QCD BND Summer School in Particle Physics 2016

Early Experimental Tests of QCD

Rick Field

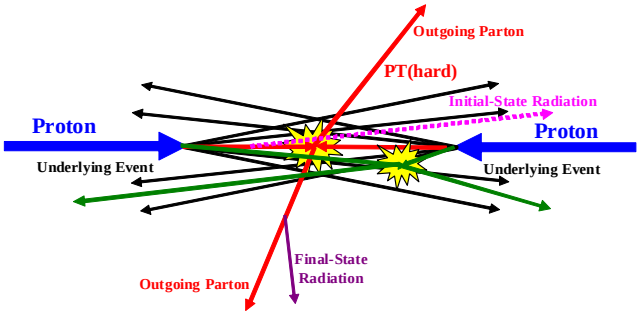
University of Florida

Quantum Chromo-Dynamics

Lecture 3

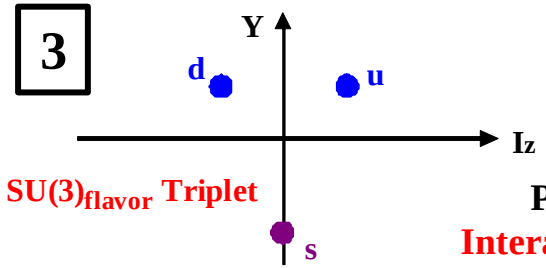
University of Antwerp August 26 – September 9, 2016

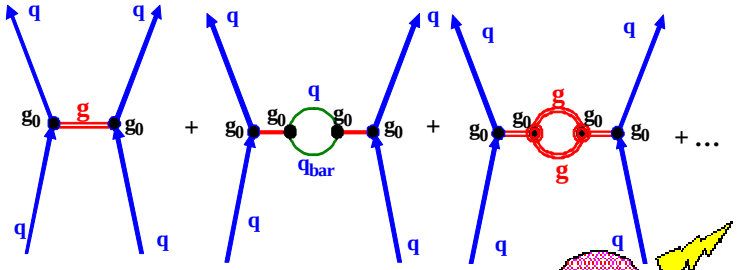
- My talk at ICHEP 1978 in Tokyo and my lectures in Boulder in 1979.
- Collider Coordinates.
- Drell-Yan Muon-Pair Production.
- Parton Intrinsic (Primordial) Transverse Momentum.
- The QCD Improved Parton Model.
- The Invariant Differential.
- Large Transverse Meson Production in Hadron-Hadron Collisions.
- Large Transverse "Jet" Production in Hadron-Hadron Collisions.



QCD My Story of QCD

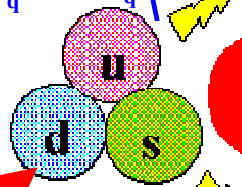
The Quark Model
The Eightfold Way





QCD Theory!

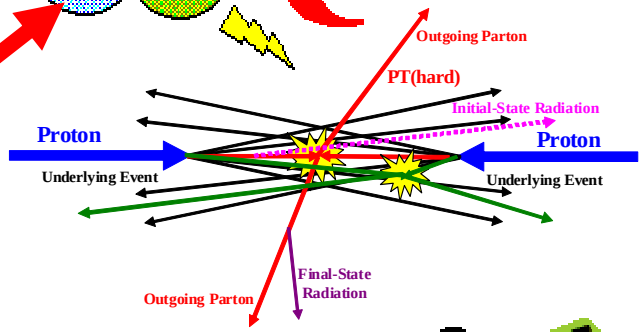
QCD



Parton Model + Perturbative QCD!

Interacting quarks, anti-quarks, and gluons!

The QCD Improved Parton Model



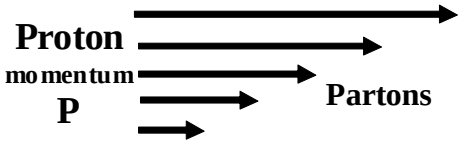
Before we knew the partons were quarks and gluons!

Non-interacting partons!

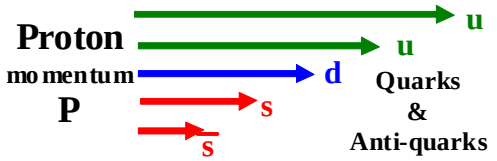
Partons = quarks, anti-quarks, and glue!

Still non-interacting partons!

The Parton Model



The Naïve Parton Model



QCD The Feynman-Field Days

1973-1983

"Feynman-Field Jet Model"

- **FF1: "Quark Elastic Scattering as a Source of High Transverse Momentum Mesons", R. D. Field and R. P. Feynman, Phys. Rev. D15, 2590-2616 (1977).**
- **FFF1: "Correlations Among Particles and Jets Produced with Large Transverse Momenta", R. P. Feynman, R. D. Field and G. C. Fox, Nucl. Phys. B128, 1-65 (1977).**
- **FF2: "A Parameterization of the properties of Quark Jets", R. D. Field and R. P. Feynman, Nucl. Phys. B136, 1-76 (1978).**
- **F1: "Can Existing High Transverse Momentum Hadron Experiments be Interpreted by Contemporary Quantum Chromodynamics Ideas?", R. D. Field, Phys. Rev. Letters 40, 997-1000 (1978).**
- **FFF2: "A Quantum Chromodynamic Approach for the Large Transverse Momentum Production of Particles and Jets", R. P. Feynman, R. D. Field and G. C. Fox, Phys. Rev. D18, 3320-3343 (1978).**

- **FW1: "A QCD Model for e^+e^- Annihilation", R. D. Field and S. Wolfram, Nucl. Phys. B213, 65-84 (1983).**

My 1st graduate student!

QCD The Feynman-Field Days

1973-1983

The Naïve Parton Model

"Feynman-Field Jet Model"

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The Naïve Parton Model

The Naïve Parton Model

QCD Improved Parton Model

QCD Improved Parton Model

QCD Improved Parton Model

My 1st graduate student!

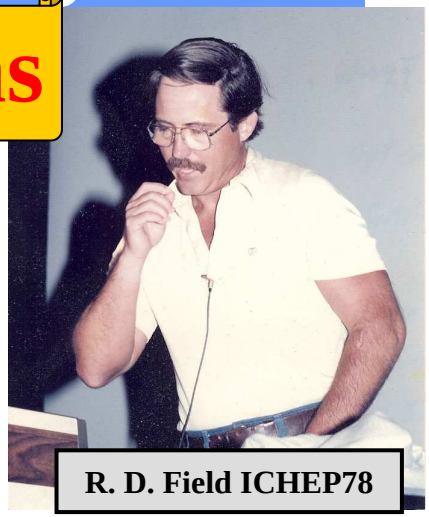
QCD ICHEP 1978 Tokyo

Dynamics of High Energy Reactions

Rick Field

California Institute of Technology

(**Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)**



R. D. Field ICHEP78

The QCD Parton Model Approach - Outline of Talk

- **The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale Λ.**
- **Quark and Gluon Distributions within Hadrons: Scale Breaking.**
  - Analysis of ep and μp Data.
  - Analysis of F_2 and xF_3 in Neutrino Processes.
- **Muon-Pair Production in pp Collisions.**
  - QCD Factorization – "Constant" Pieces.
  - Large p_T Muon-Pair Production.
  - "Scaling" in $pp \rightarrow \mu^+\mu^- + X$.
- **Quark and Gluon Fragmentation Functions: Scale Breaking.**
  - Gluon Jets.
- **Large p_T Production of Mesons and Jets in pp Collisions.**
  - Scale Breaking Effects: $Ed\sigma/d^3p$.
  - Correlations – Evidence for Gluons.
  - The Jet Cross Section..
- **A Look to the Future.**
  - $pp \rightarrow \pi^0 + X$ at W = 500 GeV.
  - Large p_T Charm Production.
  - $\gamma\gamma \rightarrow$ Jet + Jet and $e^+e^- \rightarrow e^+e^-$+Jet+Jet.
  - Three Jets: $e^+e^- \rightarrow$ q+qbar+Jet and pp→Jet+Jet+Jet+X..

QCD ICHEP 1978 Tokyo

Dynamics of High Energy Reactions

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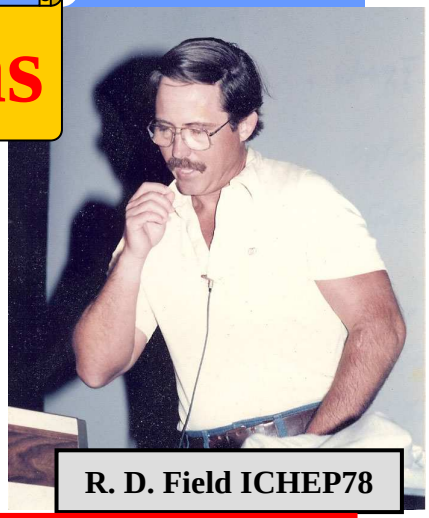

R. D. Field ICHEP78

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QCD Boulder 1979

- **The NATO Advanced Institute on Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories, held at the University of Colorado, Boulder, Colorado, July 9 – 27, 1979. (NATO Advanced Institute Series B: Physics, Volume 24, Plenum Press, 1980)**

Lecturers

G. Altarelli

A. J. Buras

C. De Tar

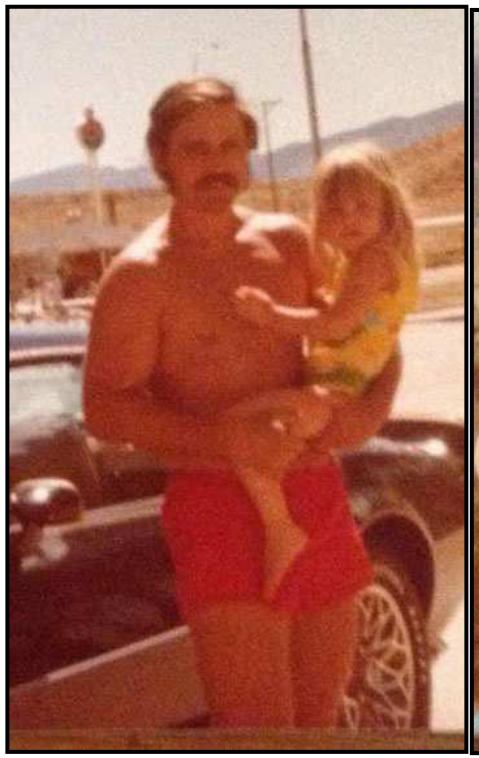
S. Ellis

R. D. Field

D. Gross

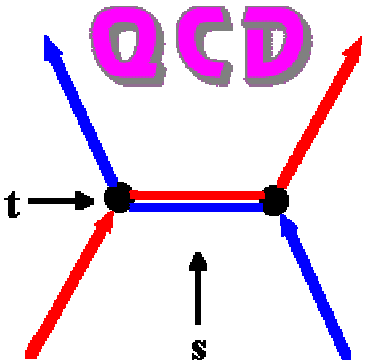
C. H. Llewellyn Smith

J. Wess

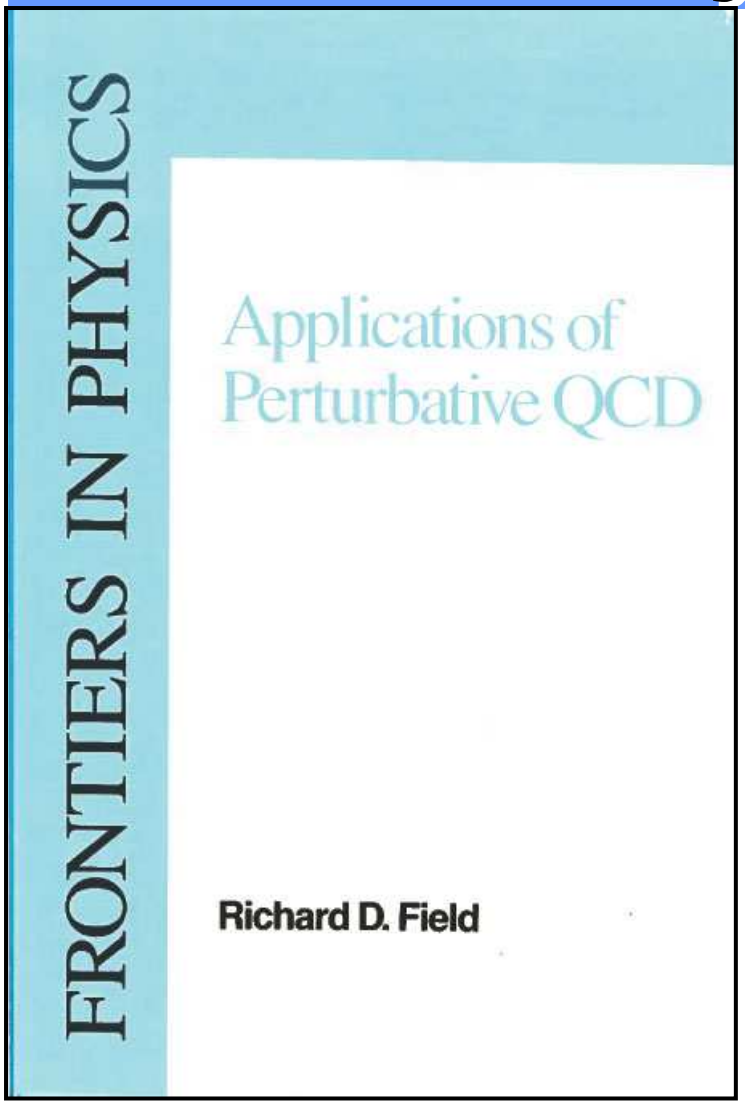


Rick Field Boulder 1979

Jimmie Field Boulder 1979



My QCD Book



- **Applications of Perturbative QCD, Addison-Wesley Publishing Company, *The Advanced Book Program,* Frontiers in Physics (Volume No. 77), 1989.**

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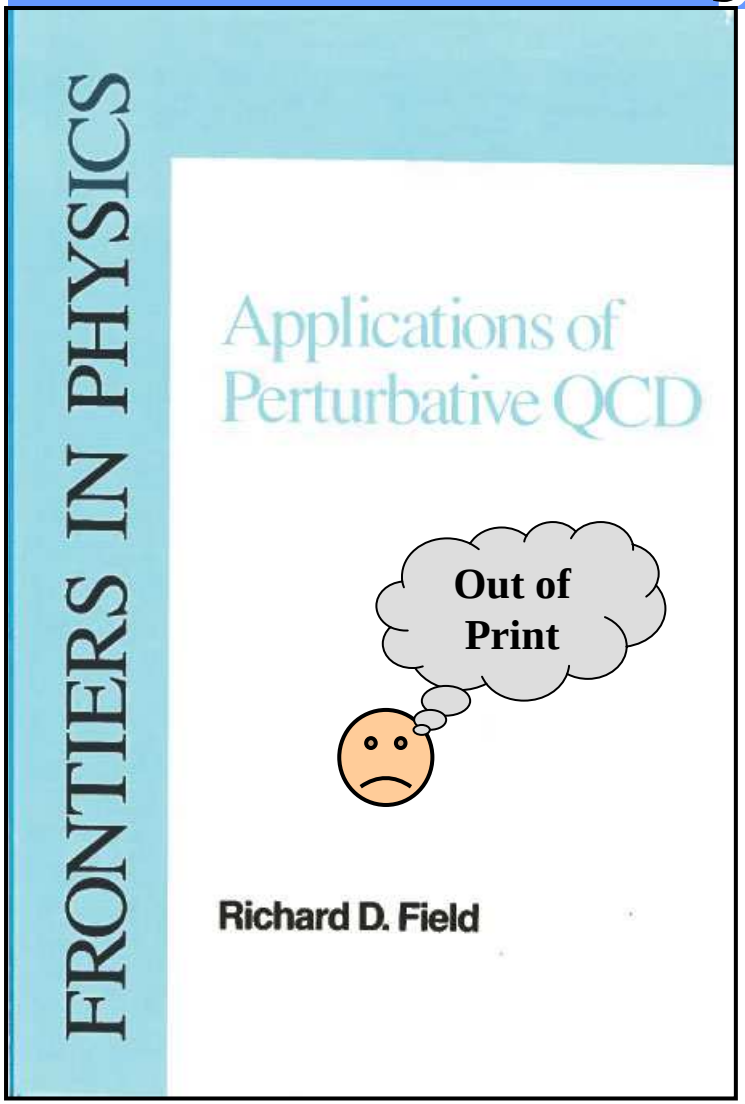

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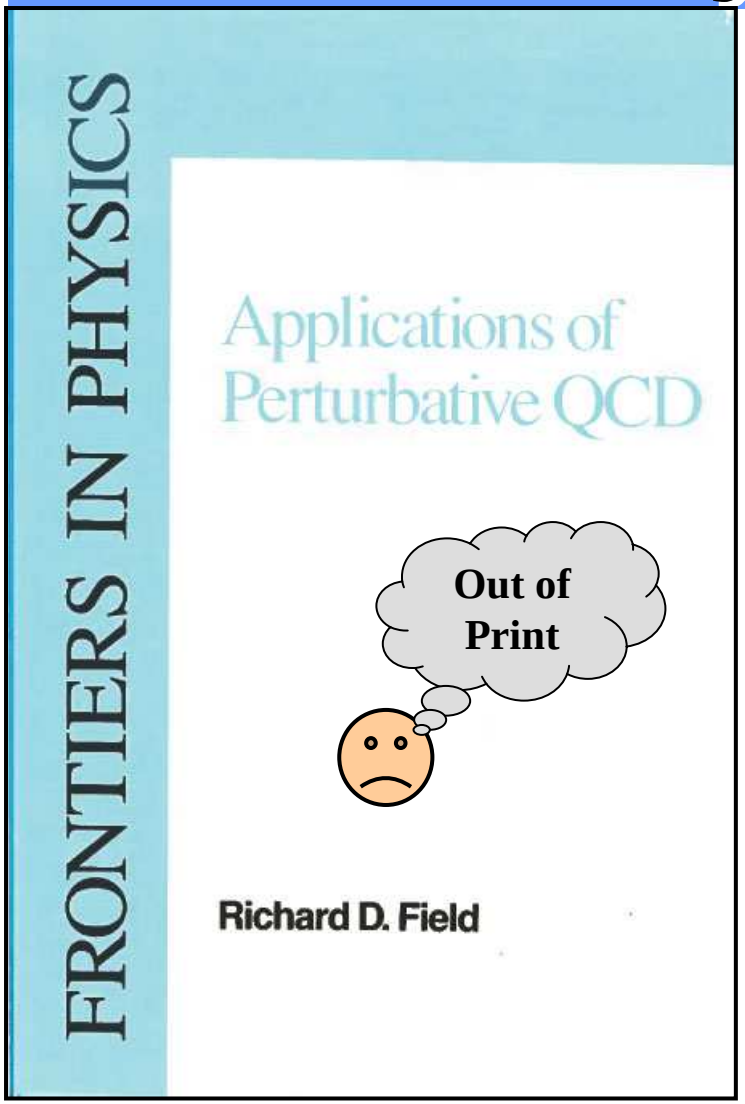

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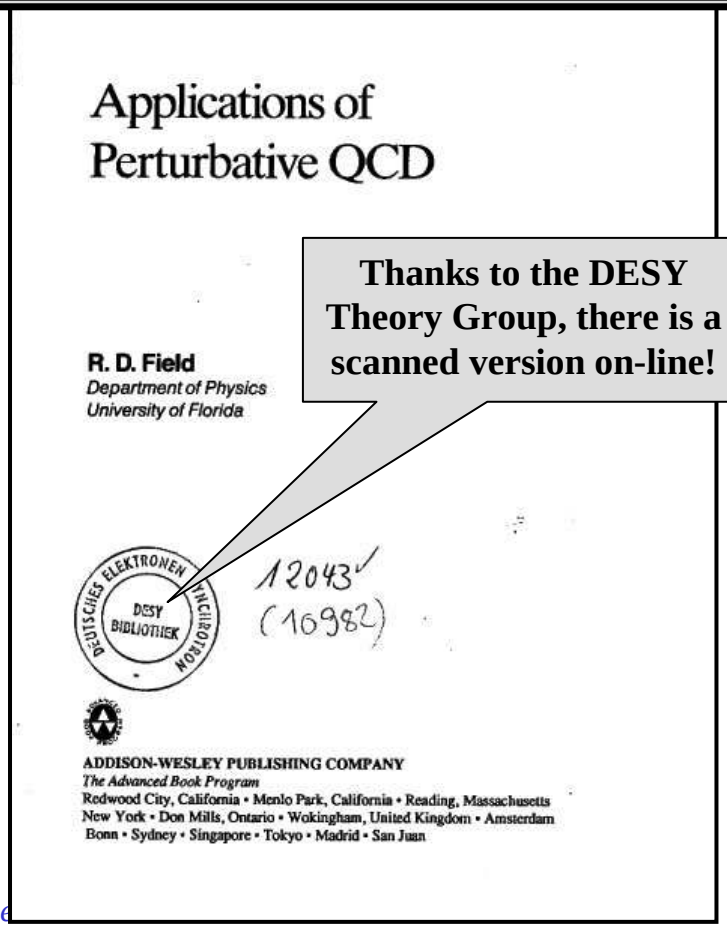
http://www.phys.ufl.edu/~rfield/cdf/RDF_QCD_Book.pdf



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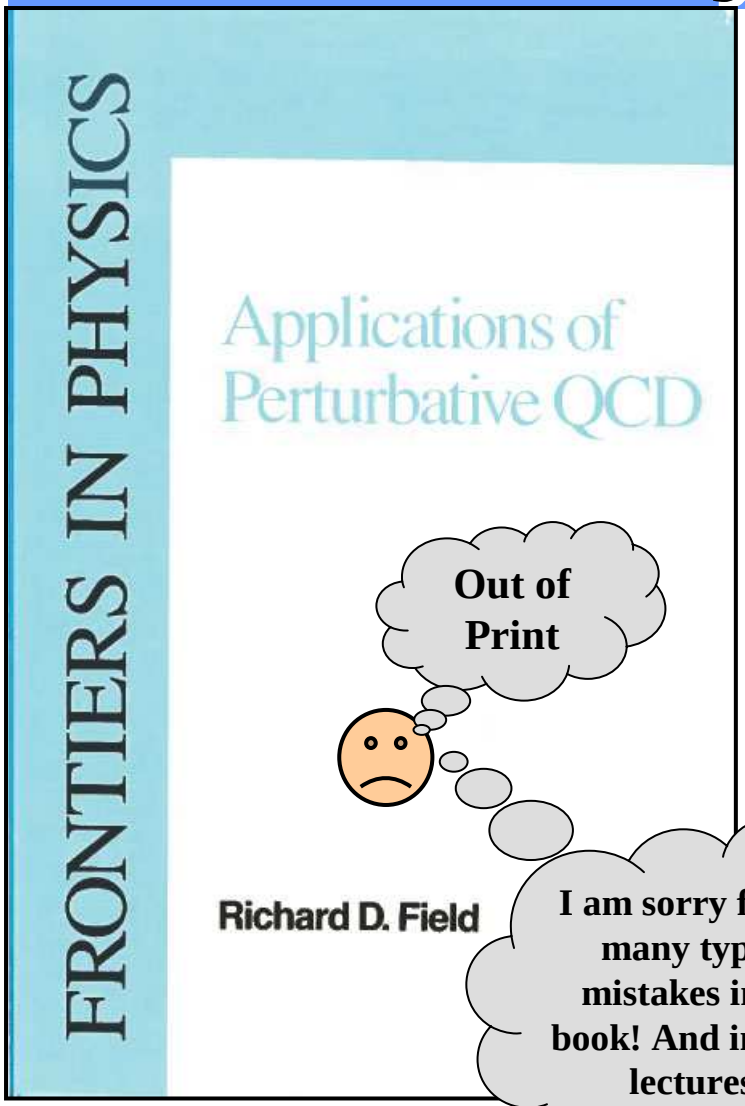

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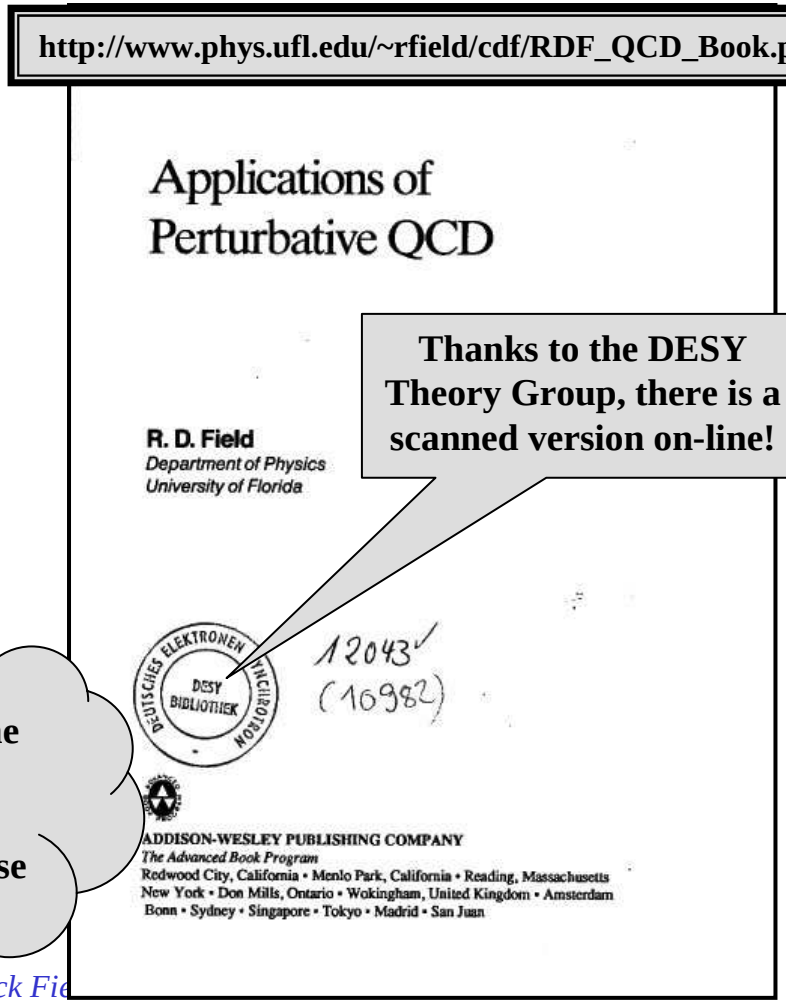


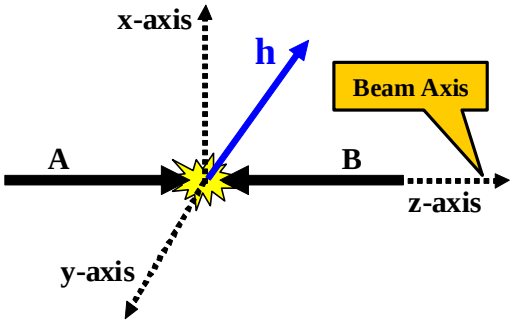
Out of Print

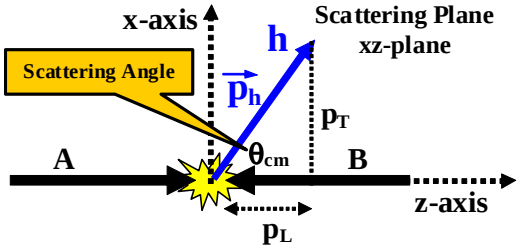
Thanks to the DESY Theory Group, there is a scanned version on-line!

