_2} \quad x_b = \frac{x_a x_2}{z_c x_a - x_1} \]

\[ x_2 = \frac{1}{2} x_T \tan(\theta_{cm} / 2) \quad x_b^{\text{min}} = \frac{x_a x_2}{x_a - x_1} \quad z_c = \frac{x_1 + x_2}{x_a x_b} \]

Quark-Quark Elastic Scattering

\[ A_{q_i + q_j \rightarrow q_i + q_j} (\hat{s}, \hat{t}) = \hat{t} \rightarrow g_s \quad \text{gluon} \]

Sum over final-state spins and color. Average over initial-state spins and color.

Parton Model Scaling

Constant at fixed \( \theta_{cm} \) and \( x_T \)!
(provided the PDF’s and the fragmentation functions scale and \( \alpha_s \) is constant)
Parton Model Scaling

Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)

\begin{align*}
1. & \quad q_i + q_j \rightarrow q_i + q_j \\
2. & \quad q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j \\
3. & \quad q_i + q_i \rightarrow q_i + q_i \\
4. & \quad q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i \\
5. & \quad q_i + \bar{q}_i \rightarrow g + g \\
6. & \quad gg \rightarrow q_i + \bar{q}_i \\
7. & \quad q_i + g \rightarrow q_i + g \\
8. & \quad \bar{q}_i + g \rightarrow \bar{q}_i + g \\
9. & \quad g + g \rightarrow g + g
\end{align*}

\[
\frac{d\hat{\sigma}_k}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2}{\hat{p}_T^4} f_k(\hat{x}_T) \quad k = 1 \text{ to } 9
\]

\[
\frac{p_T^4}{d^3 p^*} E \frac{\sigma_{A+B \rightarrow h+X}}{\text{all}} (s, p_T, \theta_{cm}) = \sum_{k=1}^{9} F_k(x_T, \theta_{cm}) \quad \text{Parton Model Scaling}
\]

\[
x_1 = \frac{1}{2} x_T \tan(\theta_{cm}/2) \quad x_2 = \frac{1}{2} x_T \tan(\theta_{cm}/2)
\]

\[
x_T = \frac{2 p_T}{\sqrt{s}}
\]

\[
x_a^\min = \frac{x_1}{1-x_2} \quad x_b = \frac{x_a x_2}{z c x_a - x_1}
\]

\[
x_b^\min = \frac{x_a x_2}{x_a - x_1} \quad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}
\]

Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)

\[
x_1 = \frac{1}{2} x_T \tan(\theta_{cm}/2) \quad x_2 = \frac{1}{2} x_T \tan(\theta_{cm}/2)
\]

\[
x_T = \frac{2 p_T}{\sqrt{s}}
\]

\[
x_a^\min = \frac{x_1}{1-x_2} \quad x_b = \frac{x_a x_2}{z c x_a - x_1}
\]

\[
x_b^\min = \frac{x_a x_2}{x_a - x_1} \quad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}
\]

\[
large \text{Large Transverse Meson Production in Hadron-Hadron Collisions}
\]

\[
\text{Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)}
\]

\[
1. q_i + q_j \rightarrow q_i + q_j \\
2. q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j \\
3. q_i + q_i \rightarrow q_i + q_i \\
4. q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i \\
5. q_i + \bar{q}_i \rightarrow g + g \\
6. gg \rightarrow q_i + \bar{q}_i \\
7. q_i + g \rightarrow q_i + g \\
8. \bar{q}_i + g \rightarrow \bar{q}_i + g \\
9. g + g \rightarrow g + g
\]

\[
\frac{d\hat{\sigma}_k}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2}{\hat{p}_T^4} f_k(\hat{x}_T) \quad k = 1 \text{ to } 9
\]

\[
\frac{p_T^4}{d^3 p^*} E \frac{\sigma_{A+B \rightarrow h+X}}{\text{all}} (s, p_T, \theta_{cm}) = \sum_{k=1}^{9} F_k(x_T, \theta_{cm}) \quad \text{Parton Model Scaling}
\]

\[
x_1 = \frac{1}{2} x_T \tan(\theta_{cm}/2) \quad x_2 = \frac{1}{2} x_T \tan(\theta_{cm}/2)
\]

\[
x_T = \frac{2 p_T}{\sqrt{s}}
\]

\[
x_a^\min = \frac{x_1}{1-x_2} \quad x_b = \frac{x_a x_2}{z c x_a - x_1}
\]

\[
x_b^\min = \frac{x_a x_2}{x_a - x_1} \quad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}
\]
High $p_T \pi$-Meson Production


The FNAL data (open circles) are from D. C. Carey et al., Fermilab Report FNAL-PUB-75120-EXP (1975).