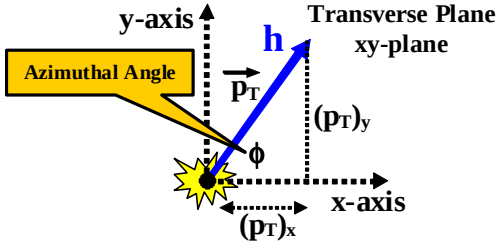
I am sorry for the many typing mistakes in the book! And in these lectures?

QCD Collider Coordinates

A+B→h+X Center-of-Mass Frame







$$p_T = p\cos\theta_{cm}$$

$$p_L = p\sin\theta_{cm}$$

$$E = \sqrt{p_L^2 + p_T^2 + m^2}$$

➡ **The z-axis is defined to be the beam axis. Let the xz-plane be the plane of the scattering with the xy-plane being the "transverse" plane.**

➡ **θ_{cm} is the center-of-mass scattering angle and ϕ is the azimuthal angle. The "transverse" momentum of a particle is given by $p_T = p\cos(\theta_{cm}) = p_x$. The "longitudinal" momentum of a particle is given by $p_L = p\sin(\theta_{cm}) = p_z$.**

➡ **The "pseudo-rapidity" η of a particle is** $\eta \equiv -\log(\tan(\theta_{cm}/2))$

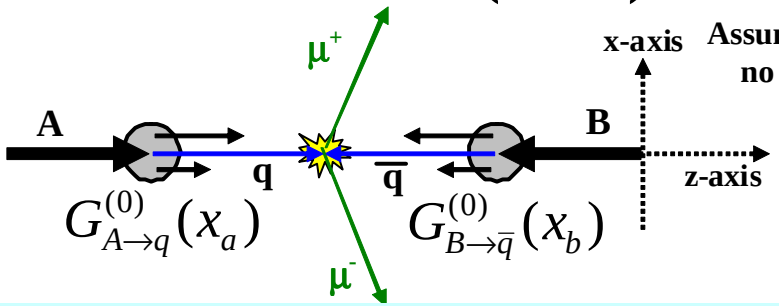
➡ **The "rapidity" y of a particle with mass m is**

$$y \equiv \frac{1}{2}\log\left(\frac{E+p_L}{E-p_L}\right) = \frac{1}{2}\log\left(\frac{\sqrt{p_L^2+p_T^2+m^2}+p_L}{\sqrt{p_L^2+p_T^2+m^2}-p_L}\right) \xrightarrow[m=0]{} \eta$$

η	θ_{cm}
0	90º
1	40º
2	15º
3	6º
4	2º

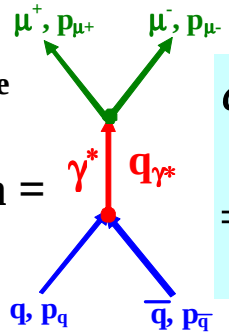
QCD Naïve Parton Model

Drell-Yan Process (1970): A+B→μ⁺μ⁻ + X



$G^{(0)}_{A\to q}(x_a)$ $G^{(0)}_{B\to\bar{q}}(x_b)$

Assume incoming partons have no transverse momentum!



Born Term =

$$\hat{\sigma}(q+\bar{q}\to\gamma^*\to\mu^+ +\mu^-) = (\tfrac{1}{3})\frac{4\pi\alpha_{em}^2 e_q^2}{3M_{\mu\mu}^2}$$

$$d\sigma^{A+B\to\mu^+\mu^- +X} = G^{(0)}_{A\to q}(x_a)dx_a G^{(0)}_{B\to\bar{q}}(x_b)dx_b\,\hat{\sigma}(q+\bar{q}\to\gamma^*\to\mu^+ +\mu^-)$$

Transverse Momentum of the Muon-Pair

$$P_T^{\mu\mu} = (\vec{p}_{\mu+}+\vec{p}_{\mu-})_T = 0$$

Longitudinal Momentum of the Muon-Pair

$$P_L^{\mu\mu} = (\vec{p}_{\mu+}+\vec{p}_{\mu-})_z = p_q - p_{\bar{q}}$$

$$\tilde{P}_A^{CM} = \begin{pmatrix} P_{CM} \\ 0 \\ 0 \\ P_{CM} \end{pmatrix} \quad \tilde{P}_B^{CM} = \begin{pmatrix} P_{CM} \\ 0 \\ 0 \\ -P_{CM} \end{pmatrix}$$

$$\tilde{p}_q = x_a\tilde{P}_A \qquad \hat{s} = (\tilde{p}_q+\tilde{p}_{\bar{q}})^2 = 2\tilde{p}_q\cdot\tilde{p}_{\bar{q}} = x_a x_b s$$

$$\tilde{p}_{\bar{q}} = x_b\tilde{P}_B \qquad \hat{s} = (\tilde{p}_{\mu+}+\tilde{p}_{\mu-})^2 = M_{\mu\mu}^2 = x_a x_b s$$

$$s = (\tilde{P}_A+\tilde{P}_B)^2 = 2\tilde{P}_A\cdot\tilde{P}_B = 2(P_{CM}^2+P_{CM}^2) = 4P_{CM}^2$$

$$x_L = 2P_L/\sqrt{s} = x_a - x_b$$
$$x_a x_b = \tau$$

$$P_{CM} = \tfrac{1}{2}\sqrt{s} \qquad \tau = M_{\mu\mu}^2/s \qquad x_E = 2E_{\mu\mu}/\sqrt{s} \qquad E_{\mu\mu}^2 = (P_L^{\mu\mu})^2 + \cancel{(P_T^{\mu\mu})^2}^{\,0} + M_{\mu\mu}^2 = (P_L^{\mu\mu})^2 + M_{\mu\mu}^2 \qquad x_E^2 = x_L^2 + 4\tau$$

$$\frac{d\sigma_{DY}^{AB}}{d\tau dx_L}(s, M_{\mu\mu}^2, x_L) = \frac{4\pi\alpha_{em}^2}{9M_{\mu\mu}^2}\frac{1}{(x_a+x_b)}P^{(0)}_{A+B\to q\bar{q}}(x_a,x_b)$$

$$x_a = \tfrac{1}{2}(x_E+x_L) \qquad x_b = \tfrac{1}{2}(x_E-x_L)$$

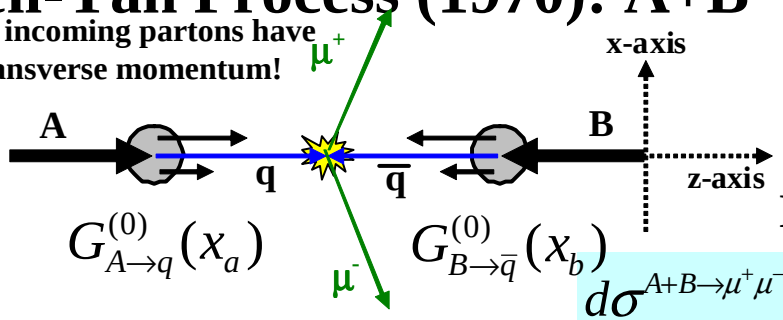
$$dx_a dx_b = \frac{d\tau dx_L}{(x_a+x_b)}$$

$$P^{(0)}_{A+B\to q\bar{q}}(x_a,x_b) = \sum_{i=1}^{nf} e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to\bar{q}_i}(x_b) + G^{(0)}_{A\to\bar{q}_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

QCD Naïve Parton Model

Drell-Yan Process (1970): A+B→μ⁺μ⁻ + X

Assume incoming partons have no transverse momentum!



Born Term

$$x_a - x_b = x_L \qquad x_a x_b = \tau$$
$$x_a = \tfrac{1}{2}(x_E + x_L) \qquad x_b = \tfrac{1}{2}(x_E - x_L)$$
$$x_E^2 = x_L^2 + 4\tau \qquad \tau = M^2_{\mu\mu}/s$$

$$d\sigma^{A+B\to\mu^+\mu^-+X} = G^{(0)}_{A\to q}(x_a)dx_a G^{(0)}_{B\to\bar{q}}(x_b)dx_b\,\hat{\sigma}(q+\bar{q}\to\gamma^*\to\mu^+ + \mu^-)$$

Rapidity of Muon-Pair

$$y_{\mu\mu} \equiv \frac{1}{2}\log\left(\frac{E_{\mu\mu}+p_L^{\mu\mu}}{E_{\mu\mu}-p_L^{\mu\mu}}\right) = \frac{1}{2}\log\left(\frac{x_E+x_L}{x_E-x_L}\right) = \frac{1}{2}\log\left(\frac{x_a}{x_b}\right)$$

$$e^{2y_{\mu\mu}} = \frac{x_E+x_L}{x_E-x_L} = \frac{(x_E+x_L)^2}{x_E^2-x_L^2} = \frac{(x_E+x_L)^2}{4\tau}$$

$$x_a = \tfrac{1}{2}(x_E+x_L) = \sqrt{\tau}e^{+y_{\mu\mu}}$$
$$x_b = \tfrac{1}{2}(x_E-x_L) = \sqrt{\tau}e^{-y_{\mu\mu}}$$

$$e^{-2y_{\mu\mu}} = \frac{x_E-x_L}{x_E+x_L} = \frac{(x_E-x_L)^2}{x_E^2-x_L^2} = \frac{(x_E-x_L)^2}{4\tau}$$

$$dx_a dx_b = d\tau dy_{\mu\mu}$$

$$\frac{d\sigma^{AB}_{DY}}{d\tau dy_{\mu\mu}}(s, M^2_{\mu\mu}, y_{\mu\mu}) = \frac{4\pi\alpha_{em}^2}{9M^2_{\mu\mu}} P^{(0)}_{A+B\to q\bar{q}}(x_a, x_b)$$

$$M^2_{\mu\mu}\frac{d\sigma^{AB}_{DY}}{d\tau dy_{\mu\mu}}(s, M^2_{\mu\mu}, y_{\mu\mu}) = \frac{4\pi\alpha_{em}^2}{9} P^{(0)}_{A+B\to q\bar{q}}(x_a, x_b) = F^{(0)}(\tau, y_{\mu\mu})$$

= Constant at fixed τ and $y_{\mu\mu}$!

(provided the PDF's scale)

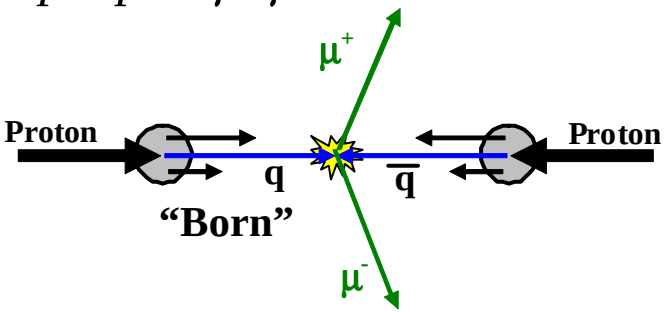
$$M^3_{\mu\mu}\frac{d\sigma^{AB}_{DY}}{dM_{\mu\mu}dy_{\mu\mu}}(s, M^2_{\mu\mu}, y_{\mu\mu}) = 2\tau F^{(0)}(\tau, y_{\mu\mu})$$

Parton Model Scaling

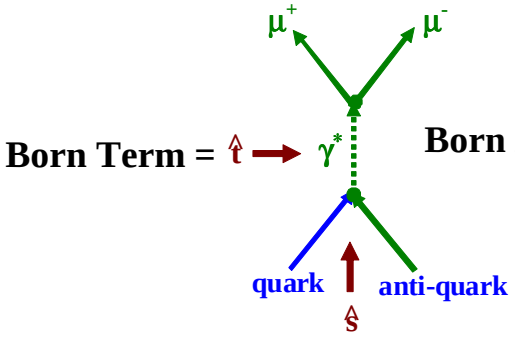
QCD Improved Parton Model

Drell-Yan Muon-Pair Production

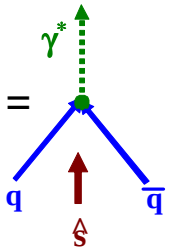
$p + p \to \mu^+\mu^- + X$



$q + \bar{q} \to \mu^+ + \mu^-$

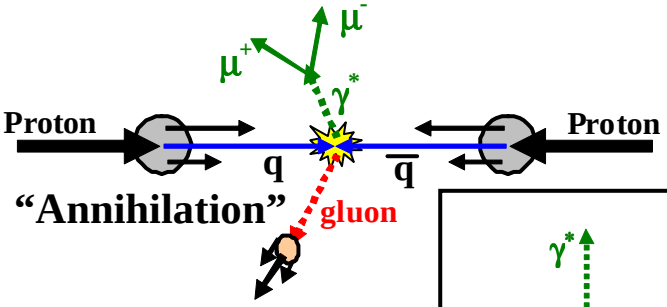


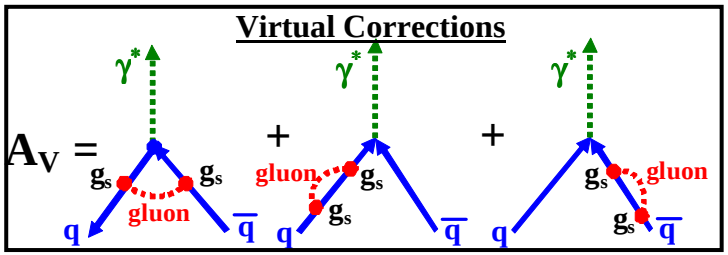
$q + \bar{q} \to \gamma^*$

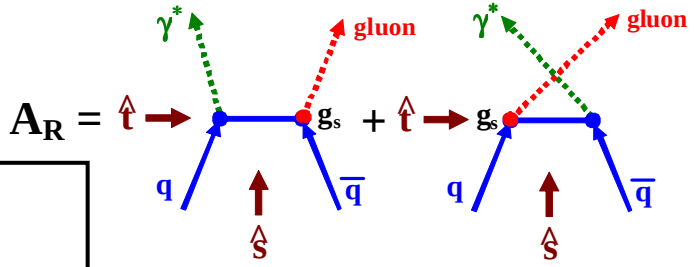


Include Leading Order QCD Corrections (Order α_s)

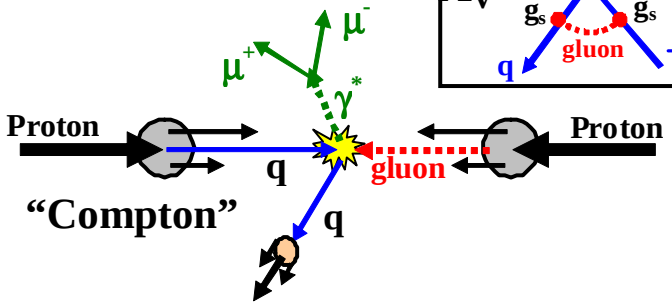
"Annihilation" $q + \bar{q} \to \gamma^* + g$

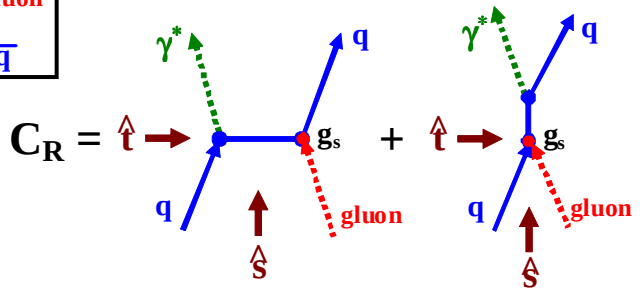


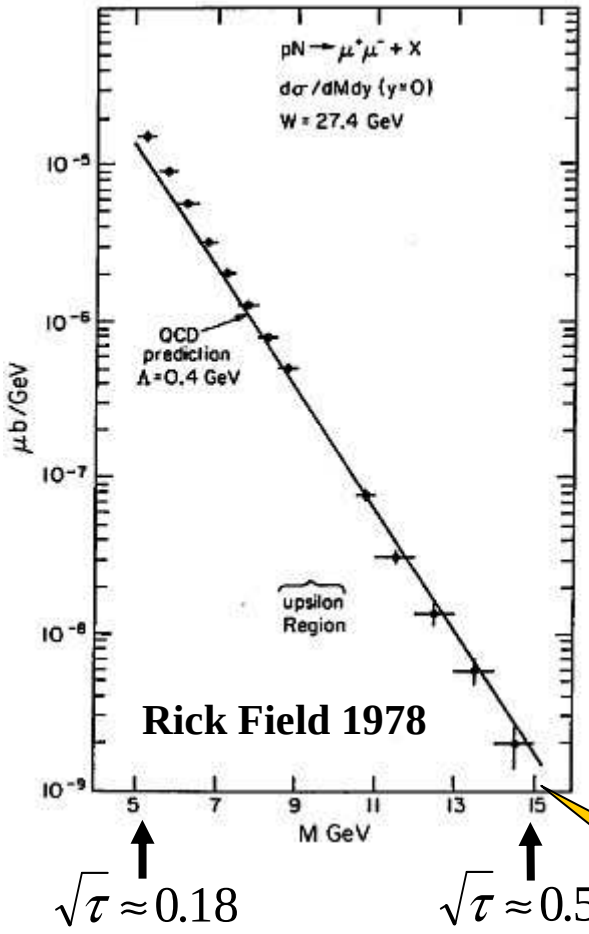




"Compton" $q + g \to \gamma^* + q$







$\sqrt{\tau} \approx 0.18$ $\sqrt{\tau} \approx 0.55$

$M_{\mu\mu}$ = 15 GeV!

The data are from D. C. Horn *et al.*, Phys. Rev. Letters 36, 1239 (1976) and 37, 1374 (1976); S. W. Herb *et al.*, Phys. Rev. Letters 39, 352 (1977); W. R. Innes et al., Phys. Rev. Letters 39, 1240 (1977); also see the talk by L. Lederman at this conference.

$$P^{DIS}_{A+B\to q\bar{q}}(x_a,x_b,Q^2)=\sum_{i=1}^{nf} e_{q_i}^2\left[G^{DIS}_{A\to q_i}(x_a,Q^2)G^{DIS}_{B\to \bar{q}_i}(x_b,Q^2)+G^{DIS}_{A\to \bar{q}_i}(x_a,Q^2)G^{DIS}_{B\to q_i}(x_b,Q^2)\right]$$

$$\frac{d\sigma^{AB}_{DY-LO}}{dM_{\mu\mu}dy_{\mu\mu}}(W,M_{\mu\mu},y_{\mu\mu}=0)=\frac{2}{W^2M_{\mu\mu}}F^{DIS}(\tau,y_{\mu\mu}=0,Q^2)$$

$$\tau = M^2_{\mu\mu}/s$$

$$x_a=\tfrac{1}{2}(x_E+x_L)=\sqrt{\tau}e^{+y_{\mu\mu}} \xrightarrow[y_{\mu\mu}=0]{} \sqrt{\tau}$$

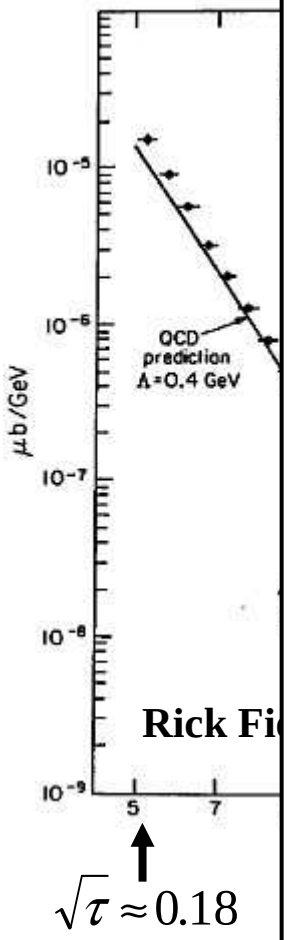
$$x_b=\tfrac{1}{2}(x_E-x_L)=\sqrt{\tau}e^{-y_{\mu\mu}} \xrightarrow[y_{\mu\mu}=0]{} \sqrt{\tau}$$

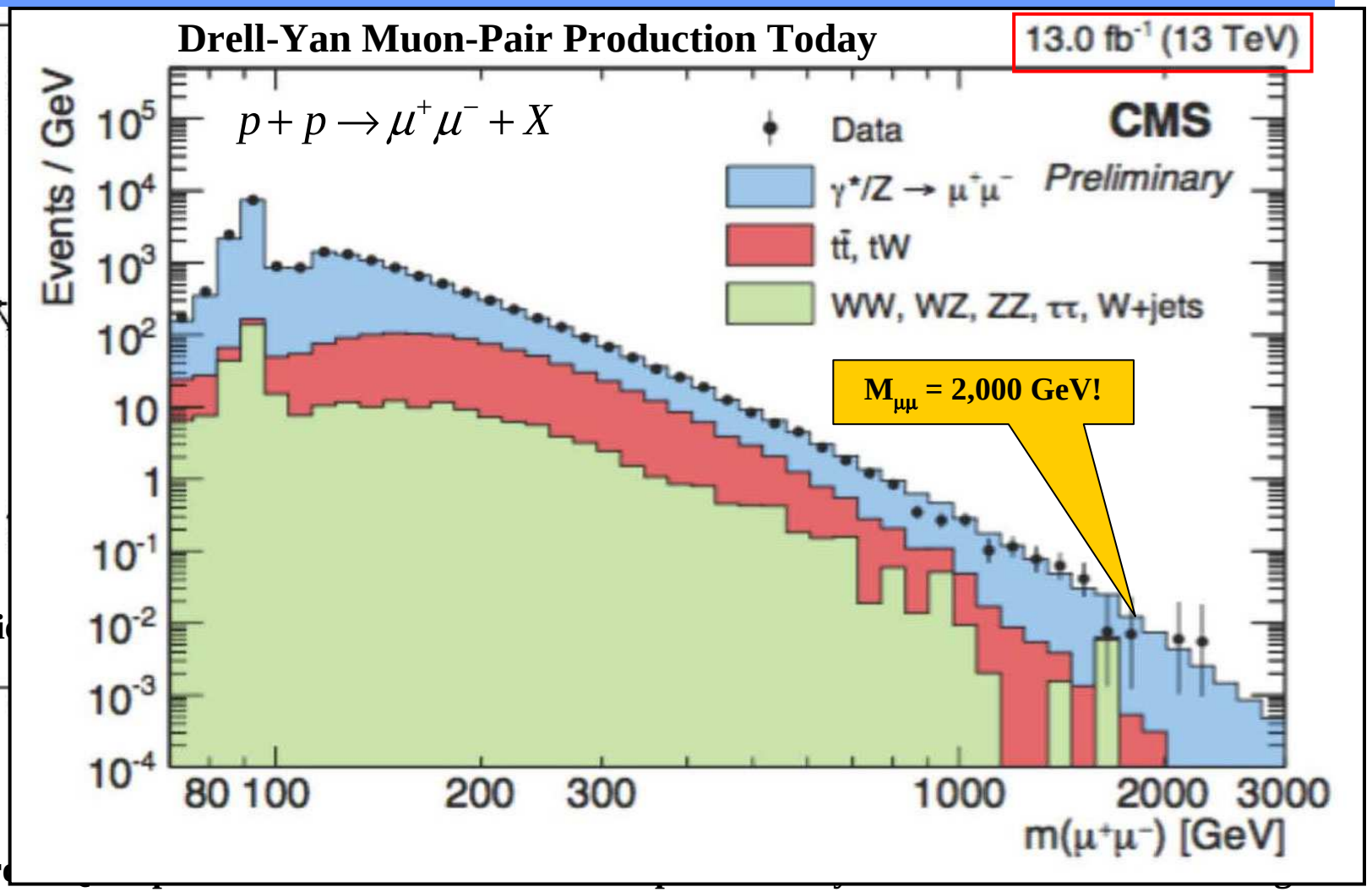
Use $Q = M_{\mu\mu}$ as the scale!

$$Q = M_{\mu\mu}$$

QCD Improved Parton Model
Use the "renormalization group improved" PDF's from DIS!

➡ **Leading order QCD prediction of the di-muon mass spectrum at y = 0 and W = 27.4 GeV using the "renormalization group" improved quark and anti-quark distributions.**





- **Leading or… the "renormalization group" improved quark and anti-quark distributions.**

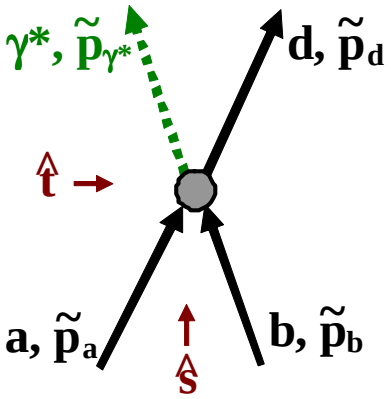
QCD Kinematics (1)

a+b→γ*+d

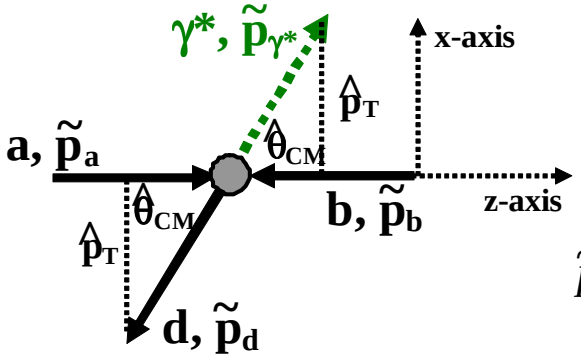
(neglect the mass of a, b, and d)

$$\tilde{p}_a^{CM} + \tilde{p}_b^{CM} = \tilde{p}_{\gamma^*}^{CM} + \tilde{p}_d^{CM}$$

2-to-2 Scattering



Center-of-Mass Frame



$$\tilde{p}_a^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ 0 \\ 0 \\ \hat{p}_{CM} \end{pmatrix} \qquad \tilde{p}_b^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ 0 \\ 0 \\ -\hat{p}_{CM} \end{pmatrix}$$

$$\tilde{p}_{\gamma^*}^{CM} = \begin{pmatrix} \sqrt{\hat{p}'^2_{CM} + M^3_{\gamma^*}} \\ \hat{p}'_{CM} \sin\hat{\theta}_{CM} \\ 0 \\ \hat{p}'_{CM} \cos\hat{\theta}_{CM} \end{pmatrix} \qquad \tilde{p}_d^{CM} = \begin{pmatrix} \hat{p}'_{CM} \\ -\hat{p}'_{CM} \sin\hat{\theta}_{CM} \\ 0 \\ -\hat{p}'_{CM} \cos\hat{\theta}_{CM} \end{pmatrix}$$

$$\hat{s}+\hat{t}+\hat{u} = M^2_{\gamma^*} \qquad \hat{s} = (\tilde{p}_a + \tilde{p}_b)^2 = 2\tilde{p}_a \cdot \tilde{p}_b = 4\hat{p}^2_{CM}$$

$$\hat{t} = (\tilde{p}_{\gamma^*} - \tilde{p}_a)^2 = (\tilde{p}_b - \tilde{p}_d)^2 = -2\tilde{p}_b \cdot \tilde{p}_d = -2\hat{p}_{CM}\hat{p}'_{CM}(1-\cos\hat{\theta}_{CM})$$

$$\hat{u} = (\tilde{p}_{\gamma^*} - \tilde{p}_b)^2 = (\tilde{p}_a - \tilde{p}_d)^2 = -2\tilde{p}_a \cdot \tilde{p}_d = -2\hat{p}_{CM}\hat{p}'_{CM}(1+\cos\hat{\theta}_{CM})$$

$$\hat{p}_T = \hat{p}'_{CM}\sin\hat{\theta}_{CM} \qquad \hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}}$$

$$\hat{p}_{CM} = \tfrac{1}{2}\sqrt{\hat{s}} \qquad \hat{p}'_{CM} = \frac{(\hat{s}-M^2_{\gamma^*})}{2\sqrt{\hat{s}}}$$

$$\hat{t} = -2p_{CM}p'_{CM}(1-\cos\hat{\theta}_{CM}) = -2\hat{p}_{CM}\hat{p}_T\frac{(1-\cos\hat{\theta}_{CM})}{\sin\hat{\theta}_{CM}} = -\tfrac{1}{2}\hat{s}\hat{x}_T\tan(\hat{\theta}_{CM}/2)$$

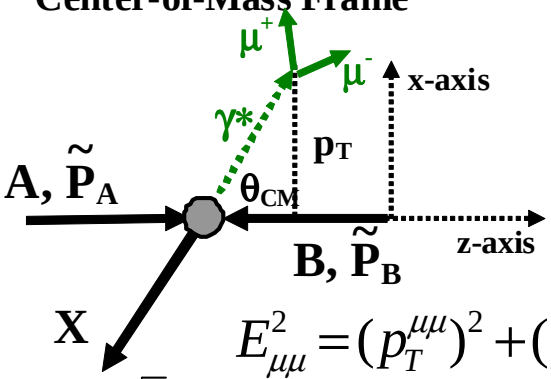
$$\hat{u} = -2p_{CM}p'_{CM}(1+\cos\hat{\theta}_{CM}) = -2\hat{p}_{CM}\hat{p}_T\frac{(1+\cos\hat{\theta}_{CM})}{\sin\hat{\theta}_{CM}} = -\tfrac{1}{2}\hat{s}\hat{x}_T\cot(\hat{\theta}_{CM}/2)$$

$$\hat{p}_T^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

QCD Kinematics (2)

$A+B \to (\gamma^* \to \mu^+\mu^-)+X$ **(neglect the mass of A and B)**

Center-of-Mass Frame



$$\tilde{P}_A^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ p_{CM} \end{pmatrix} \qquad \tilde{P}_B^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ -p_{CM} \end{pmatrix} \qquad \tilde{P}_{\mu\mu}^{CM} = \begin{pmatrix} E_{\mu\mu} \\ p_T^{\mu\mu} \\ 0 \\ p_L^{\mu\mu} \end{pmatrix}$$

$$\tau = M_{\mu\mu}^2 / s$$
$$x_T = 2p_T^{\mu\mu} / \sqrt{s}$$
$$x_L = 2p_L^{\mu\mu} / \sqrt{s}$$
$$x_E^2 = x_T^2 + x_L^2 + 4\tau$$

$$E_{\mu\mu}^2 = (p_T^{\mu\mu})^2 + (p_L^{\mu\mu})^2 + M_{\mu\mu}^2$$

$$p_{CM} = \tfrac{1}{2}\sqrt{s}$$

$$s = (\tilde{P}_A + \tilde{P}_B)^2 = 2\tilde{P}_A \cdot \tilde{P}_B = 4p_{CM}^2$$

$$t = (\tilde{P}_{\mu\mu} - \tilde{P}_A)^2 = M_{\mu\mu}^2 - 2\tilde{P}_{\mu\mu} \cdot \tilde{P}_A = M_{\mu\mu}^2 - 2p_{CM}(E_{\mu\mu} - P_L^{\mu\mu})$$

$$u = (\tilde{P}_{\mu\mu} - \tilde{P}_B)^2 = M_{\mu\mu}^2 - 2\tilde{P}_{\mu\mu} \cdot \tilde{P}_B = M_{\mu\mu}^2 - 2p_{CM}(E_{\mu\mu} + P_L^{\mu\mu})$$

Rapidity of Muon-Pair

$$y_{\mu\mu} \equiv \frac{1}{2}\log\left(\frac{E_{\mu\mu} + p_L^{\mu\mu}}{E_{\mu\mu} - p_L^{\mu\mu}}\right) = \frac{1}{2}\log\left(\frac{x_E + x_L}{x_E - x_L}\right) = \frac{1}{2}\log\left(\frac{x_1}{x_2}\right)$$

$$e^{-2y_{\mu\mu}} = \frac{x_E - x_L}{x_E + x_L} = \frac{(x_E - x_L)^2}{x_E^2 - x_L^2} = \frac{(x_E - x_L)^2}{x_E^2 + 4\tau}$$

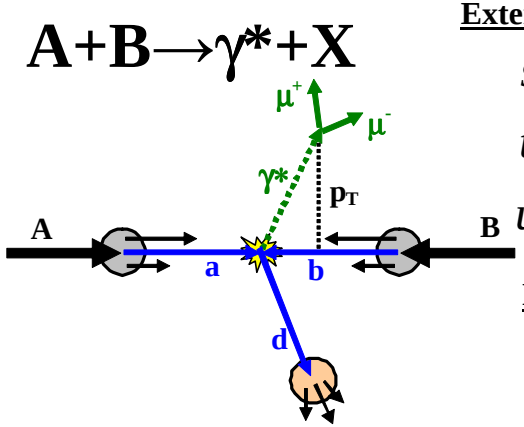
$$e^{2y_{\mu\mu}} = \frac{x_E + x_L}{x_E - x_L} = \frac{(x_E + x_L)^2}{x_E^2 - x_L^2} = \frac{(x_E + x_L)^2}{x_T^2 + 4\tau}$$

$$x_1 \equiv -(u - M^2)/s = \tfrac{1}{2}(x_E + x_L) = \tfrac{1}{2}\sqrt{x_T^2 + 4\tau}\, e^{+y_{\mu\mu}}$$

$$x_2 \equiv -(t - M^2)/s = \tfrac{1}{2}(x_E - x_L) = \tfrac{1}{2}\sqrt{x_T^2 + 4\tau}\, e^{-y_{\mu\mu}}$$

$$x_1 - x_2 = x_L \qquad x_1 x_2 = \tfrac{1}{4}x_T^2 + \tau \qquad p_T^2 = \frac{tu}{s}$$

QCD Naïve Parton Model



External Variables (neglect the mass of A and B)

$$s=(\tilde{P}_A+\tilde{P}_B)^2=2\tilde{P}_A\cdot\tilde{P}_B=4p_{CM}^2$$
$$t=(\tilde{P}_{\mu\mu}-\tilde{P}_A)^2=M_{\mu\mu}^2-2\tilde{P}_{\mu\mu}\cdot\tilde{P}_A$$
$$u=(\tilde{P}_{\mu\mu}-\tilde{P}_B)^2=M_{\mu\mu}^2-2\tilde{P}_{\mu\mu}\cdot\tilde{P}_B$$

Rapidity of Muon-Pair

$$y_{\mu\mu}=\frac{1}{2}\log\left(\frac{x_1}{x_2}\right)$$

$$x_E^2=x_T^2+x_L^2+4\tau$$

$$\tau=M_{\mu\mu}^2/s$$
$$x_T=2p_T^{\mu\mu}/\sqrt{s}$$
$$x_L=2p_L^{\mu\mu}/\sqrt{s}$$

Internal-External Connection

$$\tilde{p}_a=x_a\tilde{P}_A$$
$$\tilde{p}_b=x_b\tilde{P}_B$$

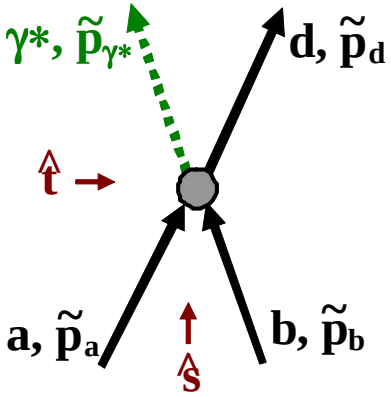
$$x_1\equiv-(u-M^2)/s=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}}$$
$$x_2\equiv-(t-M^2)/s=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$
$$x_L=x_1-x_2$$

Internal Variables (neglect the mass of a, b, and d)

$$\hat{s}=(\tilde{p}_a+\tilde{p}_b)^2=2\tilde{p}_a\cdot\tilde{p}_b=x_ax_b(2\tilde{P}_A\cdot\tilde{P}_B)=x_ax_bs$$

a+b→γ*+d

2-to-2 Scattering



$$\hat{t}=(\tilde{p}_{\gamma^*}-\tilde{p}_a)^2=M_{\mu\mu}^2-2\tilde{p}_{\gamma^*}\cdot\tilde{p}_a=M_{\mu\mu}^2-x_a(2\tilde{P}_{\mu\mu}\cdot\tilde{P}_A)=M_{\mu\mu}^2+x_a(t-M_{\mu\mu}^2)$$
$$\hat{u}=(\tilde{p}_{\gamma^*}-\tilde{p}_b)^2=M_{\mu\mu}^2-2\tilde{p}_c\cdot\tilde{p}_b=M_{\mu\mu}^2-x_b(2\tilde{P}_{\mu\mu}\cdot\tilde{P}_B)=M_{\mu\mu}^2+x_b(u-M_{\mu\mu}^2)$$

$$\hat{s}+\hat{t}+\hat{u}=M_{\mu\mu}^2 \qquad \hat{s}+(\hat{t}-M_{\mu\mu}^2)+(\hat{u}-M_{\mu\mu}^2)=-M_{\mu\mu}^2$$

$$\hat{t}-M_{\mu\mu}^2=x_a(t-M_{\mu\mu}^2)=-x_ax_2s \qquad x_ax_b-x_ax_2-x_bx_1=-\tau$$
$$\hat{u}-M_{\mu\mu}^2=x_b(u-M_{\mu\mu}^2)=-x_bx_1s$$

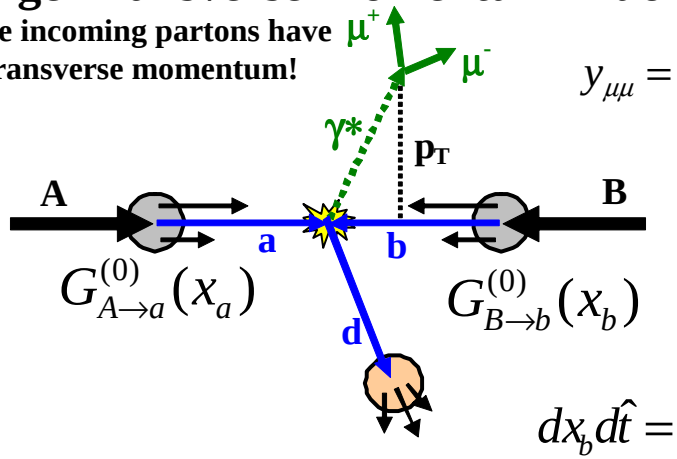
$$p_T^2=\frac{\hat{t}\hat{u}}{\hat{s}}$$

$$x_a=\frac{x_bx_1-\tau}{x_b-x_2} \qquad x_b=\frac{x_ax_2-\tau}{x_a-x_1}$$

QCD Naïve Parton Model

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$

Assume incoming partons have no transverse momentum!



$$y_{\mu\mu}=\frac{1}{2}\log\left(\frac{x_1}{x_2}\right) \qquad x_T^2=4x_1x_2+4\tau$$

$$x_1=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}} \qquad x_2=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$

$$dx_1dx_2=\tfrac{1}{4}dy_{\mu\mu}dx_T^2$$

$$x_b=\frac{x_ax_2-\tau}{x_a-x_1} \qquad \hat{t}=M^2_{\mu\mu}-x_ax_2s$$

$$dx_bd\hat{t}=\left(\frac{sx_ax_b}{x_a-x_1}\right)dx_1dx_2=\left(\frac{\frac{1}{4}sx_ax_b}{x_a-x_1}\right)dy_{\mu\mu}dx_T^2=\left(\frac{x_ax_b}{x_a-x_1}\right)dy_{\mu\mu}dp_T^2$$

$$\frac{d\sigma^{A+B\to\gamma^*+X}}{dM^2_{\mu\mu}}(s,M^2_{\mu\mu},y_{\mu\mu},p_T)=G^{(0)}_{A\to a}(x_a)dx_aG^{(0)}_{B\to b}(x_b)dx_b\left(\frac{d\hat{\sigma}^{a+b\to\gamma^*+d}}{dM^2_{\mu\mu}d\hat{t}}(\hat{s},\hat{t})\right)d\hat{t}$$

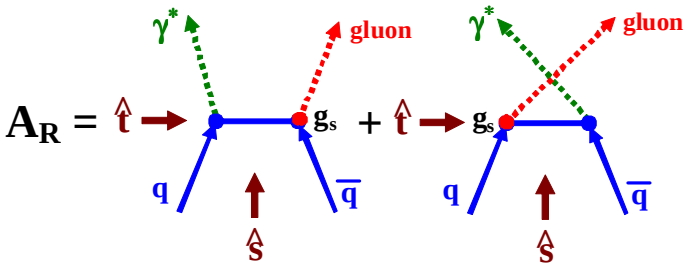
$$\frac{d\sigma^{AB}_{DY}}{dM^2_{\mu\mu}dy_{\mu\mu}dp_T^2}(s,M^2_{\mu\mu},y_{\mu\mu},p_T)=\int_{x_a^{\min}}^{1.0}dx_aG^{(0)}_{A\to a}(x_a)G^{(0)}_{B\to b}(x_b)\left(\frac{x_ax_b}{x_a-x_1}\right)\left(\frac{d\hat{\sigma}^{a+b\to\gamma^*+d}}{dM^2_{\mu\mu}d\hat{t}}(\hat{s},\hat{t})\right)$$

$$x_a=\frac{x_bx_1-\tau}{x_b-x_2} \xrightarrow{x_b=1} x_a^{\min}=\frac{x_1-\tau}{1-x_2} \qquad x_b=\frac{x_ax_2-\tau}{x_a-x_1}$$

QCD Differential Cross-Sections

Large Transverse Momentum Muon-Pair Production (Order α_s)

"Annihilation" $q+\bar{q} \to \gamma^*+g$

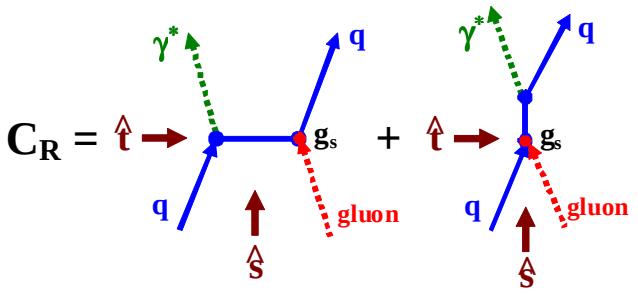


$$\frac{d\hat{\sigma}^{q+\bar{q}\to\gamma^*+g}}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t}) = \frac{8}{27}\frac{\alpha_s \alpha_{em}^2 e_q^2}{M_{\mu\mu}^2 \hat{s}^2}\left(\frac{2M_{\mu\mu}^2\hat{s}+\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}}\right)$$

$$\frac{d\hat{\sigma}^{q+\bar{q}\to\gamma^*+g}}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t}) = e_q^2 \frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})$$

$$\hat{s}+\hat{t}+\hat{u} = M_{\mu\mu}^2 \qquad p_T^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

"Compton" $q+g \to \gamma^*+q$



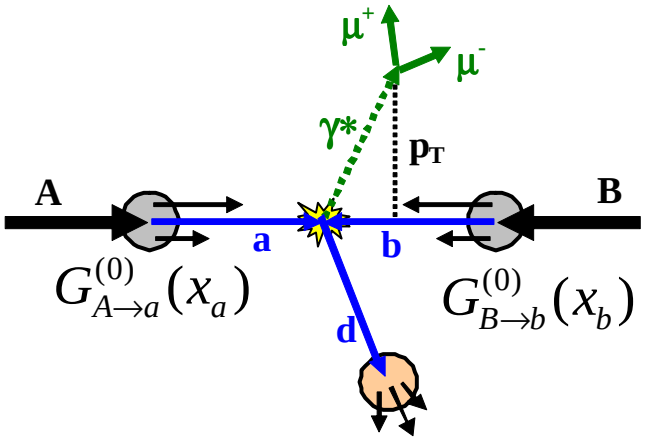
$$\frac{d\hat{\sigma}^{q+g\to\gamma^*+q}}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t}) = \frac{1}{9}\frac{\alpha_s \alpha_{em}^2 e_q^2}{M_{\mu\mu}^2 \hat{s}^2}\left(\frac{2M_{\mu\mu}^2\hat{u}+\hat{s}^2+\hat{t}^2}{-\hat{s}\hat{t}}\right)$$

$$\frac{d\hat{\sigma}^{q+g\to\gamma^*+q}}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t}) = e_q^2 \frac{d\hat{\sigma}_C}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})$$

QCD Naïve Parton Model

Large Transverse Momentum Muon-Pair Production (Order α_s)

$$A+B\to\mu^+\mu^-+X$$



Parton Flux

$$P_A^{(0)}(x_a,x_b)=\sum_{i=1}^{nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to\bar{q}_i}(x_b)+G^{(0)}_{A\to\bar{q}_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$P_C^{(0)}(x_a,x_b)=\sum_{i=1}^{2nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to g}(x_b)+G^{(0)}_{A\to g}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$\frac{d\sigma_A^{DY}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\int_{x_a^{\min}}^{1.0}dx_aP_A^{(0)}(x_a,x_b)\left(\frac{x_ax_b}{x_a-x_1}\right)\left(\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2d\hat{t}}(\hat{s},\hat{t})\right)$$

$$\frac{d\sigma_C^{DY}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\int_{x_a^{\min}}^{1.0}dx_aP_C^{(0)}(x_a,x_b)\left(\frac{x_ax_b}{x_a-x_1}\right)\left(\frac{d\hat{\sigma}_C}{dM_{\mu\mu}^2d\hat{t}}(\hat{s},\hat{t})\right)$$

$$x_1=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}}$$
$$x_2=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$

$$x_a^{\min}=\frac{x_1-\tau}{1-x_2}$$

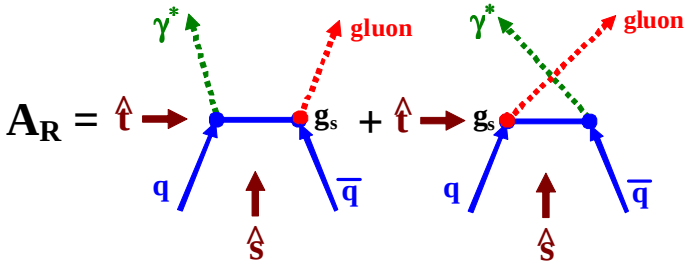
$$x_b=\frac{x_ax_2-\tau}{x_a-x_1}$$

QCD Differential Cross-Sections

Large Transverse Momentum Muon-Pair Production (Order α_s)

$A+B\to\mu^+\mu^-+X$

"Annihilation" $q+\bar{q}\to\gamma^*+g$



$$\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8}{27}\frac{\alpha_s\alpha_{em}^2}{M_{\mu\mu}^2\hat{s}^2}\left(\frac{2M_{\mu\mu}^2\hat{s}+\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}}\right)$$

$$\hat{s}+\hat{t}+\hat{u}=M_{\mu\mu}^2 \qquad p_T^2=\frac{\hat{t}\hat{u}}{\hat{s}} \qquad \hat{\tau}=M_{\mu\mu}^2/\hat{s}$$

$$\frac{\hat{t}}{\hat{s}}+\frac{\hat{u}}{\hat{s}}=-(1-\hat{\tau}) \qquad \frac{p_T^2}{\hat{s}}=\left(\frac{\hat{t}}{\hat{s}}\right)\left(\frac{\hat{u}}{\hat{s}}\right)=\left(\frac{\hat{t}}{\hat{s}}\right)\left(-(1-\hat{\tau})-\frac{\hat{t}}{\hat{s}}\right)$$

$$\hat{t}=-\frac{\hat{s}}{2}\left\{(1-\hat{\tau})-\sqrt{(1-\hat{\tau})^2-4p_T^2/\hat{s}}\right\}$$

$$\hat{u}=-\frac{\hat{s}}{2}\left\{(1-\hat{\tau})+\sqrt{(1-\hat{\tau})^2-4p_T^2/\hat{s}}\right\}$$

$$\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8}{27}\frac{\alpha_s\alpha_{em}^2}{M_{\mu\mu}^2 p_T^2\hat{s}}\left(1+\hat{\tau}^2-\frac{2p_T^2}{\hat{s}}\right) \qquad \hat{s}=x_a x_b s \qquad \hat{\tau}=\frac{\tau}{x_a x_b}$$

$$\left(\frac{x_a x_b}{x_a-x_1}\right)\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8\alpha_s\alpha_{em}^2}{27}\frac{1}{(x_a-x_1)}\frac{\tau}{M_{\mu\mu}^4 p_T^2}\left(1+\frac{\tau^2}{(x_a x_b)^2}-\frac{x_T^2}{2x_a x_b}\right)$$

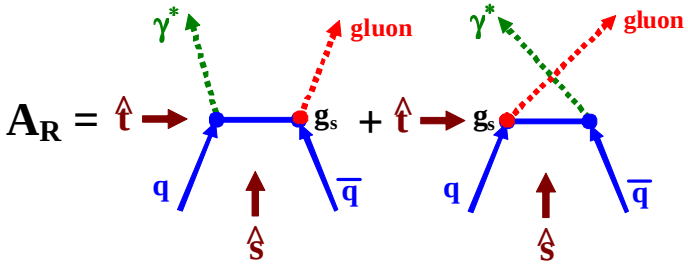
$$\tau=M_{\mu\mu}^2/s \qquad x_T=2p_T/\sqrt{s}$$

QCD Differential Cross-Sections

Large Transverse Momentum Muon-Pair Production (Order α_s)

$A+B\to\mu^+\mu^-+X$

"Annihilation" $q+\bar{q}\to\gamma^*+g$



$$\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8}{27}\frac{\alpha_s\alpha_{em}^2}{M_{\mu\mu}^2\hat{s}^2}\left(\frac{2M_{\mu\mu}^2\hat{s}+\hat{u}^2+\hat{t}^2}{\hat{t}\hat{u}}\right)$$

$$\hat{s}+\hat{t}+\hat{u}=M_{\mu\mu}^2 \qquad p_T^2=\frac{\hat{t}\hat{u}}{\hat{s}} \qquad \hat{\tau}=M_{\mu\mu}^2/\hat{s}$$

$$\frac{\hat{t}}{\hat{s}}+\frac{\hat{u}}{\hat{s}}=-(1-\hat{\tau}) \qquad \frac{p_T^2}{\hat{s}}=\left(\frac{\hat{t}}{\hat{s}}\right)\left(\frac{\hat{u}}{\hat{s}}\right)=\left(\frac{\hat{t}}{\hat{s}}\right)\left(-(1-\hat{\tau})-\frac{\hat{t}}{\hat{s}}\right)$$

$$\hat{t}=-\frac{\hat{s}}{2}\left\{(1-\hat{\tau})-\sqrt{(1-\hat{\tau})^2-4p_T^2/\hat{s}}\right\}$$

$$\hat{u}=-\frac{\hat{s}}{2}\left\{(1-\hat{\tau})+\sqrt{(1-\hat{\tau})^2-4p_T^2/\hat{s}}\right\}$$

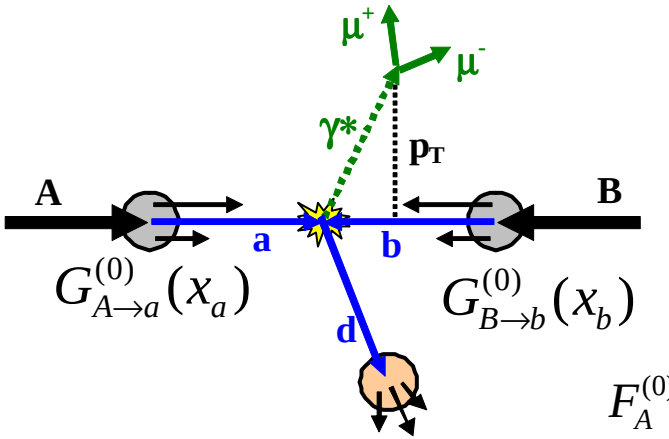
$$\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8}{27}\frac{\alpha_s\alpha_{em}^2}{M_{\mu\mu}^2 p_T^2\hat{s}}\left(1+\hat{\tau}^2-\frac{2p_T^2}{\hat{s}}\right) \qquad \hat{s}=x_a x_b s \qquad \hat{\tau}=\frac{\tau}{x_a x_b}$$

$$\left(\frac{x_a x_b}{x_a-x_1}\right)\frac{d\hat{\sigma}_A}{dM_{\mu\mu}^2 d\hat{t}}(\hat{s},\hat{t})=\frac{8\alpha_s\alpha_{em}^2}{27}\frac{1}{(x_a-x_1)}\frac{\tau}{M_{\mu\mu}^4 p_T^2}\left(1+\frac{\tau^2}{(x_a x_b)^2}-\frac{x_T^2}{2x_a x_b}\right)$$

$$\tau=M_{\mu\mu}^2/s \qquad x_T=2p_T/\sqrt{s}$$

QCD Naïve Parton Model

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$



Parton Flux

$$P_A^{(0)}(x_a,x_b)=\sum_{i=1}^{nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to\bar{q}_i}(x_b)+G^{(0)}_{A\to\bar{q}_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$P_C^{(0)}(x_a,x_b)=\sum_{i=1}^{2nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to g}(x_b)+G^{(0)}_{A\to g_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$F_A^{(0)}(\tau,y_{\mu\mu},x_T)=\frac{8\alpha_{em}^2}{27}\int_{x_a^{\min}}^{1.0}dx_a P_A^{(0)}(x_a,x_b)\frac{\tau}{(x_a-x_1)}\left(1+\frac{\tau^2}{(x_a x_b)^2}-\frac{x_T^2}{2x_a x_b}\right)$$

$$x_a^{\min}=\frac{x_1-\tau}{1-x_2}\qquad x_b=\frac{x_a x_2-\tau}{x_a-x_1}$$

$$x_1=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}}$$
$$x_2=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$

$$M_{\mu\mu}^4 p_T^2\frac{d\sigma_A^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_s F_A^{(0)}(\tau,y_{\mu\mu},x_T)$$

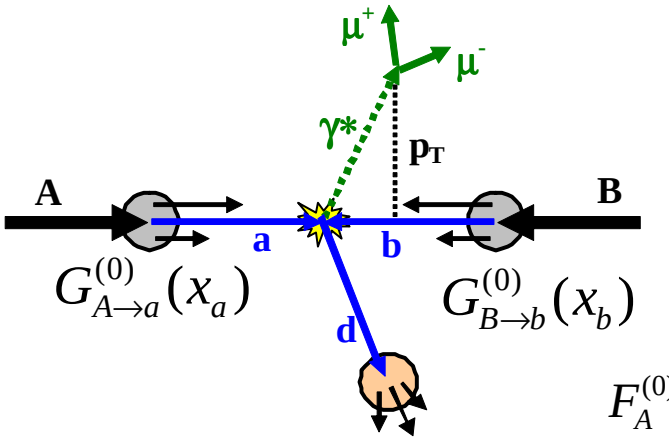
$$M_{\mu\mu}^4 p_T^2\frac{d\sigma_C^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_s F_C^{(0)}(\tau,y_{\mu\mu},x_T)$$