The ISR data (solid dots) are from B. Alper et al., Nucl. Phys. B100 (1975).

The ISR data (crosses) are from F. W. Busser et al., Nucl. Phys. B106 (1976).

\[ W = \sqrt{s}, \quad x_T = \frac{2p_T}{\sqrt{s}}, \quad p_T = \frac{1}{2} x_T W \]

\[ W = 19.4 \text{ GeV} \quad x_T = 0.2 \]
\[ p_T = \frac{1}{2} (0.2)(19.2 \text{ GeV}) = 1.92 \text{ GeV}/c \]

\[ W = 53 \text{ GeV} \quad x_T = 0.2 \]
\[ p_T = \frac{1}{2} (0.2)(53 \text{ GeV}) = 5.3 \text{ GeV}/c \]
High $p_T$ $\pi$-Meson Production


The FNAL data (open circles) are from D. C. Carey et al., Fermilab Report FNAL-PUB-75120-EXP (1975).

The ISR data (solid dots) are from B. Alper et al., Nucl. Phys. B100 (1975).

The ISR data (crosses) are from F. W. Busser et al., Nucl. Phys. B106 (1976).

The data are behaving like $1/p_T^8$ not $1/p_T^4$ at fixed $x_T$.

$W = 53$ GeV $x_T = 0.2$

$p_T = \frac{1}{2} (0.2)(53 GeV) = 5.3 GeV/c$
FF1: Black-Box Model

\[ E \frac{d\sigma^{A+B \rightarrow h+X}}{d^3p}(s, p_T, \theta_{cm}) = \]

\[ \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\min}}^{1.0} dx_a \int_{x_b^{\min}}^{1.0} dx_b \mathcal{G}^{(0)}_{A \rightarrow a}(x_a) \mathcal{G}^{(0)}_{B \rightarrow b}(x_b) D^{(0)}_{c \rightarrow h}(z_c) \frac{1}{z_c} \frac{d\hat{\sigma}^{\text{Black-Box}}}{dt}(\hat{s}, \hat{t}) \]

\[ a, b = u, d, s, \bar{u}, \bar{d}, \bar{s} \]

QCD N = 2
FF1 N = 4
CIM N = 4

\[ \frac{d\hat{\sigma}^{\text{Black-Box}}}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{A}{\hat{s}^N} f(\hat{t} / \hat{u}) \]

\[ E \frac{d\sigma^{A+B \rightarrow h+X}}{d^3p}(s, p_T, \theta_{cm}) = \frac{A}{p_T^2} F(x_T, \theta_{cm}) \]

\[ A_{FF1} = 2.3 \times 10^6 \ \mu b \cdot GeV^6 \]

\[ E \frac{d\sigma_{FF1}^{A+B \rightarrow h+X}}{d^3p}(s, p_T, \theta_{cm}) = \frac{(2.3 \times 10^6 \ \mu b \cdot GeV^6)}{p_T^8} F(x_T, \theta_{cm}) \]
CIM Model

Constituent Interchange Model (CIM)

Meson-Quark Scattering

\[ E \frac{d\sigma_{\text{CIM}}^{p+p\rightarrow\pi+X}}{d^3 p} (s, p_T, \theta_{cm}) = \frac{1}{p_T^8} F_{\text{CIM}}(x_T, \theta_{cm}) \]


Meson within the proton!
QCD Improved Parton Model

Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Quark-Quark Elastic Scattering

\[ A_{q_i+q_i\rightarrow q_i+q_i}(\hat{s},\hat{t}) = \hat{t} \]