= Constant at fixed τ, $y_{\mu\mu}$, and x_T!
(provided the PDF's scale and α_s is constant)

Parton Model Scaling

QCD Naïve Parton Model

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$



Parton Flux

$$P_A^{(0)}(x_a,x_b)=\sum_{i=1}^{nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to\bar{q}_i}(x_b)+G^{(0)}_{A\to\bar{q}_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$P_C^{(0)}(x_a,x_b)=\sum_{i=1}^{2nf}e_{q_i}^2\left[G^{(0)}_{A\to q_i}(x_a)G^{(0)}_{B\to g}(x_b)+G^{(0)}_{A\to g_i}(x_a)G^{(0)}_{B\to q_i}(x_b)\right]$$

$$F_A^{(0)}(\tau,y_{\mu\mu},x_T)=\frac{8\alpha_{em}^2}{27}\int_{x_a^{\min}}^{1.0}dx_a P_A^{(0)}(x_a,x_b)\frac{\tau}{(x_a-x_1)}\left(1+\frac{\tau^2}{(x_ax_b)^2}-\frac{x_T^2}{2x_ax_b}\right)$$

$$x_a^{\min}=\frac{x_1-\tau}{1-x_2}\qquad x_b=\frac{x_ax_2-\tau}{x_a-x_1}$$

$$x_1=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}}$$
$$x_2=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$

$$M_{\mu\mu}^4p_T^2\frac{d\sigma_A^{DY}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_sF_A^{(0)}(\tau,y_{\mu\mu},x_T)$$

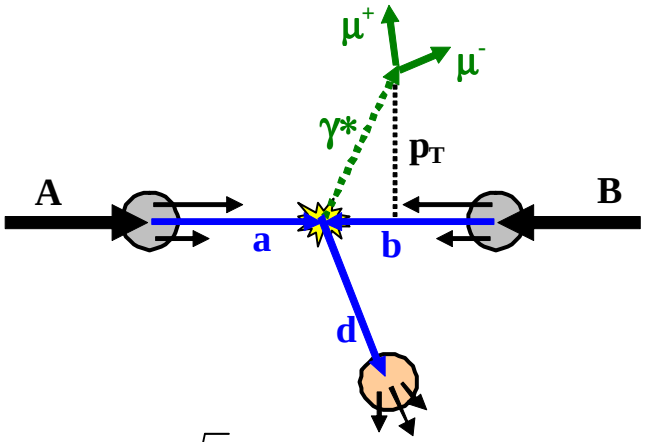
$$M_{\mu\mu}^4p_T^2\frac{d\sigma_C^{DY}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_sF_C^{(0)}(\tau,y_{\mu\mu},x_T)$$

= Constant at fixed τ, $y_{\mu\mu}$, and x_T!
(provided the PDF's scale and α_s is constant)

Parton Model Scaling

QCD Parton Model Scaling

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$



$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu} d^2 p_T}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \pi\alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$M_{\mu\mu}^3 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu} dy_{\mu\mu} d^2 p_T}(s, M_{\mu\mu}, y_{\mu\mu}, p_T) = 2\pi\alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$W = \sqrt{s}$$

$$\tau = M_{\mu\mu}^2 / s = M_{\mu\mu}^2 / W^2$$

$$x_T = 2p_T / W$$

$$M_{\mu\mu}^3 p_T^2 = (W\sqrt{\tau})^3 (\tfrac{1}{2} W x_T)^2 = \tfrac{1}{4} W^5 \tau^{\frac{3}{2}} x_T^2$$

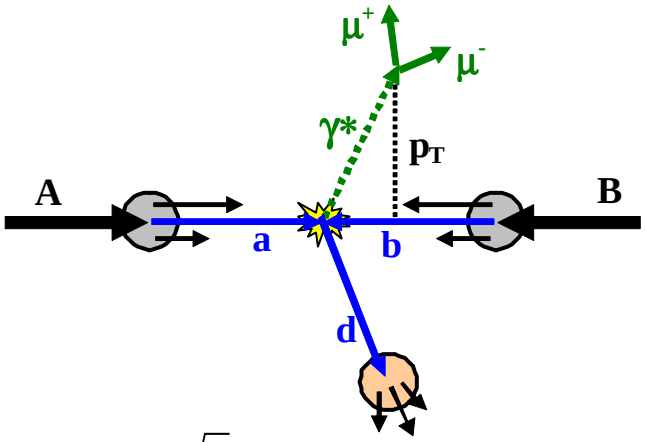
$$W^5 \frac{d\sigma^{DY}}{dM_{\mu\mu} dy_{\mu\mu} d^2 p_T}(W, M_{\mu\mu}, y_{\mu\mu}, p_T) = \frac{8\pi\alpha_s}{\tau^{\frac{3}{2}} x_T^2} F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

= Constant at fixed τ, $y_{\mu\mu}$, and x_T!
(provided the PDF's scale and α_s is constant)

Parton Model Scaling

QCD Parton Model Scaling

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$



$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu} d^2 p_T}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \pi\alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$M_{\mu\mu}^3 p_T^2 \frac{d\sigma^{DY}}{dM_{\mu\mu} dy_{\mu\mu} d^2 p_T}(s, M_{\mu\mu}, y_{\mu\mu}, p_T) = 2\pi\alpha_s F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

$$W = \sqrt{s}$$

$$\tau = M_{\mu\mu}^2 / s = M_{\mu\mu}^2 / W^2$$

$$x_T = 2p_T / W$$

$$M_{\mu\mu}^3 p_T^2 = (W\sqrt{\tau})^3 (\tfrac{1}{2} W x_T)^2 = \tfrac{1}{4} W^5 \tau^{\frac{3}{2}} x_T^2$$

$$W^5 \frac{d\sigma^{DY}}{dM_{\mu\mu} dy_{\mu\mu} d^2 p_T}(W, M_{\mu\mu}, y_{\mu\mu}, p_T) = \frac{8\pi\alpha_s}{\tau^{\frac{3}{2}} x_T^2} F^{(0)}(\tau, y_{\mu\mu}, x_T)$$

= Constant at fixed τ, $y_{\mu\mu}$, and x_T!
(provided the PDF's scale and α_s is constant)

Parton Model Scaling

QCD Improved Parton Model

Large Transverse Momentum Muon-Pair Production (Order α_s) $A+B\to\mu^+\mu^-+X$

Parton Flux

$$P_A^{DIS}(x_a,x_b,Q^2)=\sum_{i=1}^{nf}e_{q_i}^2\left[G_{A\to q_i}^{DIS}(x_a,Q^2)G_{B\to\bar{q}_i}^{DIS}(x_b,Q^2)+G_{A\to\bar{q}_i}^{DIS}(x_a,Q^2)G_{B\to q_i}^{DIS}(x_b,Q^2)\right]$$

$$P_C^{DIS}(x_a,x_b,Q^2)=\sum_{i=1}^{2nf}e_{q_i}^2\left[G_{A\to q_i}^{DIS}(x_a,Q^2)G_{B\to g}^{DIS}(x_b,Q^2)+G_{A\to g_i}^{DIS}(x_a,Q^2)G_{B\to q_i}^{DIS}(x_b,Q^2)\right]$$

$$F_A^{DIS}(\tau,y_{\mu\mu},x_T,Q^2)=\frac{8\alpha_{em}^2}{27}\int_{x_a^{\min}}^{1.0}dx_a P_A^{DIS}(x_a,x_b,Q^2)\frac{\tau}{(x_a-x_1)}\left(1+\frac{\tau^2}{(x_ax_b)^2}-\frac{x_T^2}{2x_ax_b}\right)$$

Use Q = $M_{\mu\mu}$ as the scale!

$$Q=M_{\mu\mu}$$

$$x_a^{\min}=\frac{x_1-\tau}{1-x_2}\qquad x_b=\frac{x_ax_2-\tau}{x_a-x_1}$$

$$x_1=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{+y_{\mu\mu}}\qquad x_2=\tfrac{1}{2}\sqrt{x_T^2+4\tau}e^{-y_{\mu\mu}}$$

Use the "renormalization group improved" PDF's from DIS and the running QCD coupling!

$$M_{\mu\mu}^4p_T^2\frac{d\sigma_A^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_s(Q^2)F_A^{DIS}(\tau,y_{\mu\mu},x_T,Q^2)$$

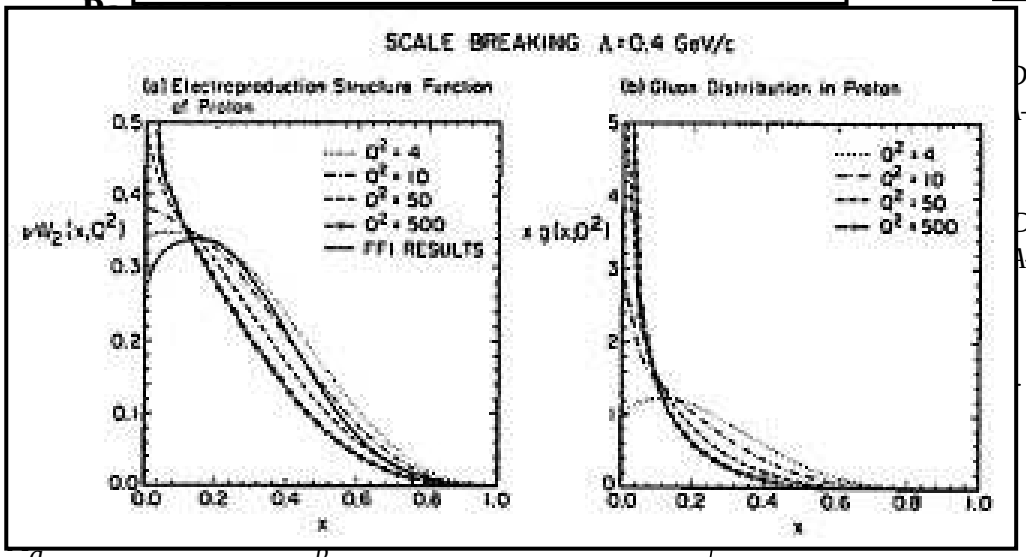
$$M_{\mu\mu}^4p_T^2\frac{d\sigma_C^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)=\alpha_s(Q^2)F_C^{DIS}(\tau,y_{\mu\mu},x_T,Q^2)$$

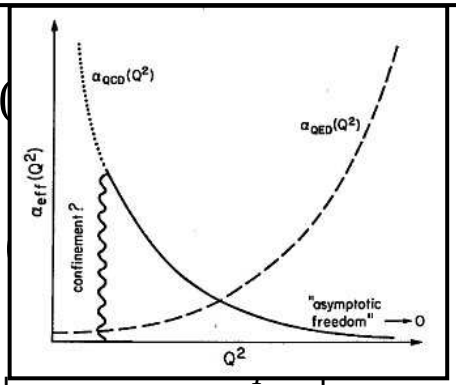
Naïve parton model scaling is now broken!

QCD Improved Parton Model

QCD Improved Parton Model

Large **Renormalization Group Improved PDF's!** **r Produ** **Running QCD Effective Coupling!** $\mu^+\mu^- + X$





Use Q = $M_{\mu\mu}$ as the scale!

$$Q = M_{\mu\mu}$$

$$\left(1+\frac{\quad}{(x_a x_b)^2}-\frac{\quad}{2x_a x_b}\right)$$

$$1-x_2 \qquad x_a - x_1 \qquad x_2 = \tfrac{1}{2}\sqrt{x_T^2 + 4\tau e^{-y_{\mu\mu}}}$$

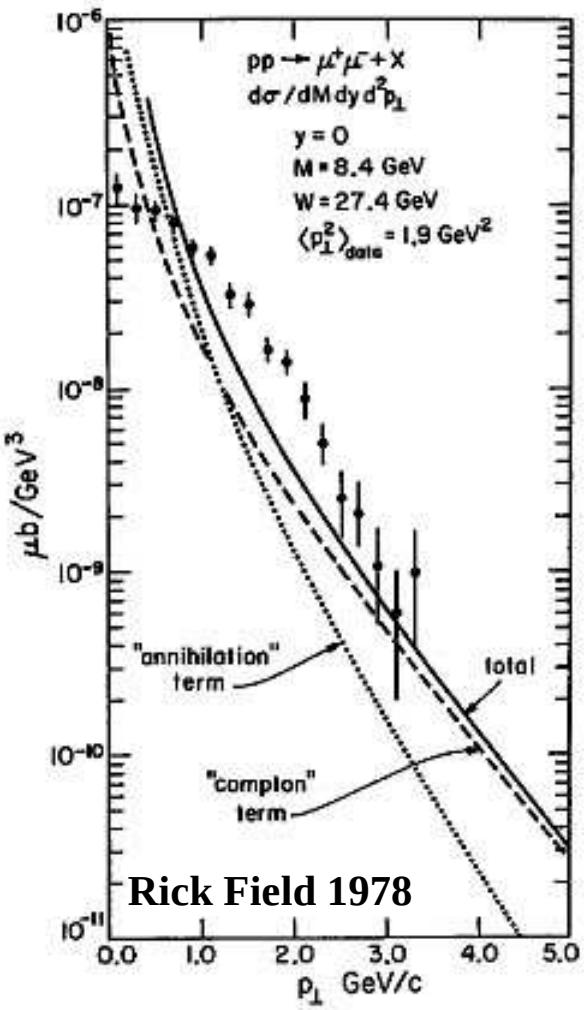
Use the "renormalization group improved" PDF's from DIS and the running QCD coupling!

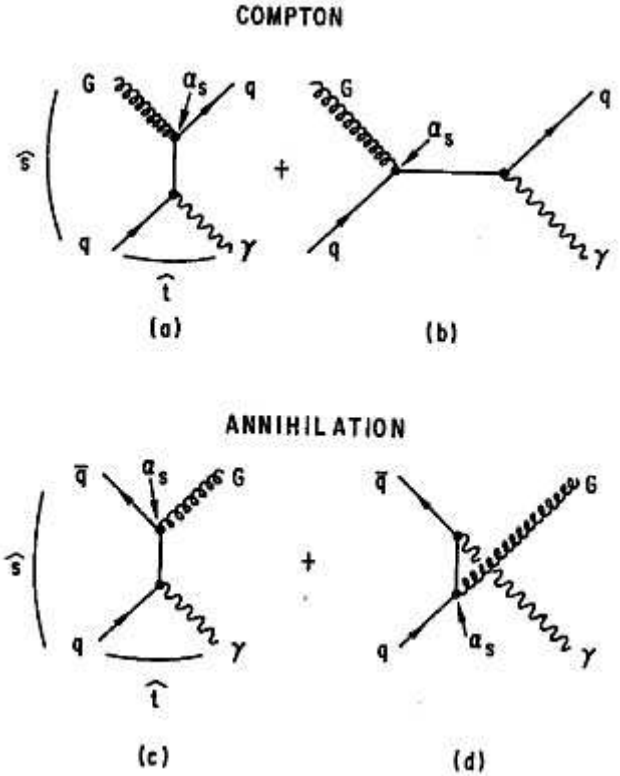
$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma_A^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \alpha_s(Q^2) F_A^{DIS}(\tau, y_{\mu\mu}, x_T, Q^2)$$

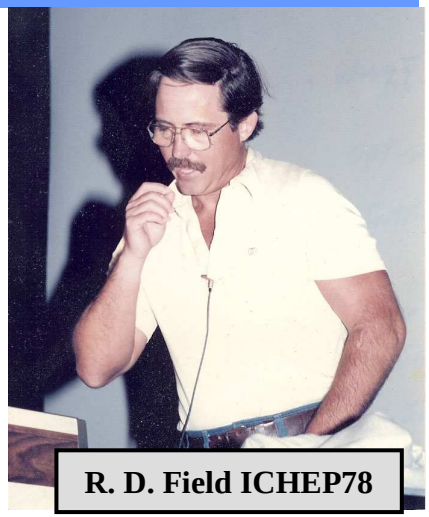
$$M_{\mu\mu}^4 p_T^2 \frac{d\sigma_C^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) = \alpha_s(Q^2) F_C^{DIS}(\tau, y_{\mu\mu}, x_T, Q^2)$$

Naïve parton model scaling is now broken!

QCD Improved Parton Model







➡ **The distribution in transverse momentum, p_T, of the $\mu^+\mu^-$ pair produced in pp collisions at W = 27.4 GeV together with the QCD perturbative predictions. The "Compton" and "annihilation" contributions are given by the dashed and dotted curves, respectively.**

QCD Improved Parton Model

Real Gluon Emissions Order α_s

$$\sigma_R^{DY}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)\equiv\frac{d\sigma_A^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)+\frac{d\sigma_C^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}dp_T^2}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T)$$

Born Term Plus Virtual Gluon Emissions Order α_s

$$\sigma_V^{DY}(s,M_{\mu\mu}^2,y_{\mu\mu})\equiv\frac{d\sigma_{B+V}^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}}(s,M_{\mu\mu}^2,y_{\mu\mu})$$

Total Cross-Section Order α_s

$$\sigma_{tot}^{DY}(s,M_{\mu\mu}^2,y_{\mu\mu})\equiv\frac{d\sigma_{B+V}^{A+B\to\gamma^*+X}}{dM_{\mu\mu}^2dy_{\mu\mu}}(s,M_{\mu\mu}^2,y_{\mu\mu})+\int\sigma_R^{DY}(s,M_{\mu\mu}^2,y_{\mu\mu},p_T^2)dp_T^2$$

Finite = Infinite + Infinite

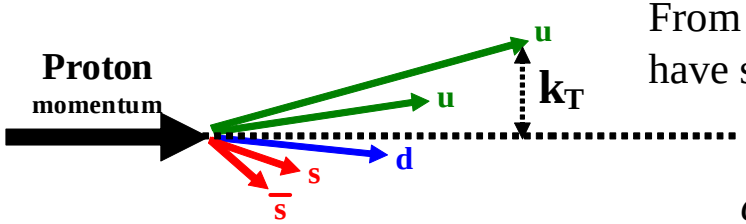
Total Cross-Section Leading Order

$$\sigma_{tot-LO}^{DY}(s,M_{\mu\mu}^2,y_{\mu\mu})=\frac{d\sigma_{tot-LO}^{DY}}{dM_{\mu\mu}^2dy_{\mu\mu}}(s,M_{\mu\mu}^2,y_{\mu\mu})=\frac{4\pi\alpha_{em}^2}{9M_{\mu\mu}^2s}P_{q\bar{q}}^{DIS}(x_a,x_b,M_{\mu\mu}^2)$$

$$P_{q\bar{q}}^{DIS}(x_a,x_b,M_{\mu\mu}^2)=\sum_{i=1}^{nf}e_{q_i}^2\left[G_{A\to q_i}^{DIS}(x_a,M_{\mu\mu}^2)G_{B\to\bar{q}_i}^{DIS}(x_b,M_{\mu\mu}^2)+G_{A\to\bar{q}_i}^{DIS}(x_a,M_{\mu\mu}^2)G_{B\to q_i}^{DIS}(x_b,M_{\mu\mu}^2)\right]$$

QCD Intrinsic Parton k_T

Parton Intrinsic (Primordial) Transverse Momentum:



From the uncertainty principle one expects that the partons will have some intrinsic (primordial) transverse momentum.

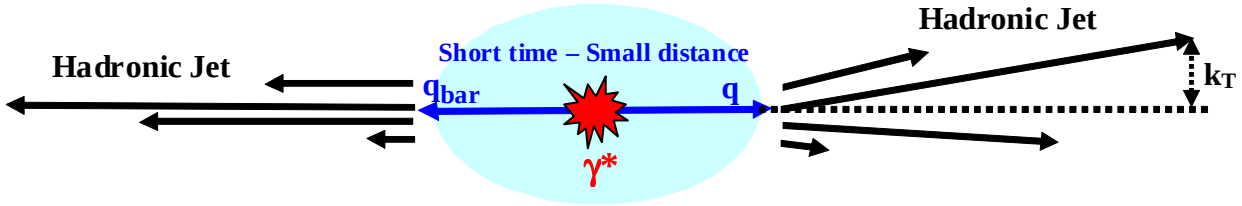
$$c\Delta p_x \approx \frac{\hbar c}{\Delta x} = \frac{197.3 MeV \cdot fm}{1\, fm} \approx 200\, MeV$$

Transverse size of the proton.

Expect intrinsic $k_T(P \rightarrow q) \approx 200 - 300$ MeV/c!

Jet Size: Also expect that the hadrons within a jet will have some intrinsic transverse momentom.

Two Jet Final State: e^+e^- to Jet + Jet



Expect intrinsic $k_T(q \rightarrow h) \approx 200 - 300$ MeV/c!

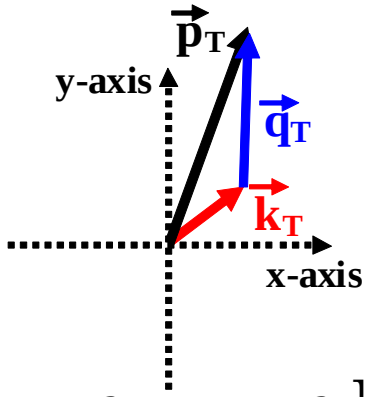
Primordial k_T Smearing

$$f(k_T^2) = \frac{1}{4\pi\sigma^2_{primordial}} e^{-\frac{k_T^2}{4\sigma^2_{primordial}}}$$

$$< k_T >_{primordial} = 2\sigma^2_{primordial}$$

Intrinsic Primordial transverse momentum!

$$\vec{q}_T = \vec{p}_T - \vec{k}_T$$



$$\sigma^{DY}_{smear}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma_{primordial}) = \int f(k_T^2)\left[\sigma^{DY}_R(s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) + \sigma^{DY}_V(s, M^2_{\mu\mu}, y_{\mu\mu})\delta(q_T^2)\right] d^2k_T$$

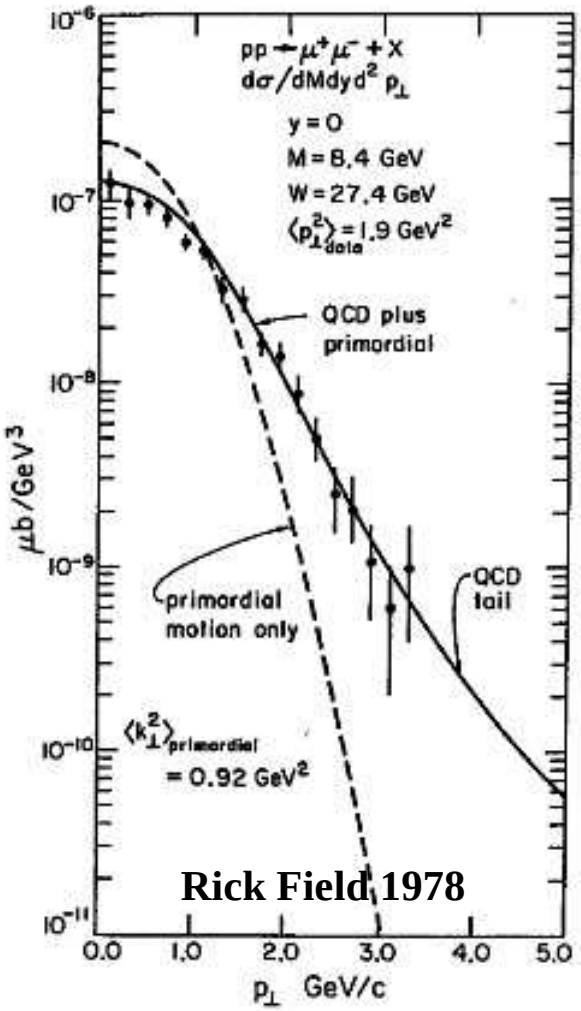
$$\sigma^{DY}_{smear}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma_{primordial}) = \int \sigma^{DY}_R(s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2)\left[f(k_T^2) - f(p_T^2)\right] d^2k_T$$

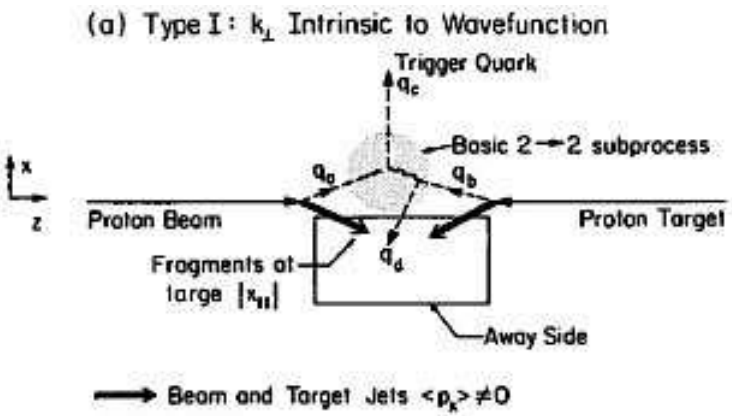
$$+ f(p_T^2)\int\left[\sigma^{DY}_R(s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2) + \sigma^{DY}_V(s, M^2_{\mu\mu}, y_{\mu\mu})\delta(q_T^2)\right] d^2q_T$$

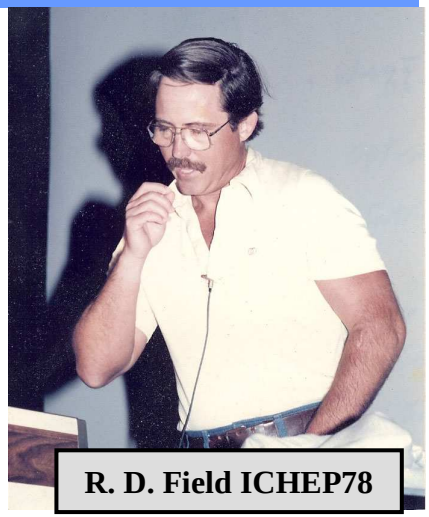
$$\sigma^{DY}_{smear}(s, M^2_{\mu\mu}, y_{\mu\mu}, p_T, \sigma) =$$

$$\int \sigma^{DY}_R(s, M^2_{\mu\mu}, y_{\mu\mu}, q_T^2)\left[f(k_T^2) - f(p_T^2)\right] d^2k_T + f(p_T^2)\sigma^{DY}_{tot}(s, M^2_{\mu\mu}, y_{\mu\mu})$$