Parton = quark, anti-quark, & gluon

- \( q+q \rightarrow q+q \)
- \( g+q \rightarrow g+q \)
- \( g+g \rightarrow q+\bar{q} \)
- \( \bar{q}+\bar{q} \rightarrow \bar{q}+\bar{q} \)
- \( g+\bar{g} \rightarrow g+\bar{g} \)
- \( g+g \rightarrow g+g \)
- \( q+\bar{q} \rightarrow q+\bar{q} \)
- \( q+\bar{q} \rightarrow q+\bar{q} \)
- \( q+\bar{q} \rightarrow g+g \)

Include Leading Order QCD 2-to-2 Parton-Parton Processes (Order \( \alpha_s^2 \))

\[
\frac{d\hat{\sigma}_{a+b\rightarrow c+d}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi \alpha_s^2}{\hat{s}^2} \left| A(a+b \rightarrow c+d) \right|^2
\]

\[
|A(q_i+q_j \rightarrow q_i+q_j)|^2 = \left( \frac{4}{9} \right) \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \left( \frac{8}{27} \right) \hat{u} \hat{t}
\]

Sum over final-state spins and color.
Average over initial-state spins and color.
QCD Improved Parton Model

Use the “renormalization group improved” PDF’s from DIS and “renormalization group improved” fragmentation functions from e⁺e⁻ annihilations and the running QCD coupling!

\[ W = E_{cm} = \sqrt{s} \]
\[ x_1 = \frac{1}{2} x_T / \tan(\theta_{cm} / 2) \]
\[ x_T = 2 p_T / W \]
\[ x_2 = \frac{1}{2} x_T \tan(\theta_{cm} / 2) \]
\[ z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b} \]

\[ \frac{d\hat{\sigma}^{a+b\to c+d}}{dt}(\hat{s},\hat{t}) = \frac{\pi \alpha_s^2(Q^2)}{s^2} \left| \mathcal{A}_{a+b\to c+d}^{\langle \hat{s},\hat{t} \rangle} \right|^2 \]

\[ E \frac{d\sigma^{A+B\to h+X}}{d^3 p}(s,p_T,\theta_{cm}) \]
\[ = \frac{1}{\pi} \sum_{a,b} \int_{x_a}^{1.0} dx_a \int_{x_b}^{1.0} dx_b G_{A\to a}^{DIS}(x_a, Q^2) G_{B\to b}^{DIS}(x_b, Q^2) D_{c\to h}^{e^+e^-}(z_c, Q^2) \frac{1}{z_c} d\hat{\sigma}^{a+b\to c+d}(x_a x_b s, x_b t / z_c) \]

Q^2 dependent PDF’s!
Q^2 dependent fragmentation!
QCD Improved Parton Model

Use Renormalization Group Improved PDF’s!

Renormalization Group Improved Fragmentation!

Renormalization Group Improved Fragmentation!

Renormalization Group Improved Fragmentation!

Renormalization Group Improved Fragmentation!

Q^2 dependent PDF’s!

Q^2 dependent fragmentation!

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Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)

1. $q_i + q_j \rightarrow q_i + q_j$
2. $q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j$
3. $q_i + q_i \rightarrow q_i + q_i$
4. $q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i$
5. $q_i + \bar{q}_i \rightarrow g + g$
6. $gg \rightarrow q_i + \bar{q}_i$
7. $q_i + g \rightarrow q_i + g$
8. $\bar{q}_i + g \rightarrow \bar{q}_i + g$
9. $g + g \rightarrow g + g$

QCD Improved Parton Model

$$\frac{d\sigma_k}{dt}(s, \hat{t}) = \pi \alpha_s^2(Q^2) \frac{s^{2k}}{\hat{p}_T^4} f_k(\hat{x}_T) \quad k = 1 \text{ to } 9$$

Possible Choices for the Scale

$Q^2 = 4\hat{p}_T^2$

$Q^2 = \frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2 + \hat{t}^2 + \hat{u}^2}$

Naïve parton model scaling is now broken!
Comparison of a QCD model (normalized absolutely) with data on large $p_T$ pion production in proton-proton collisions at $W = 19.4$ and 53 GeV with $\theta_{cm} = 90^\circ$. The dot-dashed and solid curves are the results before and after smearing, respectively, with $\Lambda = 0.4$ GeV/c and $<k_T>_{primordial} = 848$ MeV/c and the dashed curves are with $\Lambda = 0.6$ GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark (\textit{i.e.} no gluons) is shown by the dotted curves.
Comparison of a QCD model (normalized absolutely) with data on large pion production in proton-proton collisions at $W = 19.4$ and 53 GeV with $\theta_{cm} = 90^\circ$. The dot-dashed and solid curves are the results before and after smearing, respectively, with $\Lambda = 0.4$ GeV/c and $<k_T>_{primordial} = 848$ MeV/c and the dashed curves are with $\Lambda = 0.6$ GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark (i.e. no gluons) is shown by the dotted curves.

Primordial $k_T$ has a big effect at low $p_T$ and low $W$ where the cross section is very steep!

Early Experimental Test of QCD

**F1 (1978)**

**FFF2 (1978)**
Data on $p_T^4 E d\sigma / d^3 p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$. 

QCD Improved Parton Model

\[ p_T^4 E d\sigma / d^3 p \]

\[ p_T^4(1/p_T^8) \]
Data on $p_T^4 E d\sigma / d^3 p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$. 

Approaches a constant at large $p_T$!
Data on $p_T^8 E d\sigma/d^3 p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2$, 0.35, and 0.5 versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.
Data on $p_T^8 E d\sigma / d^3 p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2, 0.35, \text{and} 0.5$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{\text{primordial}} = 848 \text{ MeV}$, respectively, with $\Lambda = 0.4 \text{ GeV/c}$ and the dashed curves are the results of using $\Lambda = 0.6 \text{ GeV/c}$ (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.
The behavior of $p_T^8 E d\sigma / d^3 p$, at $x_T = 0.05$ versus $p_T$ calculated from the QCD approach with $\Lambda = 0.4 \text{ GeV/c}$ (solid curves) and $\Lambda = 0.6 \text{ GeV/c}$ (dashed curves). The two low $p_T$ data points are at 53 and 63 GeV. The predictions are a factor of 100 (1000) times larger than the flat horizontal $(1/p_T^8)$ extrapolation to $W = 500 \text{ GeV (1000 GeV)}$. 

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R. Field - Florida/CDF/CMS

Early QCD Predictions

\[ W = 1.96 \text{ TeV} \]

\[ x_T = 0.05 \]

\[ p_T = \frac{1}{2}(0.05)(1960\text{GeV}) = 49\text{GeV}/c \]

\[ \frac{d\sigma^h}{dp_T dy d\phi} (x_T = 0.05, p_T = 49\text{GeV}/c) \approx 10^{-9} \text{mb}/(\text{GeV}/c)^2 \]

\[ p_T^8 E \frac{d\sigma^{\pi^0}}{d^3 p} (x_T = 0.05, p_T = 49\text{GeV}/c) \approx (0.5)(49\text{GeV}/c)^8 \times 10^{-9} \text{mb}/(\text{GeV}/c)^2 \]

\[ \approx 1.66 \times 10^7 \mu\text{b} \cdot (\text{GeV}/c)^6 \]
Amazing!

factor of
≈ 10,000
QCD “Jet” Production

\[ A + B \rightarrow c + X \]

QCD Improved Parton Model

Large Transverse Momentum Parton Production

\[ D_{e^+e^-}(z_c, Q^2) = \delta(1-z_c) \]

\[ x_a = \frac{x_b x_1}{z_c x_b - x_2} \]
\[ x_b = \frac{x_a x_2}{z_c x_a - x_1} \]
\[ z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b} \]
\[ \frac{dz_c}{dx_b} = -\frac{x_2}{x_b} \]

\[ E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3 p}(s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\text{min}}}^{1} dx_a \int_{x_b^{\text{min}}}^{1} dx_b G^{\text{DIS}}_{A \rightarrow a}(x_a, Q^2) G^{\text{DIS}}_{B \rightarrow b}(x_b, Q^2) \delta(1-z_c) \frac{1}{z_c} \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}}(\hat{s}, \hat{t}) \]