Finite **Finite**

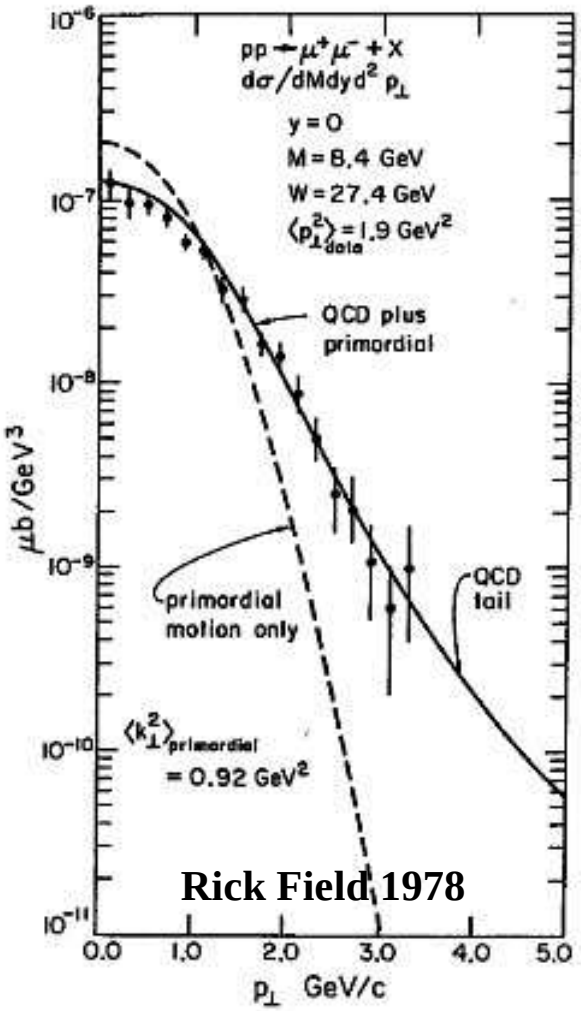


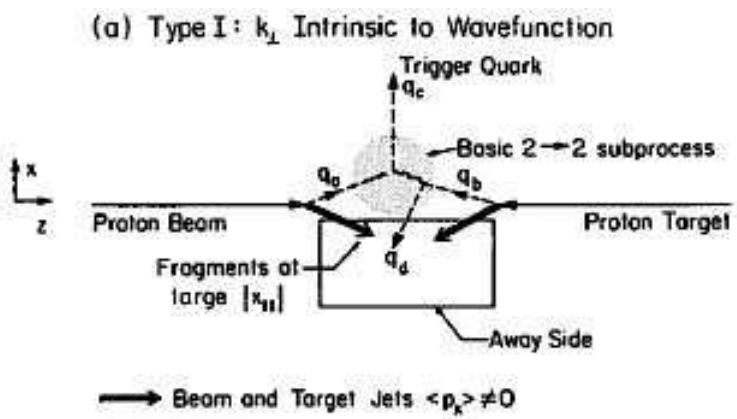


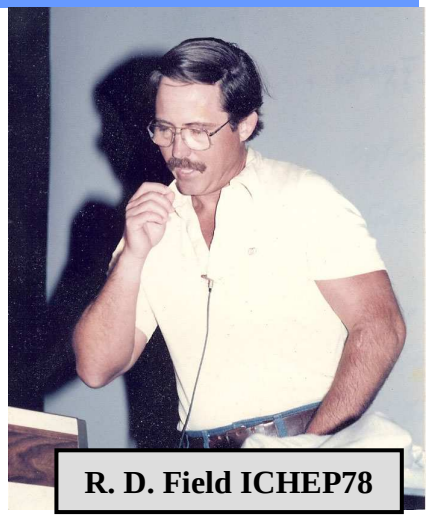


➡ **The distribution in transverse momentum, p_T, of the $\mu^+\mu^-$ pair produced in pp collisions at W = 27.4 GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle$ = 600 MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.**

Early Experimental Test of QCD

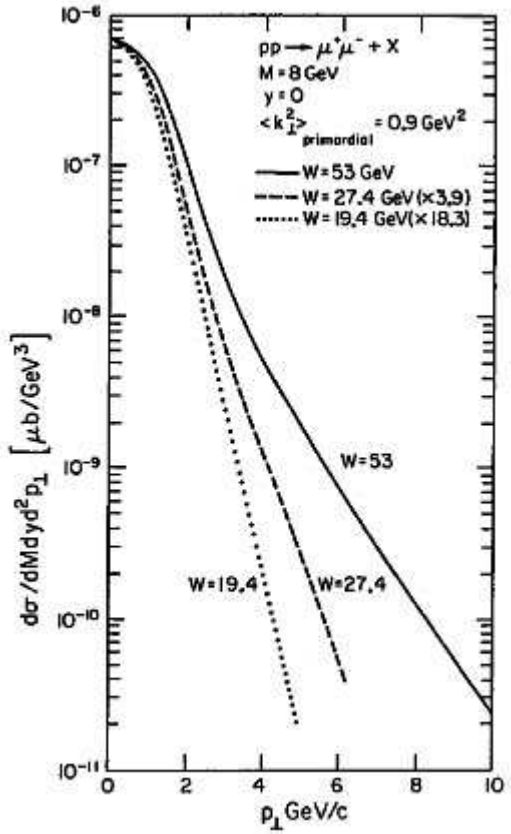




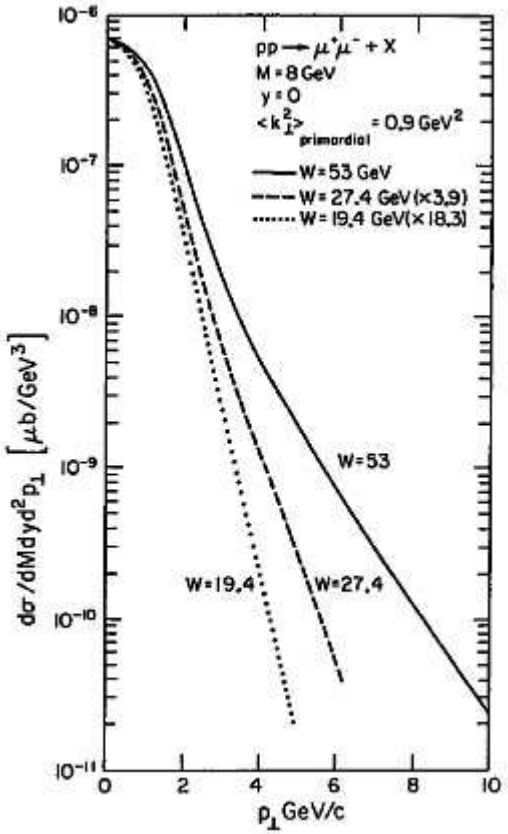


Large primordial k_T!

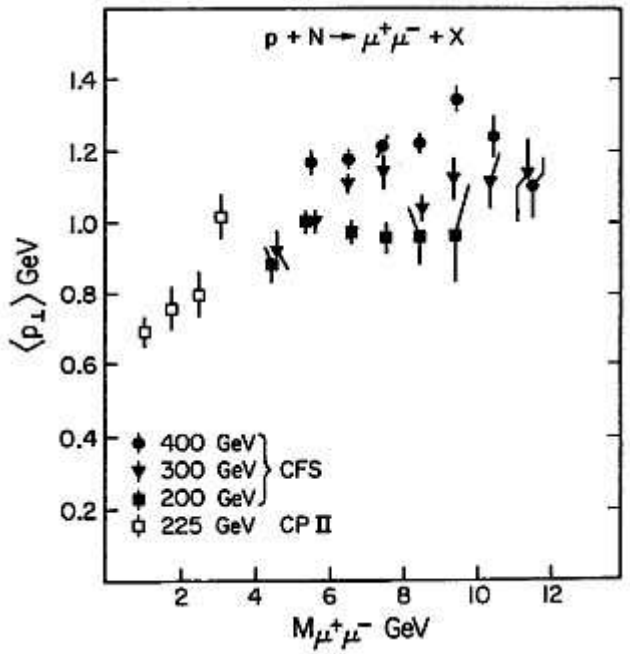
➡ **The distribution in transverse momentum, p_T, of the $\mu^+\mu^-$ pair produced in pp collisions at W = 27.4 GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle$ = 600 MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.**



- **Energy dependence of the large p_T tail expected for pp -> μ+μ- + X from the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle$ = 600 MeV.**



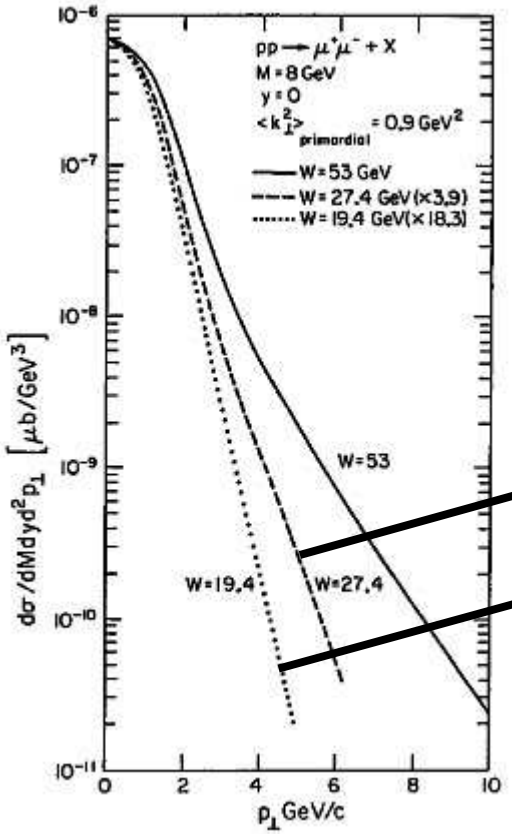
The data are from D. C. Horn *et al.*, Phys. Rev. Letters 36, 1239 (1976) and 37, 1374 (1976); S. W. Herb *et al.*, Phys. Rev. Letters 39, 352 (1977); W. R. Innes et al., Phys. Rev. Letters 39, 1240 (1977); also see the talk by L. Lederman at this conference.

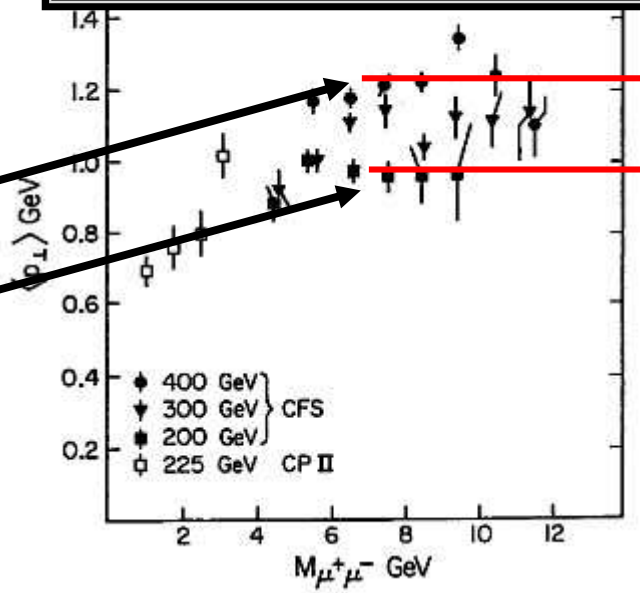


- Energy dependence of the large p_T tail expected for pp -> μ+μ- + X from the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle$ = 600 MeV.
- Data on the energy dependence of the $\langle p_T \rangle$ of the μ+μ- pair in proton-nucleon collisions which indicate that the p_T distribution is becoming broader as the energy increases.

The data are from D. C. Horn *et al.*, Phys. Rev. Letters 36, 1239 (1976) and 37, 1374 (1976); S. W. Herb *et al.*, Phys. Rev. Letters 39, 352 (1977); W. R. Innes et al., Phys. Rev. Letters 39, 1240 (1977); also see the talk by L. Lederman at this conference.

Early Experimental Test of QCD



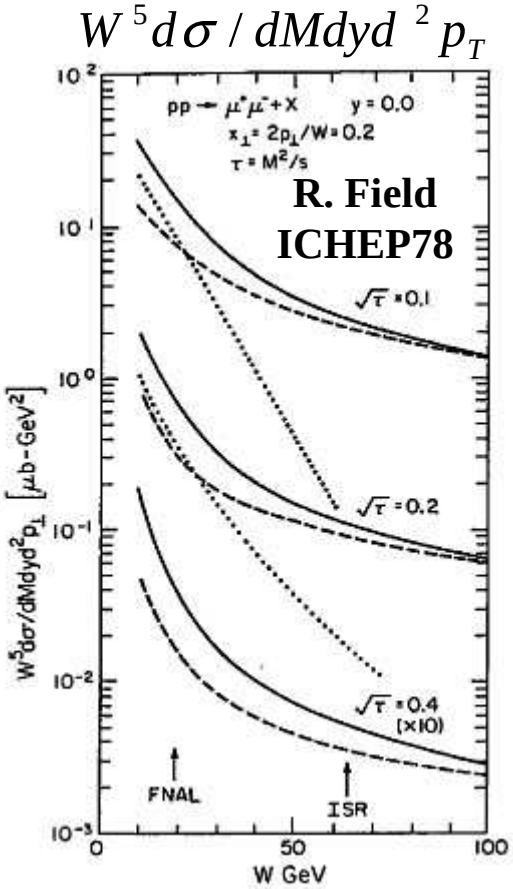


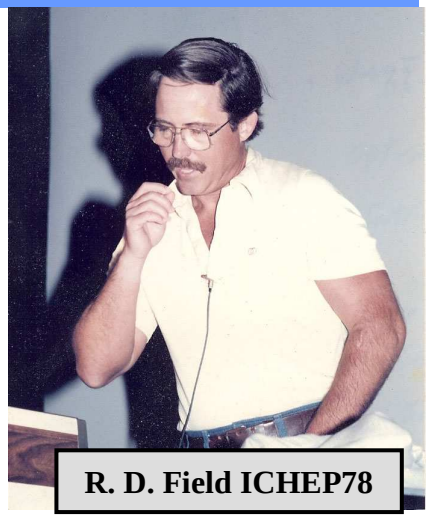
W = 27.4 GeV

W = 19.4 GeV

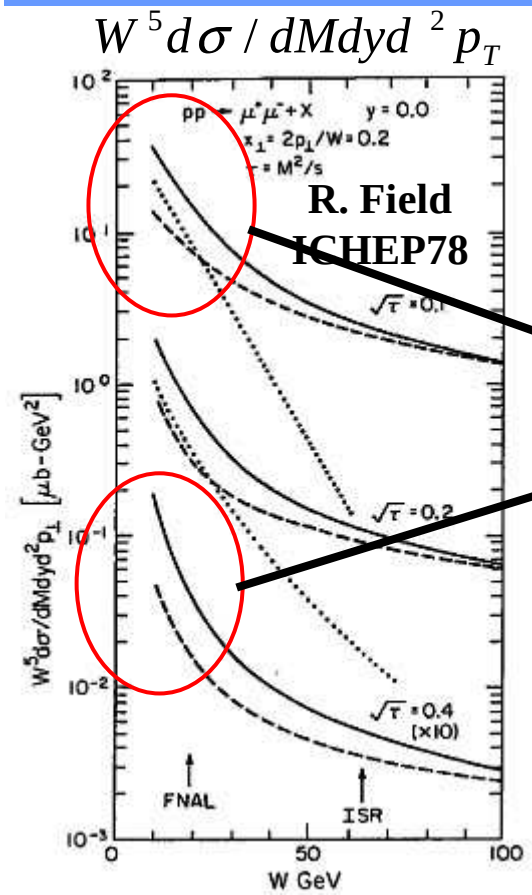
- Energy dependence of the large p_T tail expected for pp -> μ+μ- + X from the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle$ = 600 MeV.
- Data on the energy dependence of the $\langle p_T \rangle$ of the μ+μ- pair in proton-nucleon collisions which indicate that the p_T distribution is becoming broader as the energy increases.

QCD R. Field ICHEP78



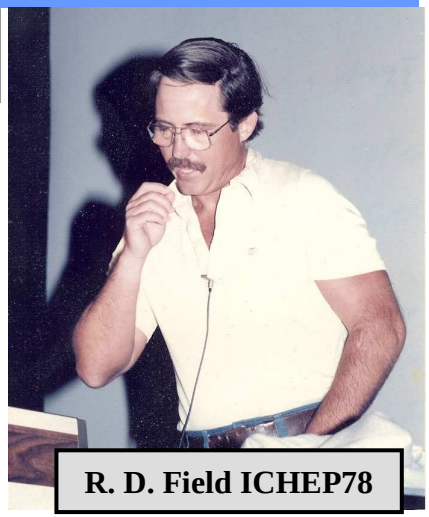


➡ **Expected "scale breaking" of the quantity $W^5 d\sigma/dMdyd^2p_T$ for pp->μ+μ- +X at y = 0 and x_T = 0.2 from QCD with Λ = 0.4 GeV/c. The solid (dashed) curves are the results after (before) smearing with a parton primordial transverse momentun with $\langle k_T \rangle$ = 600 MeV. The dotted curves would be expected if primordial motion alone were responsible for the muon pair p_T. Scaling would predict this quantity to be independent of W at fixed τ and x_T.**



Early QCD Predictions

Smearing breaks naïve parton model scaling!

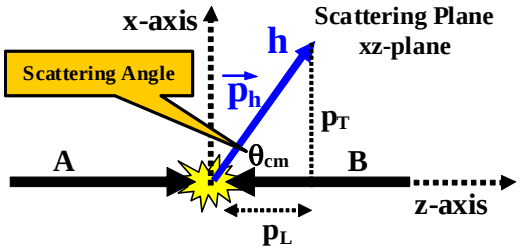


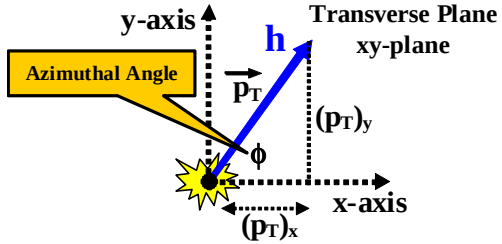
R. D. Field ICHEP78

➡ **Expected "scale breaking" of the quantity $W^5 d\sigma/dMdyd^2p_T$ for pp->μ+μ- +X at y = 0 and x_T = 0.2 from QCD with Λ = 0.4 GeV/c. The solid (dashed) curves are the results after (before) smearing with a parton primordial transverse momentun with $\langle k_T \rangle$ = 600 MeV. The dotted curves would be expected if primordial motion alone were responsible for the muon pair p_T. Scaling would predict this quantity to be independent of W at fixed τ and x_T.**

The Invariant Differential

A+B→h+X Center-of-Mass Frame





$$p_T = p\cos\theta_{cm}$$

$$p_L = p\sin\theta_{cm}$$

$$E = \sqrt{p_L^2 + p_T^2 + m^2}$$

$$p_x^T = p_T\cos\phi$$

$$p_y^T = p_T\sin\phi$$

$$y \equiv \frac{1}{2}\log\left(\frac{E+p_L}{E-p_L}\right)$$

$$\frac{\partial(p_x^T, p_y^T)}{\partial(p_T,\phi)} = \begin{vmatrix} \partial p_x^T/\partial p_T & \partial p_y^T/\partial p_T \\ \partial p_x^T/\partial\phi & \partial p_y^T/\partial\phi \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi \\ -p_T\sin\phi & p_T\cos\phi \end{vmatrix} = p_T$$

Jacobian

Integrate over ϕ

Invariant Differential

$$\frac{d^3p}{E} = \frac{dp_L d^2p_T}{E}$$

$$d^2p_T = dp_x^T dp_y^T = \frac{\partial(p_x^T, p_y^T)}{\partial(p_T,\phi)} dp_T d\phi = p_T dp_T d\phi = \tfrac{1}{2} dp_T^2 d\phi = \pi dp_T^2$$

$$\frac{dE}{dp_L} = \frac{p_L}{E}$$

$$\frac{dy}{dp_L} = \frac{1}{2}\left(\frac{\frac{p_L}{E}+1}{E+p_L} - \frac{\frac{p_L}{E}-1}{E-p_L}\right) = \frac{1}{E} \quad \rightarrow \quad \frac{dp_L}{E} = dy$$

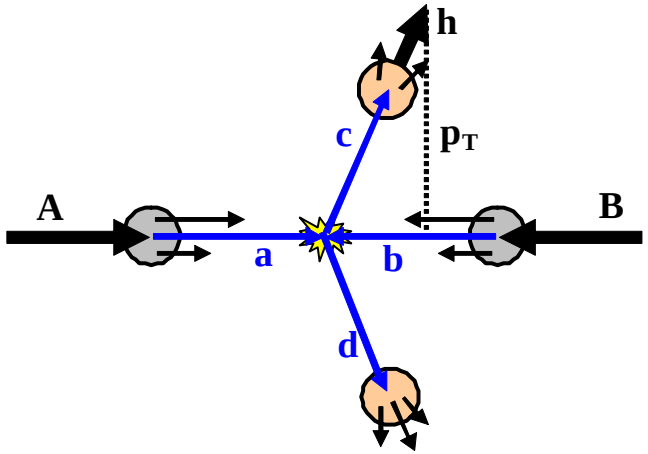
$$\frac{d^3p}{E} = dyd^2p_T = \pi dydp_T^2 = \tfrac{1}{4}\pi s dydx_T^2$$

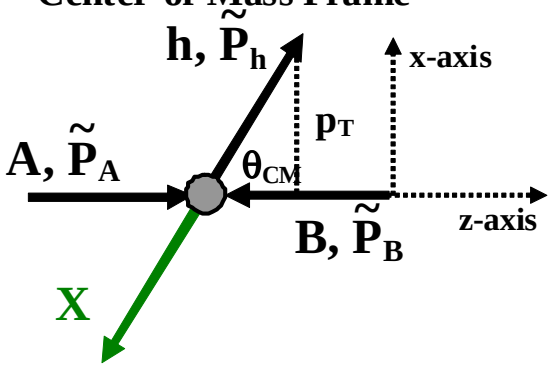
$$x_T = 2p_T/\sqrt{s}$$

$$x_T^2 = 4p_T^2/s$$

QCD Kinematics (1)

A+B→h+X (neglect the mass of A, B, and h)





$$\tilde{P}_A^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ p_{CM} \end{pmatrix} \qquad \tilde{P}_B^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ -p_{CM} \end{pmatrix}$$

$$\tilde{P}_h^{CM} = \begin{pmatrix} p'_{CM} \\ p'_{CM}\sin\hat{\theta}_{CM} \\ 0 \\ p'_{CM}\cos\hat{\theta}_{CM} \end{pmatrix}$$

$$s = (\tilde{P}_A + \tilde{P}_B)^2 = 2\tilde{P}_A \cdot \tilde{P}_B = 4p_{CM}^2 \qquad p_{CM} = \tfrac{1}{2}\sqrt{s} \qquad p'_{CM} = \frac{s - M_X^2}{2\sqrt{s}}$$

$$t = (\tilde{P}_h - \tilde{P}_A)^2 = -2\tilde{P}_h \cdot \tilde{P}_A = -2p_{CM}p'_{CM}(1-\cos\theta_{CM})$$

$$u = (\tilde{P}_h - \tilde{P}_B)^2 = -2\tilde{P}_h \cdot \tilde{P}_B = -2p_{CM}p'_{CM}(1+\cos\theta_{CM})$$

$$s + t + u = M_X^2 \qquad p_T = p'_{CM}\sin\theta_{CM}$$

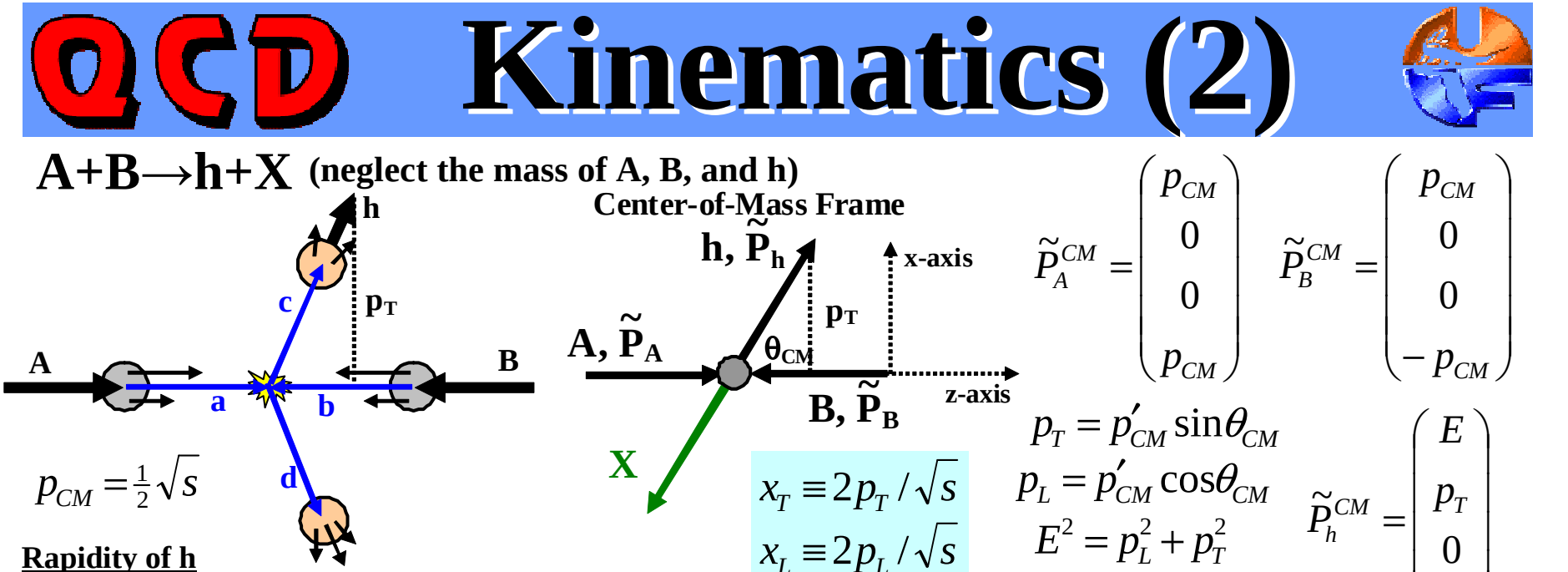
$$x_T \equiv 2p_T/\sqrt{s}$$

$$t = -2p_{CM}p'_{CM}(1-\cos\hat{\theta}_{CM}) = -2p_{CM}p_T\frac{(1-\cos\theta_{CM})}{\sin\theta_{CM}} = -\tfrac{1}{2}sx_T\tan(\theta_{CM}/2)$$

$$u = -2p_{CM}p'_{CM}(1+\cos\hat{\theta}_{CM}) = -2p_{CM}p_T\frac{(1+\cos\theta_{CM})}{\sin\theta_{CM}} = -\tfrac{1}{2}sx_T\cot(\theta_{CM}/2)$$

$$p_T^2 = \frac{tu}{s}$$

QCD Kinematics (2)



$$p_{CM} = \tfrac{1}{2}\sqrt{s}$$

$$\tilde{P}_A^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ p_{CM} \end{pmatrix} \qquad \tilde{P}_B^{CM} = \begin{pmatrix} p_{CM} \\ 0 \\ 0 \\ -p_{CM} \end{pmatrix}$$

$$p_T = p'_{CM}\sin\theta_{CM}$$
$$p_L = p'_{CM}\cos\theta_{CM}$$
$$E^2 = p_L^2 + p_T^2$$
$$x_E^2 = x_L^2 + x_T^2$$

$$\tilde{P}_h^{CM} = \begin{pmatrix} E \\ p_T \\ 0 \\ p_L \end{pmatrix}$$

$$x_T \equiv 2p_T/\sqrt{s}$$
$$x_L \equiv 2p_L/\sqrt{s}$$
$$x_E \equiv 2p_E/\sqrt{s}$$

Rapidity of h

$$y \equiv \frac{1}{2}\log\left(\frac{E+p_L}{E-p_L}\right) = \frac{1}{2}\log\left(\frac{x_E+x_L}{x_E-x_L}\right) = \frac{1}{2}\log\left(\frac{x_1}{x_2}\right)$$

$$t = (\tilde{P}_h - \tilde{P}_A)^2 = -2\tilde{P}_h\cdot\tilde{P}_A = -2p_{CM}(E-p_L) = -\tfrac{1}{2}s(x_E - x_L)$$
$$u = (\tilde{P}_h - \tilde{P}_B)^2 = -2\tilde{P}_h\cdot\tilde{P}_B = -2p_{CM}(E+p_L) = -\tfrac{1}{2}s(x_E + x_L)$$

$$t = -x_2 s = -\tfrac{1}{2}sx_T e^{-y} = -\tfrac{1}{2}sx_T\tan(\theta_{CM}/2)$$
$$u = -x_1 s = -\tfrac{1}{2}sx_T e^{+y} = -\tfrac{1}{2}sx_T\cot(\theta_{CM}/2)$$
$$y = \eta = -\log(\tan(\theta_{cm}/2))$$

$$e^{+2y} = \frac{x_E+x_L}{x_E-x_L} = \frac{(x_E+x_L)^2}{x_E^2-x_L^2} = \frac{(x_E+x_L)^2}{x_T^2}$$