\[ E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3 p}(s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{z_c^{\text{min}}}^{1} dz_c G^{\text{DIS}}_{A \rightarrow a}(x_a, Q^2) G^{\text{DIS}}_{B \rightarrow b}(x_b, Q^2) \delta(1-z_c) \frac{x_b^2}{x_2 z_c} \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}}(\hat{s}, \hat{t}) \]

\[ E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3 p}(s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\text{min}}}^{1} dx_a \frac{x_b^2}{x_2} G^{\text{DIS}}_{A \rightarrow a}(x_a, Q^2) G^{\text{DIS}}_{B \rightarrow b}(x_b, Q^2) \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}}(x_a x_b s, x_b t) \]

\[ x_a^{\text{min}} = \frac{x_1}{1-x_2} \]
\[ x_b = \frac{x_a x_2}{x_a - x_1} \]
Comparison of the results on the 90° π° cross section, Edσ/d³p, from the QCD approach with Λ = 0.4 GeV/c (solid curves) and the quark-quark “black-box” parton model of FF1 (dotted curves). Both models agree with the data at W = 53 GeV. The QCD approach results in much larger cross sections than the FF1 model at W = 500 and 1000 GeV. The FF1 results at 1000 GeV (not shown) are only slightly larger than at 500 GeV. Also shown are the cross sections for producing a jet at 90° (divided by 1000) as predicted by the QCD approach (dashed curves) and the FF1 model (dot-dashed curve).
Comparison of the jet and single π⁰ cross sections measured at 200 GeV (W = 19.4 GeV) and θ_{cm} = 90°. The jet data are from two FNAL experiments, E260 and E395, where a jet is defined as the sum of all the particles into their respective detectors. Also shown is the QCD prediction for the cross section of producing a parton (quark, antiquark, or gluon) at W = 19.4 GeV and θ_{cm} = 90°.

The FNAL data (E260) are from C. Bromberg et al., Phys Rev Letters 38 (1977); C. Bromberg et al., CALT-68-613 (to be published in Nucl. Phys.).

The FNAL data (E395) are from the Fermilab-Lehigh-Pennsylvania-Wisconsin Collaboration, talk given by W. Selove at this conference.
High $P_T$ Jets

Feynman, Field, & Fox (1978)

Predict large "jet" cross-section

$30 \text{ GeV/c!}$

CDF (2006)

Midpoint ($R_{cone}=0.7$, $f_{merge}=0.75$, $R_{sep}=1.3$)

$\int L=1.04 \text{ fb}^{-1}$

600 GeV/c Jets!
High $P_T$ Jets

**Feynman, Field, & Fox (1978)**

Predict large “jet” cross-section

30 GeV/c!

CDF (2006)

Feynman quote from FFF

“At the time of this writing, there is still no sharp quantitative test of QCD. An important test will come in connection with the phenomena of high $P_T$ discussed here.”

Lecturers

G. Altarelli
A. J. Buras
C. De Tar
S. Ellis
R. D. Field
D. Gross
C. H. Llewellyn Smith
J. Wess

Rick Field Boulder 1979
Jimmie Field Boulder 1979
At low $x_T$ the dominate QCD sub-process is gluon production which fragments equally into $\pi^+$ and $\pi^-!$

To produce an outgoing $\pi^+$ in the CIM approach one must find a $\pi^+$ within the incoming $\pi$ or the incoming proton (with small probability)!
At low $x_T$ the dominate QCD sub-process is gluon production which fragments equally into $\pi^+$ and $\pi^-$.


Direct CIM Contribution $\pi^+ + $d-quark $\rightarrow \pi^- + $d-quark

To produce an outgoing $\pi^+$ in the CIM approach one must find a $\pi^+$ within the incoming $\pi^-$ or the incoming proton (with small probability)!
The NATO Advanced Institute on Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories, held at the University of Colorado, Boulder, Colorado, July 9 – 27, 1979.
CDF (2006)

Midpoint \( R_{cone} = 0.7, R_{merge} = 0.75, R_{sep} = 1.3 \)

1.96 TeV

Amazing!