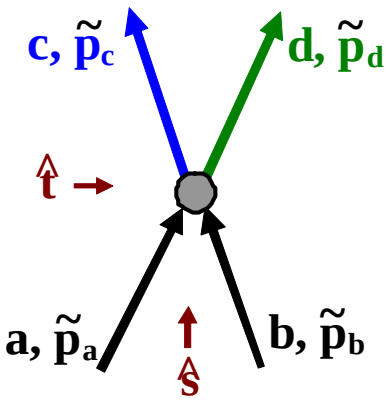
$$e^{-2y} = \frac{x_E-x_L}{x_E+x_L} = \frac{(x_E-x_L)^2}{x_E^2-x_L^2} = \frac{(x_E-x_L)^2}{x_T^2}$$

$$x_1 \equiv -u/s = \tfrac{1}{2}(x_E+x_L) = \tfrac{1}{2}x_T e^{+y}$$
$$x_2 \equiv -t/s = \tfrac{1}{2}(x_E-x_L) = \tfrac{1}{2}x_T e^{-y}$$

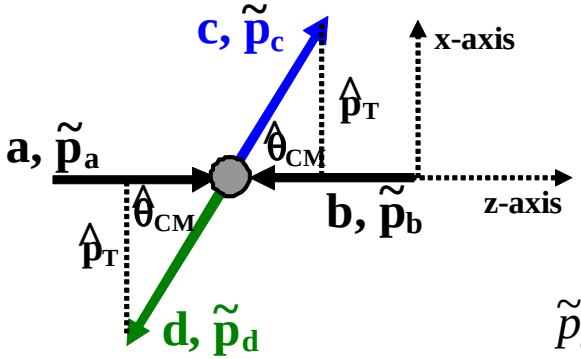
QCD 2-to-2 Kinematics

a+b→c+d (neglect masses)

2-to-2 Scattering



Center-of-Mass Frame



$$\tilde{p}_a^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ 0 \\ 0 \\ \hat{p}_{CM} \end{pmatrix} \qquad \tilde{p}_b^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ 0 \\ 0 \\ -\hat{p}_{CM} \end{pmatrix}$$

$$\tilde{p}_c^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ \hat{p}_{CM}\sin\hat{\theta}_{CM} \\ 0 \\ \hat{p}_{CM}\cos\hat{\theta}_{CM} \end{pmatrix} \qquad \tilde{p}_d^{CM} = \begin{pmatrix} \hat{p}_{CM} \\ -\hat{p}_{CM}\sin\hat{\theta}_{CM} \\ 0 \\ -\hat{p}_{CM}\cos\hat{\theta}_{CM} \end{pmatrix}$$

$$\hat{s} = (\tilde{p}_a + \tilde{p}_b)^2 = 2\tilde{p}_a \cdot \tilde{p}_b = 4\hat{p}_{CM}^2$$

$$\hat{t} = (\tilde{p}_c - \tilde{p}_a)^2 = -2\tilde{p}_c \cdot \tilde{p}_a = -2\hat{p}_{CM}^2(1-\cos\hat{\theta}_{CM})$$

$$\hat{u} = (\tilde{p}_c - \tilde{p}_b)^2 = -2\tilde{p}_c \cdot \tilde{p}_b = -2\hat{p}_{CM}^2(1+\cos\hat{\theta}_{CM})$$

$$\hat{p}_{CM} = \tfrac{1}{2}\sqrt{\hat{s}} \qquad \hat{p}_T = \hat{p}_{CM}\sin\hat{\theta}_{CM} = \tfrac{1}{2}\sqrt{\hat{s}}\sin\hat{\theta}_{CM}$$

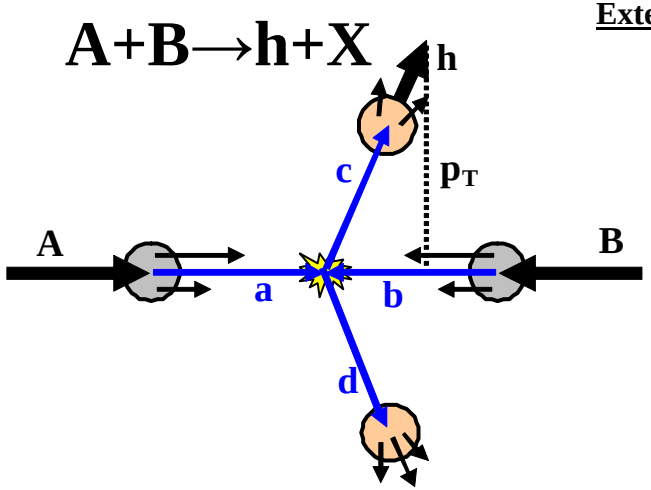
$$\hat{s}+\hat{t}+\hat{u} = 0 \qquad \hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}}$$

$$\hat{t} = -2\hat{p}_{CM}^2(1-\cos\hat{\theta}_{CM}) = -2\hat{p}_{CM}\hat{p}_T\frac{(1-\cos\hat{\theta}_{CM})}{\sin\hat{\theta}_{CM}} = -\tfrac{1}{2}\hat{s}\hat{x}_T\tan(\hat{\theta}_{CM}/2)$$

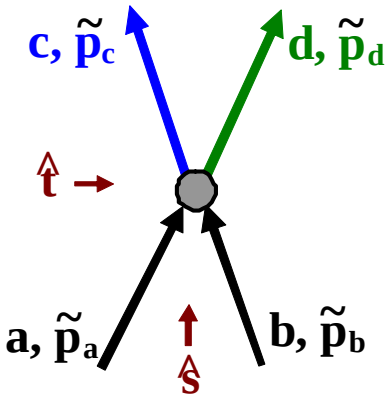
$$\hat{u} = -2\hat{p}_{CM}^2(1+\cos\hat{\theta}_{CM}) = -2\hat{p}_{CM}\hat{p}_T\frac{(1+\cos\hat{\theta}_{CM})}{\sin\hat{\theta}_{CM}} = -\tfrac{1}{2}\hat{s}\hat{x}_T\cot(\hat{\theta}_{CM}/2)$$

$$\hat{p}_T^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

QCD Naïve Parton Model



2-to-2 Scattering



External Variables (neglect the mass of A, B, and h)

$$s=(\tilde{P}_A+\tilde{P}_B)^2=2\tilde{P}_A\cdot\tilde{P}_B$$
$$t=(\tilde{P}_h-\tilde{P}_A)^2=-2\tilde{P}_h\cdot\tilde{P}_A$$
$$u=(\tilde{P}_h-\tilde{P}_B)^2=-2\tilde{P}_h\cdot\tilde{P}_B$$

$$x_1=-u/s$$
$$x_2=-t/s$$
$$x_T=2p_T/W$$

Internal-External Connection

$$\tilde{p}_a=x_a\tilde{P}_A$$
$$\tilde{p}_b=x_b\tilde{P}_B$$
$$\tilde{P}_h=z_c\tilde{p}_c$$

Internal Variables (neglect masses)

$$\hat{s}=(\tilde{p}_a+\tilde{p}_b)^2=2\tilde{p}_a\cdot\tilde{p}_b=2x_ax_b(\tilde{P}_A\cdot\tilde{P}_B)=x_ax_bs$$
$$\hat{t}=(\tilde{p}_c-\tilde{p}_a)^2=-2\tilde{p}_c\cdot\tilde{p}_a==-2x_a(\tilde{P}_h\cdot\tilde{P}_A)/z_c=x_at/z_c$$
$$\hat{u}=(\tilde{p}_c-\tilde{p}_b)^2=-2\tilde{p}_c\cdot\tilde{p}_b=-2x_b(\tilde{P}_h\cdot\tilde{P}_B)/z_c=x_bu/z_c$$

$$\hat{s}+\hat{t}+\hat{u}=0 \longrightarrow x_ax_b-\frac{x_ax_2}{z_c}-\frac{x_bx_1}{z_c}=0$$

$$\hat{p}_T^2=\frac{\hat{t}\hat{u}}{\hat{s}}=\frac{1}{z_c^2}\frac{tu}{s}=\frac{p_T^2}{z_c^2}$$

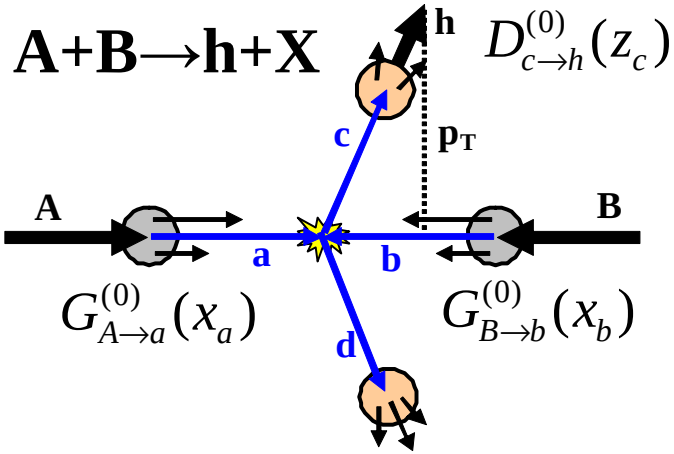
$$\hat{p}_T=p_T/z_c$$

$$x_a=\frac{x_bx_1}{z_cx_b-x_2}$$

$$x_b=\frac{x_ax_2}{z_cx_a-x_1}$$

$$z_c=\frac{x_1}{x_a}+\frac{x_2}{x_b}$$

QCD Naïve Parton Model



External Variables

$$W = E_{cm} = \sqrt{s} \qquad x_T = 2p_T/W$$

$$t = -x_2 s = -\tfrac{1}{2} s x_T e^{-y}$$

$$u = -x_1 s = -\tfrac{1}{2} s x_T e^{+y}$$

$$x_1 \equiv -u/s = \tfrac{1}{2}(x_E + x_L) = \tfrac{1}{2} x_T e^{+y}$$

$$x_2 \equiv -t/s = \tfrac{1}{2}(x_E - x_L) = \tfrac{1}{2} x_T e^{-y}$$

Internal Variables

$$\hat{s} = x_a x_b s$$

$$\hat{t} = x_a t/z_c = -x_a x_2 s/z_c$$

$$\hat{u} = x_b u/z_c = -x_b x_1 s/z_c$$

$$\hat{s} + \hat{t} + \hat{u} = 0$$

$$\hat{p}_T = p_T/z_c$$

$$d\sigma^{A+B\to h+X}(s,t) = G^{(0)}_{A\to a}(x_a)dx_a G^{(0)}_{B\to b}(x_b)dx_b \left(\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t})\right) d\hat{t} D^{(0)}_{c\to h}(z_c)dz_c$$

$$z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}$$

$$d\hat{t}\,dz_c = \frac{\partial(\hat{t}, z_c)}{\partial(x_1, x_2)} dx_1 dx_2 = \frac{s}{z_c} dx_1 dx_2 \qquad dx_1 dx_2 = \frac{\partial(x_1, x_2)}{\partial(y, x_T)} dy dx_T = \tfrac{1}{2} x_T dy dx_T$$

$$dz_c d\hat{t} = \frac{1}{\pi z_c}\frac{d^3p}{E}$$

$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s, p_T, \theta_{cm}) = \frac{1}{\pi}\sum_{a,b}\int_{x_a^{\min}}^{1.0} dx_a \int_{x_b^{\min}}^{1.0} dx_b G^{(0)}_{A\to a}(x_a) G^{(0)}_{B\to b}(x_b) D^{(0)}_{c\to h}(z_c) \frac{1}{z_c}\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t})$$

$$x_a = \frac{x_b x_1}{z_c x_b - x_2} \xrightarrow[z_c = 1]{x_b = 1} x_a^{\min} = \frac{x_1}{1 - x_2} \qquad x_b = \frac{x_a x_2}{z_c x_a - x_1} \xrightarrow{z_c = 1} x_b^{\min} = \frac{x_a x_2}{x_a - x_1}$$

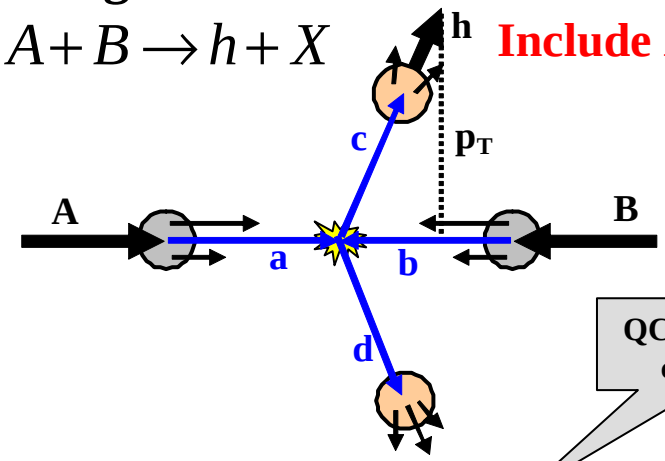
QCD Parton-Parton Scattering

Large Transverse Meson Production in Hadron-Hadron Collisions

$A + B \to h + X$

Include All The QCD 2-to-2 Parton-Parton Processes (Order α_s^2)

Parton = quark, anti-quark, & gluon



$$\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2(Q^2)}{\hat{s}^2}\left|\overline{A}^{a+b\to c+d}(\hat{s},\hat{t})\right|^2$$

(1) $q_i + q_j \xrightarrow[i\neq j]{} q_i + q_j$

(2) $q_i + \bar{q}_j \xrightarrow[i\neq j]{} q_i + \bar{q}_j$

(3) $q_i + q_i \to q_i + q_i$

(4) $q_i + \bar{q}_i \to q_i + \bar{q}_i$

(5) $q_i + \bar{q}_i \to g + g$

(6) $gg \to q_i + \bar{q}_i$

(7) $q_i + g \to q_i + g$

(8) $\bar{q}_i + g \to \bar{q}_i + g$

(9) $g + g \to g + g$

TABLE I. Cross sections for the various constituent quark-quark, quark-gluon, and gluon-gluon subprocesses. [a] The differential cross section is given by $d\hat{\sigma}/d\hat{t} = \pi\alpha_s^2(Q^2)|A|^2/\hat{s}^2$, where $\alpha_s(Q^2)$ is the effective coupling given by Eq. (3.1).

	Subprocess	$\lvert A\rvert^2$
1.	$q_iq_j \to q_iq_j$ $q_i\bar{q}_j \to q_i\bar{q}_j$ $(i \neq j)$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
2.	$q_iq_i \to q_iq_i$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}$
3.	$q_i\bar{q}_i \to q_i\bar{q}_i$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}$
4.	$q_i\bar{q}_i \to gg$	$\frac{32}{27}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}}\right)-\frac{8}{3}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}\right)$
5.	$gg \to q_i\bar{q}_i$	$\frac{1}{6}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{u}\hat{t}}\right)-\frac{3}{8}\left(\frac{\hat{u}^2+\hat{t}^2}{\hat{s}^2}\right)$
6.	$q_ig \to q_ig$	$-\frac{4}{9}\left(\frac{\hat{u}^2+\hat{s}^2}{\hat{u}\hat{s}}\right)+\left(\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}\right)$
7.	$gg \to gg$	$\frac{9}{2}\left(3-\frac{\hat{u}\hat{t}}{\hat{s}^2}-\frac{\hat{u}\hat{s}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$

QCD Parton Model Scaling

$$\hat{s}+\hat{t}+\hat{u}=0 \qquad \hat{p}_T^2=\frac{\hat{t}\hat{u}}{\hat{s}} \qquad \hat{x}_T=2\hat{p}_T/\sqrt{\hat{s}} \qquad \hat{s}=4p_T^2/\hat{x}_T^2$$

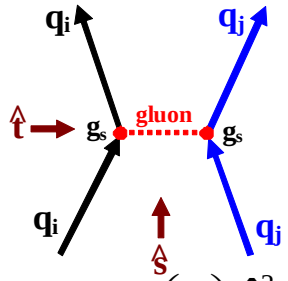
$$\hat{t}=-\tfrac{1}{2}\hat{s}\left(1-\sqrt{1-\hat{x}_T^2}\right) \qquad \hat{u}=-\tfrac{1}{2}\hat{s}\left(1+\sqrt{1-\hat{x}_T^2}\right)$$

$$\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s},\hat{t})=\frac{4}{9}\frac{\pi\alpha_s^2}{\hat{s}^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)=\frac{\pi\alpha_s^2}{\hat{p}_T^4}\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{t}\hat{u}}{\hat{s}^2}\right)\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

$$\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{t}\hat{u}}{\hat{s}^2}\right)\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}=\left(\frac{\hat{x}_T^2}{9}\right)\left[\frac{\hat{u}}{\hat{t}}+\left(\frac{\hat{u}}{\hat{s}}\right)^2\frac{\hat{u}}{\hat{t}}\right]=\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{u}}{\hat{t}}\right)\left(1+\left(\frac{\hat{u}}{\hat{s}}\right)^2\right)=f_1(\hat{x}_T)$$

Quark-Quark Elastic Scattering

$$A_{q_i+q_j\to q_i+q_j}(\hat{s},\hat{t})=$$



$$\left|\overline{A}(q_i+q_j\to q_i+q_j)\right|^2=\left(\frac{4}{9}\right)\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Sum over final-state spins and color.
Average over initial-state spins and color.

$$P_1^{(0)}(x_a,x_b,z_c)=\sum_{i\neq j}G_{A\to q_i}^{(0)}(x_a)G_{B\to q_j}^{(0)}(x_b)D_{q_i\to h}^{(0)}(z_c)$$

Parton Model Scaling

$$p_T^4E\frac{d\sigma_1^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm})=\int_{x_a^{\min}}^{1.0}dx_a\int_{x_b^{\min}}^{1.0}dx_bP_1^{(0)}(x_a,x_b,z_c)\frac{\alpha_s^2}{z_c}f_1(x_a,x_b,x_T)=F_1(x_T,\theta_{cm})$$

Constant at fixed θ_{cm} and x_T!
(provided the PDF's and the fragmentation functions scale and α_s is constant)

$$x_1=\tfrac{1}{2}x_T/\tan(\theta_{cm}/2) \qquad x_a^{\min}=\frac{x_1}{1-x_2} \qquad x_b=\frac{x_ax_2}{z_cx_a-x_1}$$

$$x_2=\tfrac{1}{2}x_T\tan(\theta_{cm}/2)$$

$$x_T=2p_T/\sqrt{s} \qquad x_b^{\min}=\frac{x_ax_2}{x_a-x_1} \qquad z_c=\frac{x_1}{x_a}+\frac{x_2}{x_b}$$

QCD Parton Model Scaling

$$\hat{s}+\hat{t}+\hat{u}=0 \qquad \hat{p}_T^2=\frac{\hat{t}\hat{u}}{\hat{s}} \qquad \hat{x}_T=2\hat{p}_T/\sqrt{\hat{s}} \qquad \hat{s}=4p_T^2/\hat{x}_T^2$$

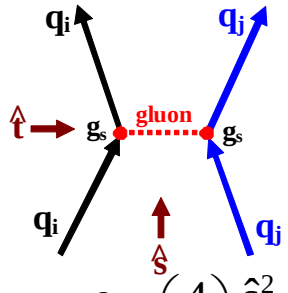
$$\hat{t}=-\tfrac{1}{2}\hat{s}\left(1-\sqrt{1-\hat{x}_T^2}\right) \qquad \hat{u}=-\tfrac{1}{2}\hat{s}\left(1+\sqrt{1-\hat{x}_T^2}\right)$$

$$\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s},\hat{t})=\frac{4}{9}\frac{\pi\alpha_s^2}{\hat{s}^2}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)=\frac{\pi\alpha_s^2}{\hat{p}_T^4}\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{t}\hat{u}}{\hat{s}^2}\right)\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

$$\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{t}\hat{u}}{\hat{s}^2}\right)\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}=\left(\frac{\hat{x}_T^2}{9}\right)\left[\frac{\hat{u}}{\hat{t}}+\left(\frac{\hat{u}}{\hat{s}}\right)^2\frac{\hat{u}}{\hat{t}}\right]=\left(\frac{\hat{x}_T^2}{9}\right)\left(\frac{\hat{u}}{\hat{t}}\right)\left(1+\left(\frac{\hat{u}}{\hat{s}}\right)^2\right)=f_1(\hat{x}_T)$$

Quark-Quark Elastic Scattering

$$A_{q_i+q_j\to q_i+q_j}(\hat{s},\hat{t})=$$



$$\left|\overline{A}(q_i+q_j\to q_i+q_j)\right|^2=\left(\frac{4}{9}\right)\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Sum over final-state spins and color.
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$$P_1^{(0)}(x_a,x_b,z_c)=\sum_{i\neq j}G_{A\to q_i}^{(0)}(x_a)G_{B\to q_j}^{(0)}(x_b)D_{q_i\to h}^{(0)}(z_c)$$

Parton Model Scaling

$$p_T^4E\frac{d\sigma_1^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm})=\int_{x_a^{\min}}^{1.0}dx_a\int_{x_b^{\min}}^{1.0}dx_bP_1^{(0)}(x_a,x_b,z_c)\frac{\alpha_s^2}{z_c}f_1(x_a,x_b,x_T)=F_1(x_T,\theta_{cm})$$

Constant at fixed θ_{cm} and x_T!
(provided the PDF's and the fragmentation functions scale and α_s is constant)

$$x_1=\tfrac{1}{2}x_T/\tan(\theta_{cm}/2) \qquad x_a^{\min}=\frac{x_1}{1-x_2} \qquad x_b=\frac{x_ax_2}{z_cx_a-x_1}$$

$$x_2=\tfrac{1}{2}x_T\tan(\theta_{cm}/2)$$

$$x_T=2p_T/\sqrt{s} \qquad x_b^{\min}=\frac{x_ax_2}{x_a-x_1} \qquad z_c=\frac{x_1}{x_a}+\frac{x_2}{x_b}$$

QCD Parton Model Scaling

Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order α_s^2)

(1) $q_i + q_j \xrightarrow[i \neq j]{} q_i + q_j$

(2) $q_i + \bar{q}_j \xrightarrow[i \neq j]{} q_i + \bar{q}_j$

(3) $q_i + q_i \to q_i + q_i$

(4) $q_i + \bar{q}_i \to q_i + \bar{q}_i$

(5) $q_i + \bar{q}_i \to g + g$

(6) $gg \to q_i + \bar{q}_i$

(7) $q_i + g \to q_i + g$

(8) $\bar{q}_i + g \to \bar{q}_i + g$

(9) $g + g \to g + g$

$$\frac{d\hat{\sigma}_k}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2}{\hat{p}_T^4} f_k(\hat{x}_T) \qquad \textbf{k = 1 to 9}$$

$$p_T^4 E \frac{d\sigma_{all}^{A+B\to h+X}}{d^3p}(s, p_T, \theta_{cm}) = \sum_{k=1}^{9} F_k(x_T, \theta_{cm})$$

Parton Model Scaling

= Constant at fixed θ_{cm} and x_T!

(provided the PDF's and the fragmentation functions scale and α_s is constant)

$$x_1 = \tfrac{1}{2} x_T / \tan(\theta_{cm}/2) \qquad x_a^{\min} = \frac{x_1}{1 - x_2} \qquad x_b = \frac{x_a x_2}{z_c x_a - x_1}$$

$$x_2 = \tfrac{1}{2} x_T \tan(\theta_{cm}/2)$$

$$x_T = 2p_T/\sqrt{s} \qquad x_b^{\min} = \frac{x_a x_2}{x_a - x_1} \qquad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}$$

QCD Parton Model Scaling

Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order α_s^2)

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$$\frac{d\hat{\sigma}_k}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2}{\hat{p}_T^4} f_k(\hat{x}_T) \qquad \textbf{k = 1 to 9}$$

$$p_T^4 E \frac{d\sigma_{all}^{A+B\to h+X}}{d^3p}(s, p_T, \theta_{cm}) = \sum_{k=1}^{9} F_k(x_T, \theta_{cm})$$

Parton Model Scaling

= Constant at fixed θ_{cm} and x_T!

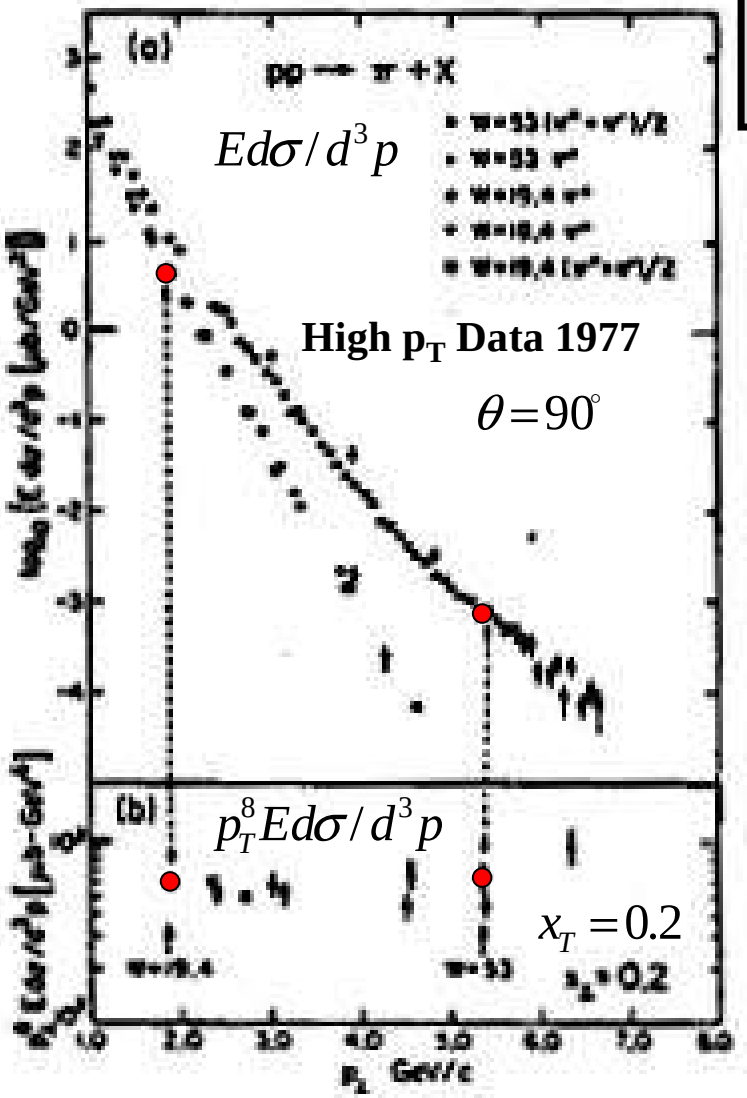
(provided the PDF's and the fragmentation functions scale and α_s is constant)

$$x_1 = \tfrac{1}{2} x_T / \tan(\theta_{cm}/2) \qquad x_a^{\min} = \frac{x_1}{1-x_2} \qquad x_b = \frac{x_a x_2}{z_c x_a - x_1}$$

$$x_2 = \tfrac{1}{2} x_T \tan(\theta_{cm}/2)$$

$$x_T = 2p_T/\sqrt{s} \qquad x_b^{\min} = \frac{x_a x_2}{x_a - x_1} \qquad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}$$

QCD High p_T π-Meson Production



The FNAL data (open squares) are from J. W. Cronin *et al.* (CP Collaboration), Phys. Rev. D11 (1975); D. Antreasyan *et al.*, Phys. Rev. Letters 38 (1977); Phys. Rev. Letters 38 (1977).

The FNAL data at W = 19.4 GeV (solid triangles) are from G. Donaldson *et al.*, Phys Rev. Letters 36 (1976).

The FNAL data (open circles) are from D. C. Carey *et al.*, Fermilab Report FNAL-PUB-75120-EXP (1975).

The ISR data (solid dots) are from B. Alper *et al.* (BS Collaboration), Nucl. Phys. B100 (1975).

The ISR data (crosses) are from F. W. Busser *et al.*, Nucl. Phys. B106 (1976).

$$W = \sqrt{s} \qquad x_T = 2p_T/\sqrt{s} \qquad p_T = \tfrac{1}{2} x_T W$$

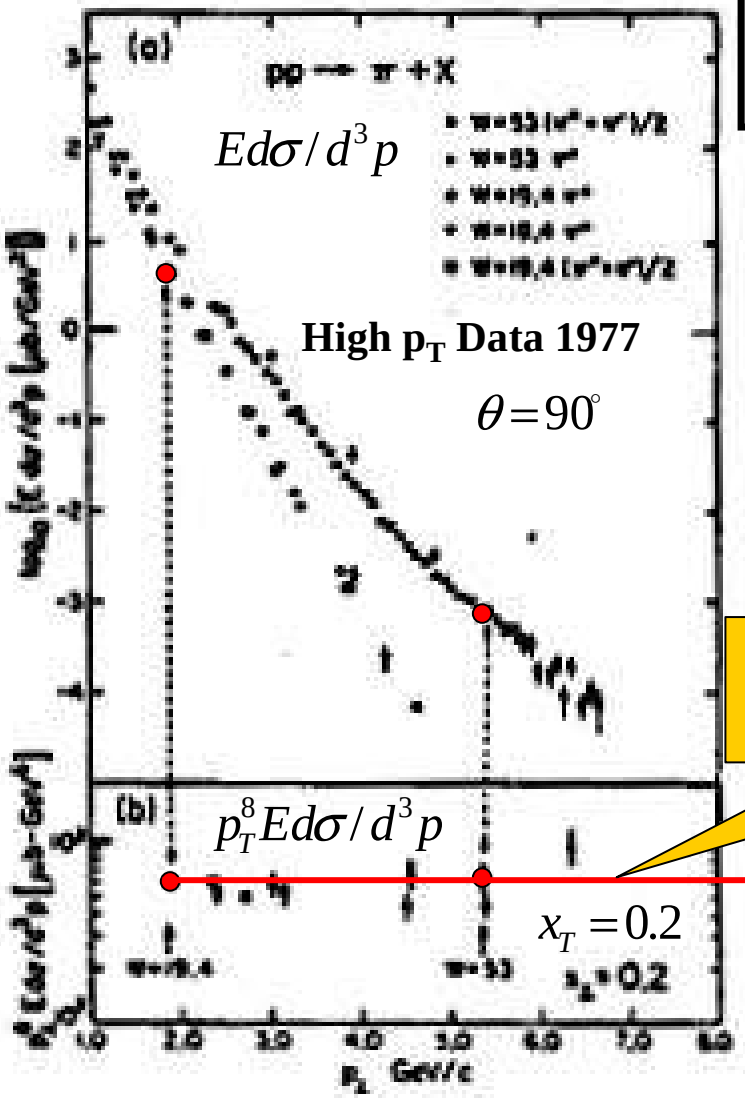
W = 19.4 GeV x_T = 0.2

$$p_T = \tfrac{1}{2}(0.2)(19.2GeV) = 1.92GeV/c$$

W = 53 GeV x_T = 0.2

$$p_T = \tfrac{1}{2}(0.2)(53GeV) = 5.3GeV/c$$

QCD High p_T π-Meson Production



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The ISR data (crosses) are from F. W. Busser *et al.*, Nucl. Phys. B106 (1976).

$p_T = \frac{1}{2} x_T W$

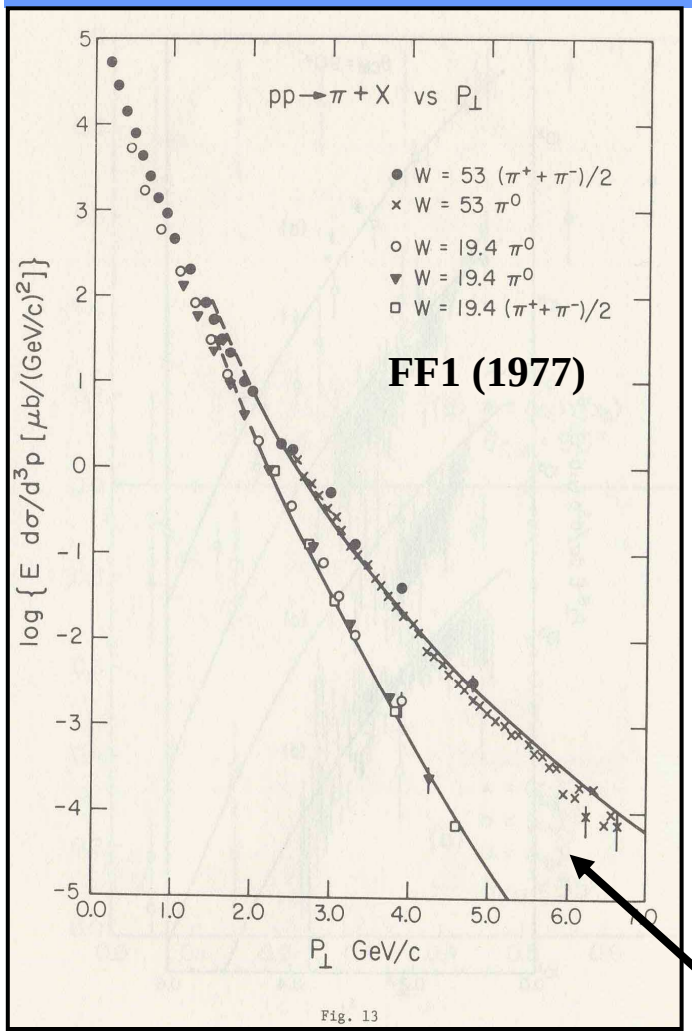
Yikes!! The data is behaving like $1/p_T^8$ not $1/p_T^4$ at fixed x_T.

$\frac{1}{2}(0.2)(19.2GeV) = 1.92GeV/c$

W = 53 GeV x_T = 0.2

$p_T = \frac{1}{2}(0.2)(53GeV) = 5.3GeV/c$

FF1: Black-Box Model



Naïve Parton-Parton Black-Box Model

$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm})=$$

$$\frac{1}{\pi}\sum_{a,b}\int_{x_a^{\min}}^{1.0}dx_a\int_{x_b^{\min}}^{1.0}dx_b G^{(0)}_{A\to a}(x_a)G^{(0)}_{B\to b}(x_b)D^{(0)}_{c\to h}(z_c)\frac{1}{z_c}\frac{d\hat{\sigma}^{Black-Box}}{d\hat{t}}(\hat{s},\hat{t})$$

$$a,b=u,d,s,\bar{u},\bar{d},\bar{s}$$

$$\frac{d\hat{\sigma}^{Black-Box}}{d\hat{t}}(\hat{s},\hat{t})=\frac{A}{\hat{s}^N}\,f(\hat{t}/\hat{u})$$

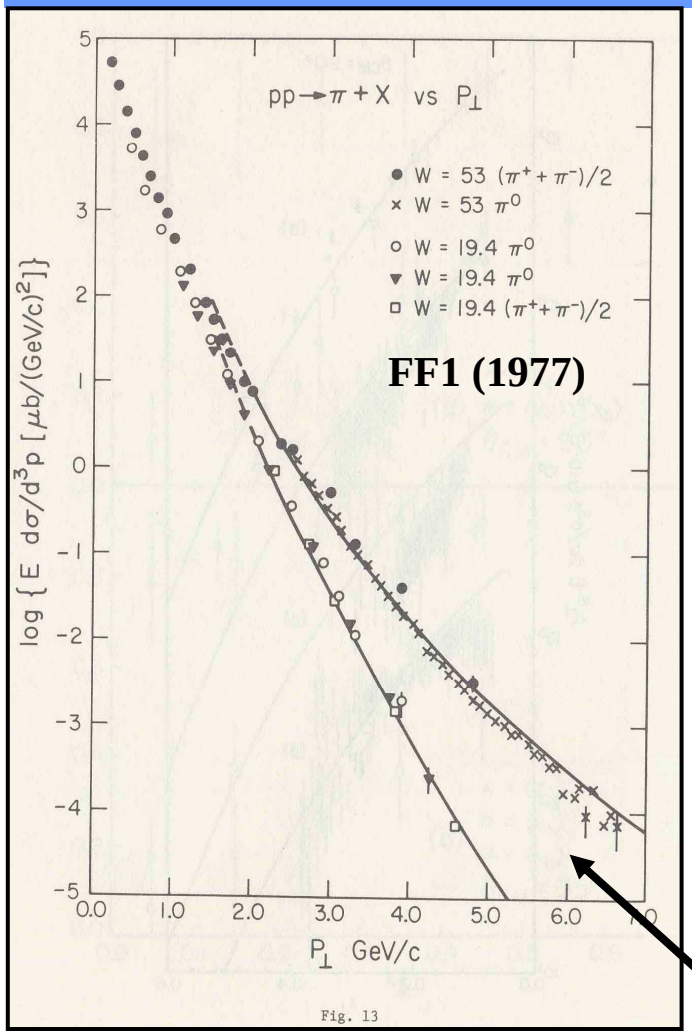
$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm})=\frac{A}{p_T^{2N}}F(x_T,\theta_{cm})$$

$$\frac{d\hat{\sigma}^{Black-Box}_{FF1}}{d\hat{t}}(\hat{s},\hat{t})=\frac{A}{\hat{s}^4}\left(\frac{\hat{s}}{-\hat{t}}\right)^3=\frac{A}{-\hat{s}\hat{t}}$$

$$A_{FF1}=2.3\times10^6\,\mu b\cdot GeV^6$$

$$E\frac{d\sigma^{A+B\to h+X}_{FF1}}{d^3p}(s,p_T,\theta_{cm})=\frac{(2.3\times10^6\,\mu b\cdot GeV^6)}{p_T^8}F(x_T,\theta_{cm})$$

QCD FF1: Black-Box Model



Naïve Parton-Parton Black-Box Model

$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm}) =$$

$$\frac{1}{\pi}\sum_{a,b}\int_{x_a^{min}}^{1.0}dx_a\int_{x_b^{min}}^{1.0}dx_b G^{(0)}_{A\to a}(x_a)G^{(0)}_{B\to b}(x_b)D^{(0)}_{c\to h}(z_c)\frac{1}{z_c}\frac{d\hat{\sigma}^{Black-Box}}{d\hat{t}}(\hat{s},\hat{t})$$

$$a,b = u,d,s,\bar{u},\bar{d},\bar{s}$$

$$\frac{d\hat{\sigma}^{Black-Box}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{A}{\hat{s}^N}\,f(\hat{t}/\hat{u})$$

QCD N = 2
FF1 N = 4
CIM N = 4

$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm}) = \frac{A}{p_T^{2N}}F(x_T,\theta_{cm})$$

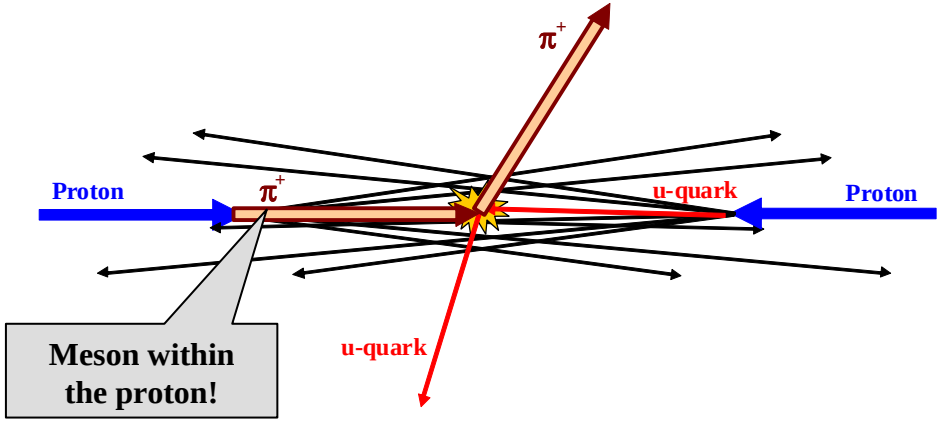
$$\frac{d\hat{\sigma}^{Black-Box}_{FF1}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{A}{\hat{s}^4}\left(\frac{\hat{s}}{-\hat{t}}\right)^3 = \frac{A}{-\hat{s}\hat{t}}$$

$$A_{FF1} = 2.3\times10^6\,\mu b\cdot GeV^6$$

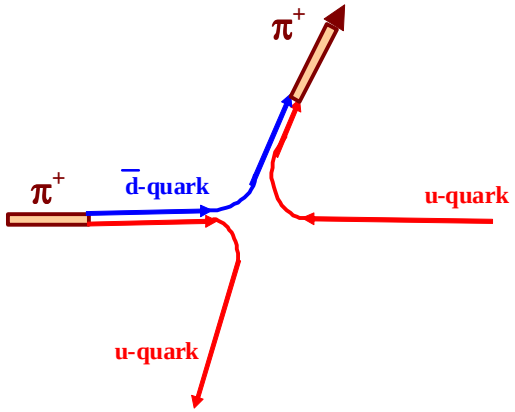
$$E\frac{d\sigma^{A+B\to h+X}_{FF1}}{d^3p}(s,p_T,\theta_{cm}) = \frac{(2.3\times10^6\,\mu b\cdot GeV^6)}{p_T^8}F(x_T,\theta_{cm})$$

CIM Model

Constituent Interchange Model (CIM)



Meson-Quark Scattering



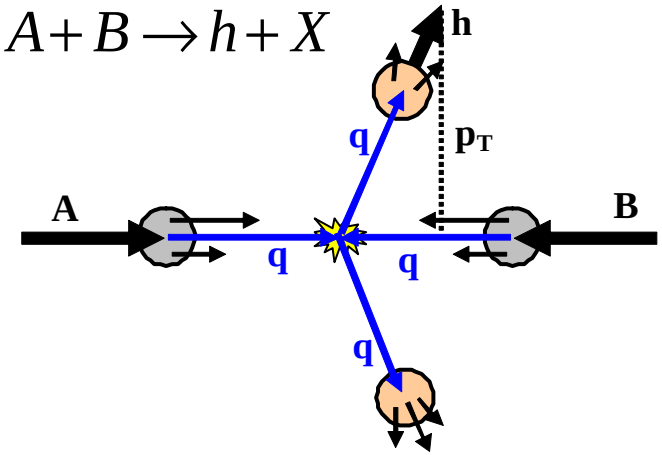
D. Sivers, S. J. Brodsky, and R. Blankenbecler, Phys. Reports 23C, 1 (1976); R. Blankenbecler, S. J. Brodsky, and J. F. Gunion, "The Magnitudes of Large Transverse Momentum Cross Sections", SLAC-PUB-2057 (1977)

$$E\frac{d\sigma_{CIM}^{p+p\to\pi+X}}{d^3p}(s,p_T,\theta_{cm})=\frac{1}{p_T^8}F_{CIM}(x_T,\theta_{cm})$$

QCD Improved Parton Model

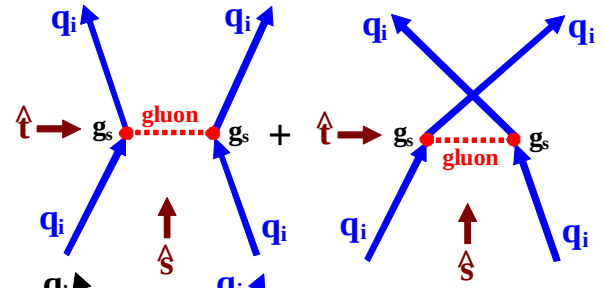
Large Transverse Meson Production in Hadron-Hadron Collisions

$A + B \to h + X$

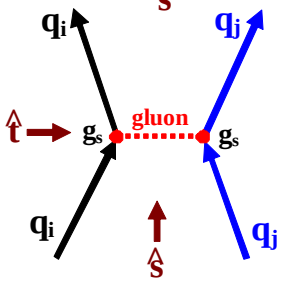


Quark-Quark Elastic Scattering

$A_{q_i+q_i \to q_i+q_i}(\hat{s},\hat{t}) =$



$A_{q_i+q_j \to q_i+q_j}(\hat{s},\hat{t}) =$



Include Leading Order QCD 2-to-2 Parton-Parton Processes (Order α_s^2)

$$\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2}{\hat{s}^2}\left|\overline{A}(a+b\to c+d)\right|^2$$

$$\left|\overline{A}(q_i+q_j \to q_i+q_j)\right|^2 = \left(\frac{4}{9}\right)\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

Parton = quark, anti-quark, & gluon

$$\left|\overline{A}(q_i+q_i \to q_i+q_i)\right|^2 = \left(\frac{4}{9}\right)\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)-\left(\frac{8}{27}\right)\frac{\hat{s}^2}{\hat{u}\hat{t}}$$

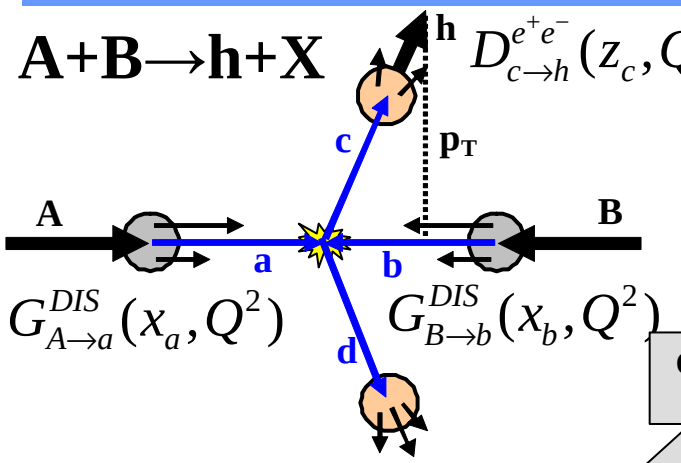
$q+q \to q+q$ $\quad g+q \to g+q$ $\quad g+g \to q+\bar{q}$

$\bar{q}+\bar{q} \to \bar{q}+\bar{q}$ $\quad g+\bar{q} \to g+\bar{q}$ $\quad g+g \to g+g$

$q+\bar{q} \to q+\bar{q}$ $\quad q+\bar{q} \to g+g$

Sum over final-state spins and color.
Average over initial-state spins and color.

QCD Improved Parton Model



Use the "renormalization group improved" PDF's from DIS and "renormalization group improved" fragmentation functions from e^+e^- annihilations and the running QCD coupling!

$$W = E_{cm} = \sqrt{s} \qquad x_T = 2p_T/W$$

$$x_1 = \tfrac{1}{2} x_T / \tan(\theta_{cm}/2) \qquad x_2 = \tfrac{1}{2} x_T \tan(\theta_{cm}/2)$$

QCD effective coupling

$$\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2(Q^2)}{\hat{s}^2}\left|\overline{A}^{a+b\to c+d}(\hat{s},\hat{t})\right|^2$$

$$x_a^{\min} = \frac{x_1}{1-x_2} \qquad x_b^{\min} = \frac{x_a x_2}{x_a - x_1} \qquad z_c = \frac{x_1}{x_b} + \frac{x_2}{x_a}$$

$$E\frac{d\sigma^{A+B\to h+X}}{d^3p}(s,p_T,\theta_{cm})$$

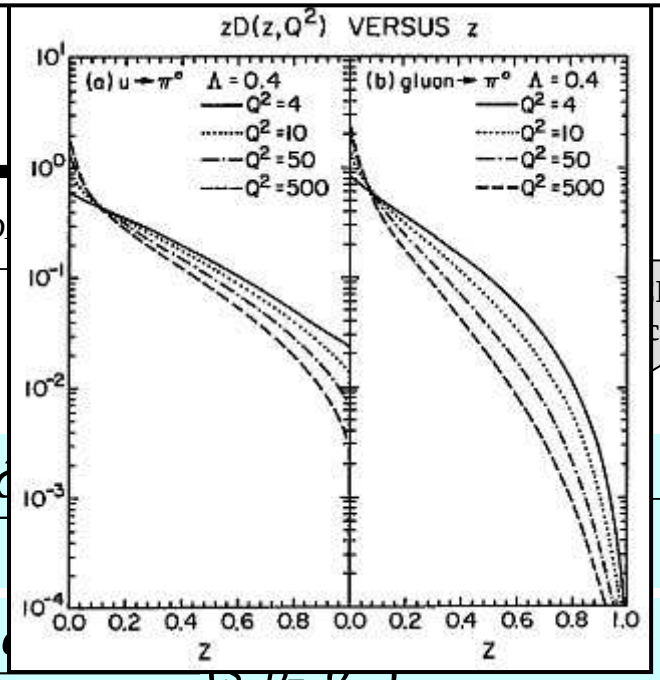
$$= \frac{1}{\pi}\sum_{a,b}\int_{x_a^{\min}}^{1.0} dx_a \int_{x_b^{\min}}^{1.0} dx_b G^{DIS}_{A\to a}(x_a,Q^2) G^{DIS}_{B\to b}(x_b,Q^2) D^{e^+e^-}_{c\to h}(z_c,Q^2) \frac{1}{z_c}\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(x_a x_b s, x_b t/z_c)$$

Q^2 dependent PDF's!

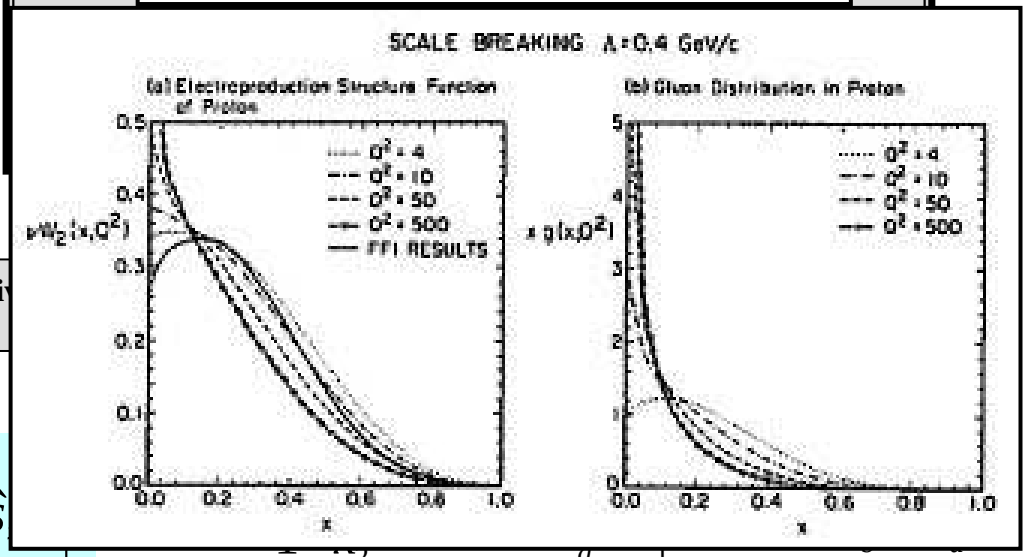
Q^2 dependent fragmentation!

QCD Improved Parton Model

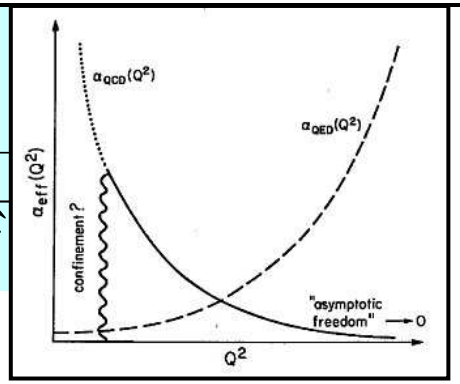
Renormalization Group Improved Fragmentation!



Renormalization Group Improved PDF's!



Running QCD Effective Coupling!



$$E\frac{d^3\sigma}{d^3p}(s,p_T,\theta_{cm}) = \frac{1}{\pi}\sum_{a,b}\int_{x_a^{min}}^{1.0}dx_a\int_{x_b^{min}}^{1.0}dx_b\,G_{A\to a}^{DIS}(x_a,Q^2)\,G_{B\to b}^{DIS}(x_b,Q^2)\,D_{c\to h}^{e^+e^-}(z_c,Q^2)\,\frac{1}{z_c}\,\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}$$

Q^2 dependent PDF's!

Q^2 dependent fragmentation!