2 TeV

27 Years!

R. Field BND 1979

2 TeV

400 GeV/c Jets!

\[ E \frac{d\sigma}{d^3 p} (pp \rightarrow \text{Jet} + X) \approx 10^{-14} (\mu b / GeV^2) \]

\[ \frac{d^2 \sigma}{dp_T dy} = 2\pi p_T \left( E \frac{d\sigma}{d^3 p} \right) \]

\[ \frac{d^2 \sigma}{dp_T dy} (pp \rightarrow \text{Jet} + X) \approx 2.51 \times 10^{-5} (nb / GeV) \]
The QCD Parton Model Approach – Summary & Conclusions

- No evidence against the (perturbative) QCD parton model approach.
- Lots of qualitative support for the approach:
  - $e p \rightarrow e + X$, $\mu p \rightarrow \mu + X$, $\nu p \rightarrow \mu + X$.
  - $p p \rightarrow \mu + \mu + X$, $p p \rightarrow \pi + X$, $p p \rightarrow \text{Jet} + X$.
  - $e + e^\prime \rightarrow \text{Jet} + \text{Jet} + X$, $\nu p \rightarrow \mu - + \text{Jet} + X$.
- The applications of the theory are still quite crude and should be considered as qualitative first guesses.
- Many of the most dramatic and definitive predictions of the theory have not yet been observed experimentally.
- Our knowledge of what the theory predicts is improving and soon (two years) we should have a more quantitative picture (i.e. predictions to say 20% level).
- Care must be taken in using the (perturbative) aspects of the theory only where they apply (i.e. large $Q^2$). There are in many cases large corrections to the asymptotic forms at low $Q^2$ (i.e. $1/Q^2$ effects, non-perturbative effects) which cannot at present be calculated.
- This is a very exciting and fun time. We have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong interactions.
The QCD Parton Model Approach – Summary & Conclusions

- No evidence against the (perturbative) QCD parton model approach.
- Lots of qualitative support for the approach:
  - ep → e + X, μp → μ + X, νp → μ- + X.
  - pp → μ + μ- + X, pp → π+ X, pp → Jet + X.
  - e + e- → Jet + Jet + X, νp → μ- + Jet + X.

The applications of the theory are still quite crude and should be considered as qualitative first guesses.

- Many of the most dramatic and definitive predictions of the theory have not yet been observed experimentally.
- Care must be taken in using the (perturbative) aspects of the theory only where they apply (i.e. large Q^2). There are in many cases large corrections to the asymptotic forms at low Q^2 (i.e. 1/Q^2 effects, non-perturbative effects) which cannot at present be calculated.

QCD is not just “another theory”.
If it is not the correct description of nature, then it will be quite some time before another candidate theory emerges.

- R. Field, ICHEP78

This is a very exciting and fun time. We have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong interactions.
Three-Jet Final State:

Not observed until after Boulder 1979!

Observed by the JADE detector at PETRA at $E_{CM} = 30$ GeV (1980?).
7 GeV $\pi^0$'s $\rightarrow$ 2 TeV Jets

Field-Feynman (1977)
$7 \text{ GeV } \pi^0\text{s} \rightarrow 2 \text{ TeV Jets}$
7 GeV $\pi^0$'s $\rightarrow$ 2 TeV Jets

QCD Improved Parton Model

CMS

72 pb$^{-1}$ (13 TeV)

$\log_{10}\{E\sigma/dE/dp_{T}\} [\mu b/(GeV/c)^2]\}$

Single $\pi$ Rate

6 GeV/c Jets!

FFF2 (1978)

CMS Preliminary

2 TeV Jets!
7 GeV $\pi^0$'s $\rightarrow$ 2 TeV Jets

QCD Improved Parton Model

CMS

72 pb$^{-1}$ (13 TeV)

CMS

Rick & Jimmie ISMD - Chicago 2013

2 TeV Jets!

Rick & Jimmie CALTECH 1973

CMS

Rick Field – Florida/CDF/CMS

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7 GeV $\pi^0$'s $\to$ 2 TeV Jets

QCD Improved Parton Model

$72 \text{ pb}^{-1} (13 \text{ TeV})$

Rick & Jimmie CERN - June 2017

Rick & Jimmie CALTECH 1973

Rick & Jimmie ISMD - Chicago 2013

Rick & Jimmie CERN - June 2017

TeV Jets!