QCD Improved Parton Model

Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order α_s^2)

(1) $q_i + q_j \xrightarrow[i\neq j]{} q_i + q_j$

(2) $q_i + \bar{q}_j \xrightarrow[i\neq j]{} q_i + \bar{q}_j$

(3) $q_i + q_i \to q_i + q_i$

(4) $q_i + \bar{q}_i \to q_i + \bar{q}_i$

(5) $q_i + \bar{q}_i \to g + g$

(6) $gg \to q_i + \bar{q}_i$

(7) $q_i + g \to q_i + g$

(8) $\bar{q}_i + g \to \bar{q}_i + g$

(9) $g + g \to g + g$

QCD effective coupling

$$\frac{d\hat{\sigma}_k}{d\hat{t}}(\hat{s},\hat{t}) = \frac{\pi\alpha_s^2(Q^2)}{\hat{p}_T^4} f_k(\hat{x}_T) \qquad \textbf{k = 1 to 9}$$

Use the "renormalization group improved" PDF's from DIS and "renormalization group improved" fragmentation functions from e^+e^- annihilations and the running QCD coupling!

$$p_T^4 E \frac{d\sigma_{all}^{A+B\to h+X}}{d^3p}(s, p_T, \theta_{cm}) = \sum_{k=1}^{9} F_k(s, p_T, \theta_{cm})$$

QCD Improved Parton Model

Naïve parton model scaling is now broken!

$$x_1 = \tfrac{1}{2} x_T / \tan(\theta_{cm}/2) \qquad x_a^{\min} = \frac{x_1}{1-x_2} \qquad x_b = \frac{x_a x_2}{z_c x_a - x_1}$$

$$x_2 = \tfrac{1}{2} x_T \tan(\theta_{cm}/2) \qquad x_b^{\min} = \frac{x_a x_2}{x_a - x_1} \qquad z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b}$$

$$x_T = 2p_T/\sqrt{s}$$

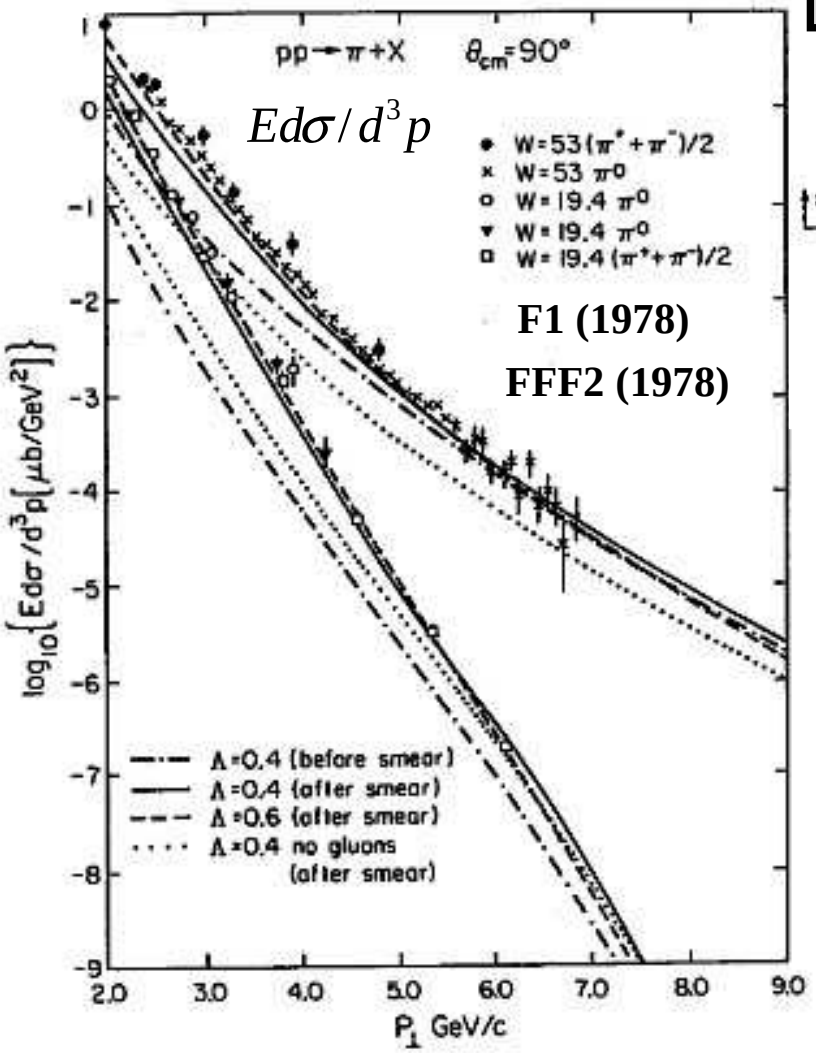
Possible Choices for the Scale

$$Q^2 = 4\hat{p}_T^2$$

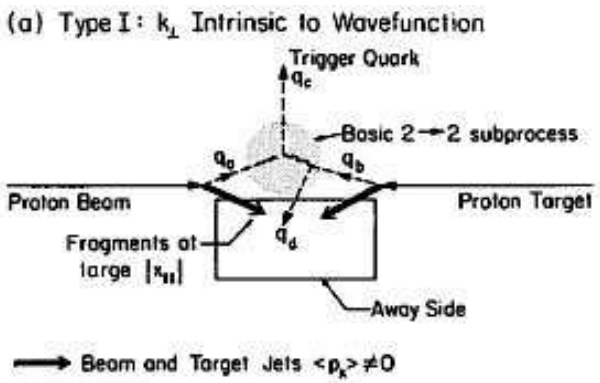
$$Q^2 = \frac{2\hat{s}\hat{t}\hat{u}}{\hat{s}^2 + \hat{t}^2 + \hat{u}^2}$$

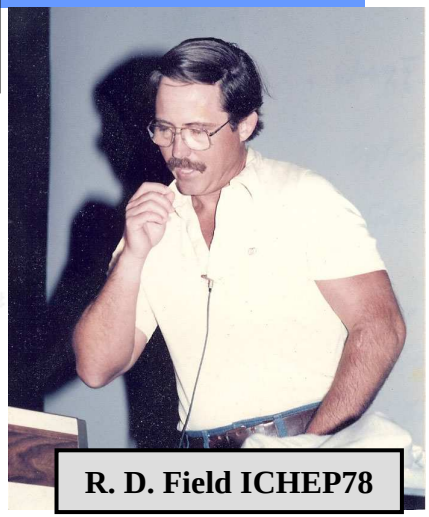
QCD R. Field ICHEP78

QCD Improved Parton Model



Early Experimental Test of QCD



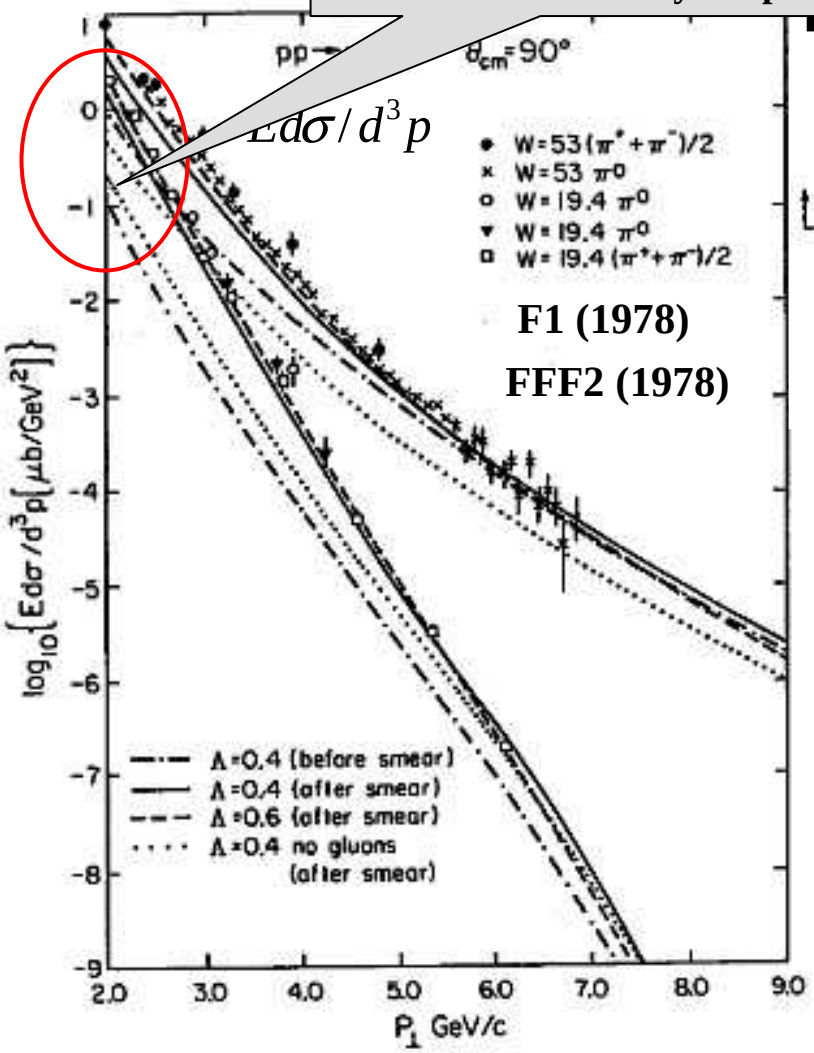

R. D. Field ICHEP78

➡ **Comparison of a QCD model (normalized absolutely) with data on large p_T pion production in proton-proton collisions at W = 19.4 and 53 GeV with $\theta_{cm} = 90^o$. The dot-dashed and solid curves are the results before and after smearing, respectively, with $\Lambda = 0.4$ GeV/c and $\langle k_T \rangle_{primordial} = 848$ MeV/c and the dashed curves are with $\Lambda = 0.6$ GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark (*i.e.* no gluons) is shown by the dotted curves.**

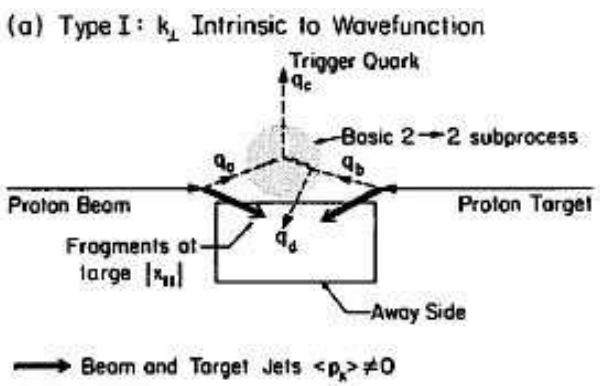
QCD R. Field ICHEP78

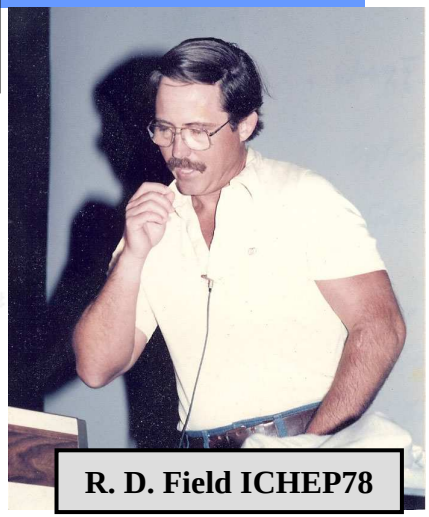
Primordial k_T has a big effect at low p_T and low W where the cross section is very steep!

QCD Imp



arly Experimental Test of QCD

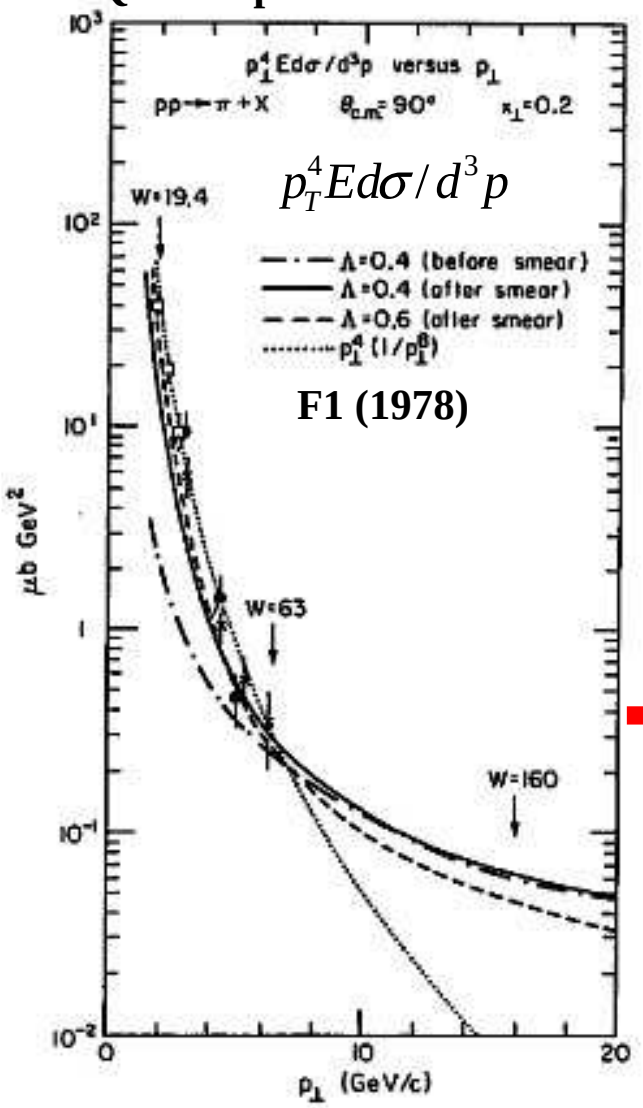


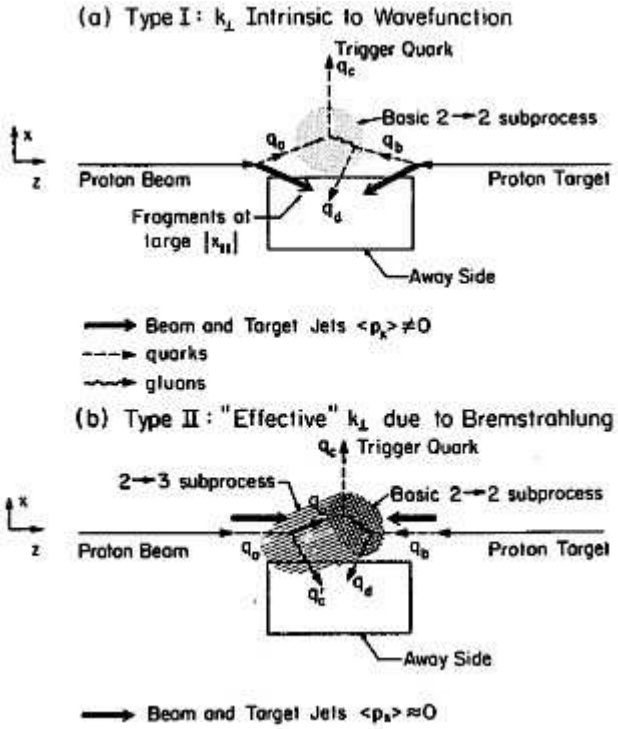

R. D. Field ICHEP78

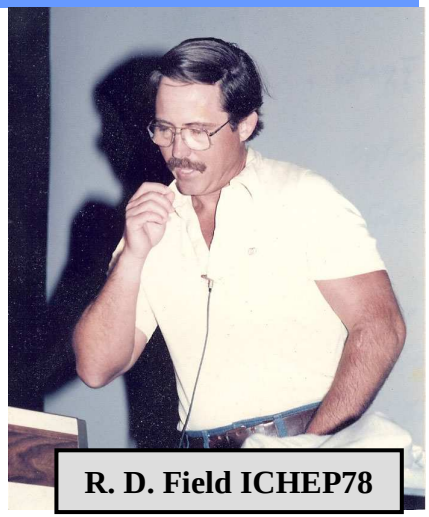
Large primordial k_T!

- Comparison of a QCD m (normalized absolutely) with data on lar pion production in proton-proton collisions at = 19.4 and 53 GeV with θ_{cm} = 90°. The dot-dashed an solid curves are the results before and after smearing, respectively, with Λ = 0.4 GeV/c and $\langle k_T \rangle_{primordial}$ = 848 MeV/c and the dashed curves are with Λ = 0.6 GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark (*i.e.* no gluons) is shown by the dotted curves.

QCD Improved Parton Model

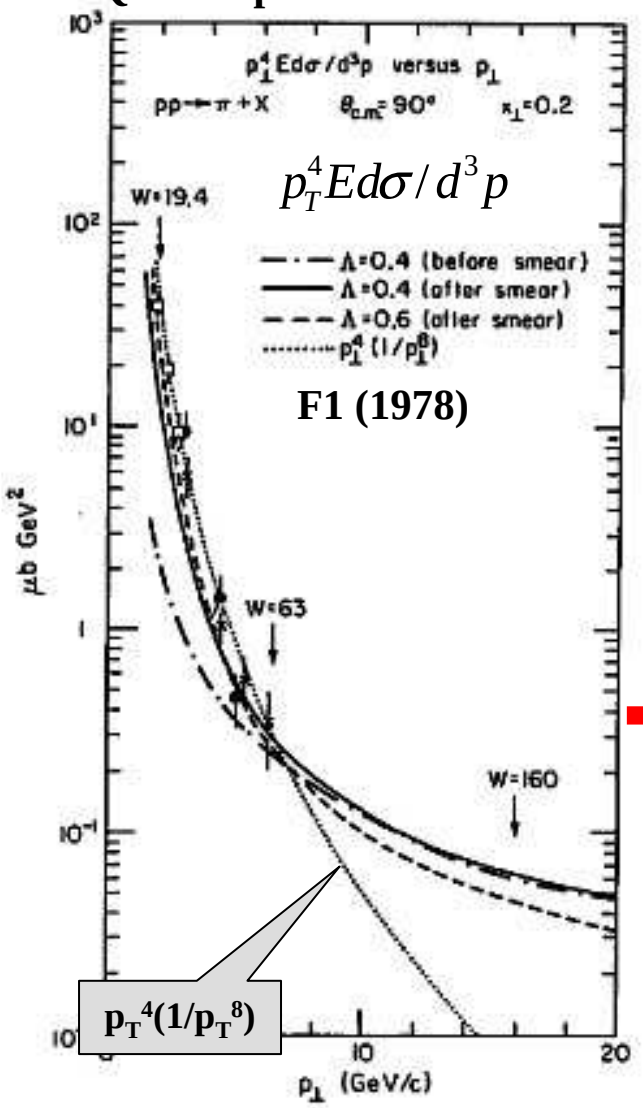


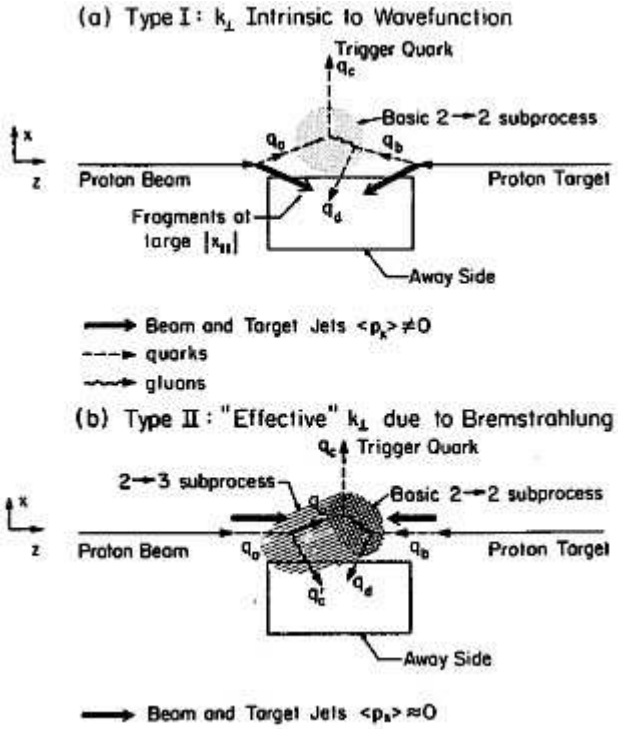


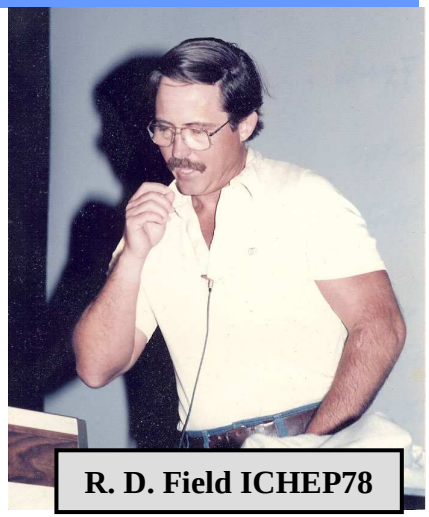


Data on p_T^4 times $Ed\sigma/d^3p$ for large p_T pion production at θ_{cm} = 90° and fixed x_T = 0.2 versus p_T compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{primordial}$ = 848 MeV, respectively, with Λ = 0.4 GeV/c and the dashed curves are the results of using Λ = 0.6 GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$.

QCD Improved Parton Model

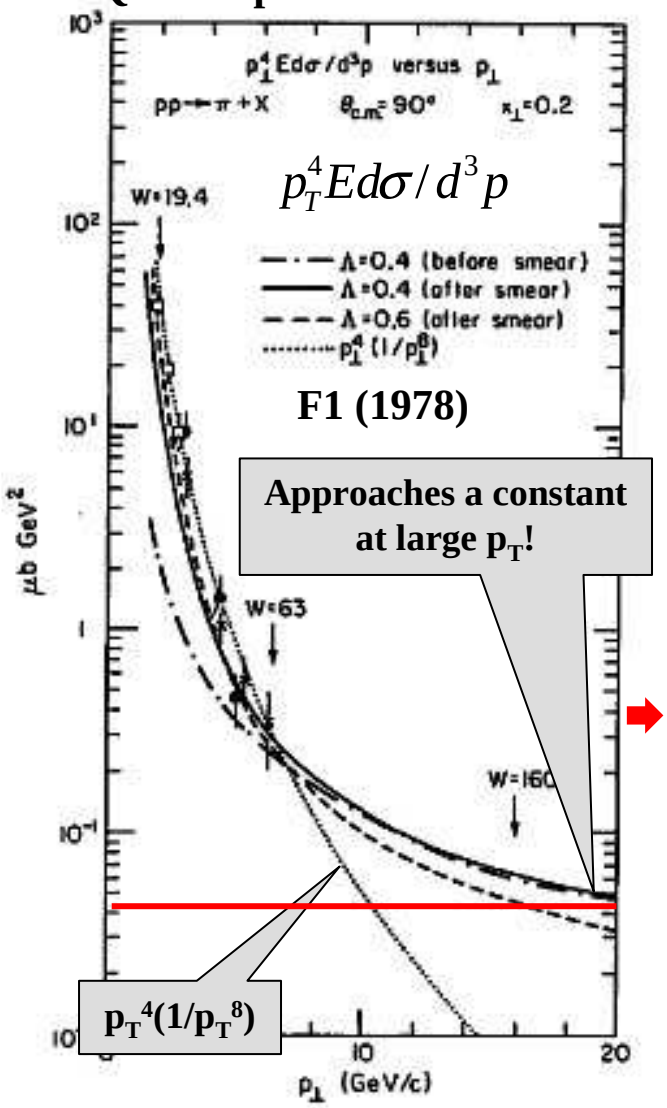


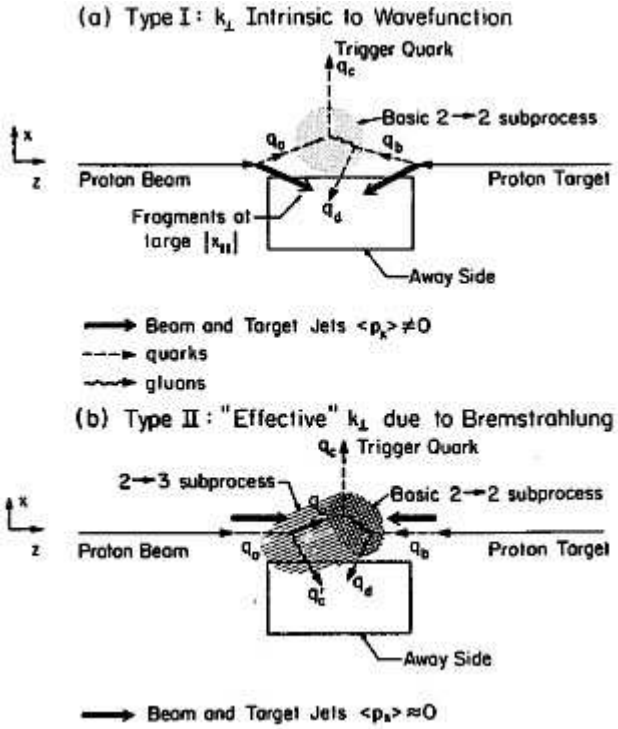


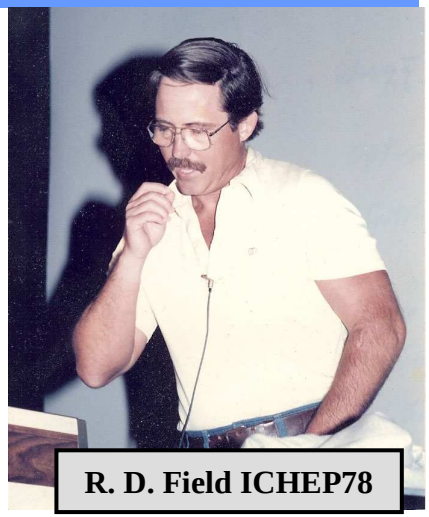


Data on p_T^4 times $Ed\sigma/d^3p$ for large p_T pion production at θ_{cm} = 90° and fixed x_T = 0.2 versus p_T compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{primordial}$ = 848 MeV, respectively, with Λ = 0.4 GeV/c and the dashed curves are the results of using Λ = 0.6 GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$.

QCD Improved Parton Model

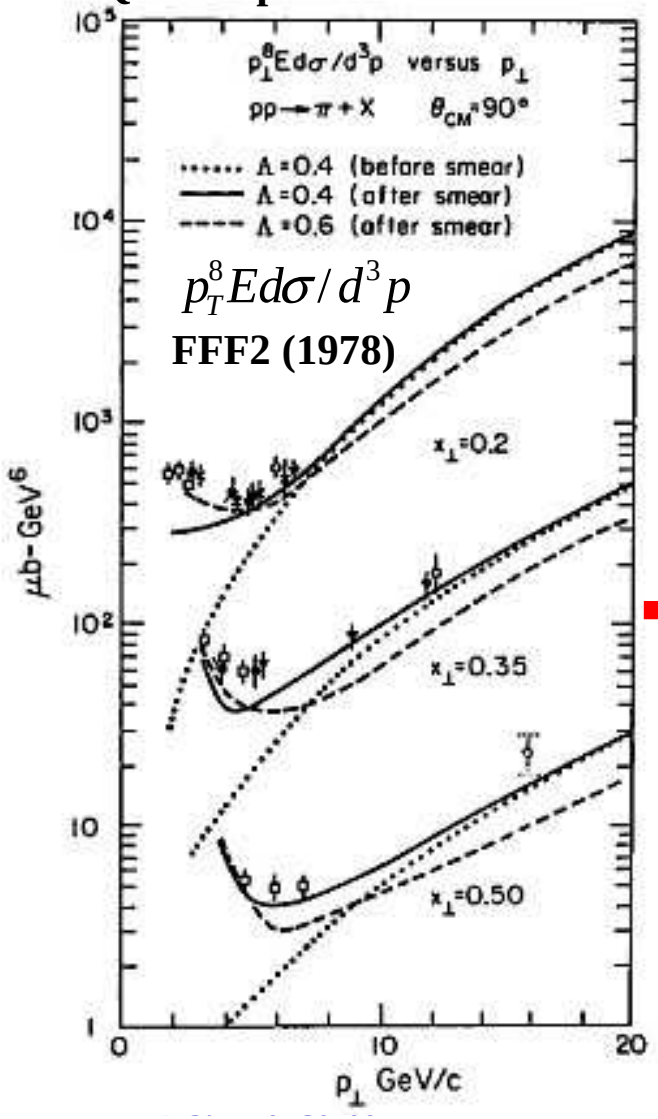


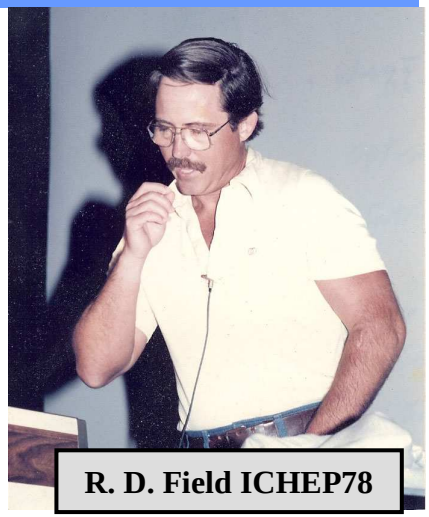




Data on p_T^4 times $Ed\sigma/d^3p$ for large p_T pion production at θ_{cm} = 90° and fixed x_T = 0.2 versus p_T compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{primordial}$ = 848 MeV, respectively, with Λ = 0.4 GeV/c and the dashed curves are the results of using Λ = 0.6 GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$.

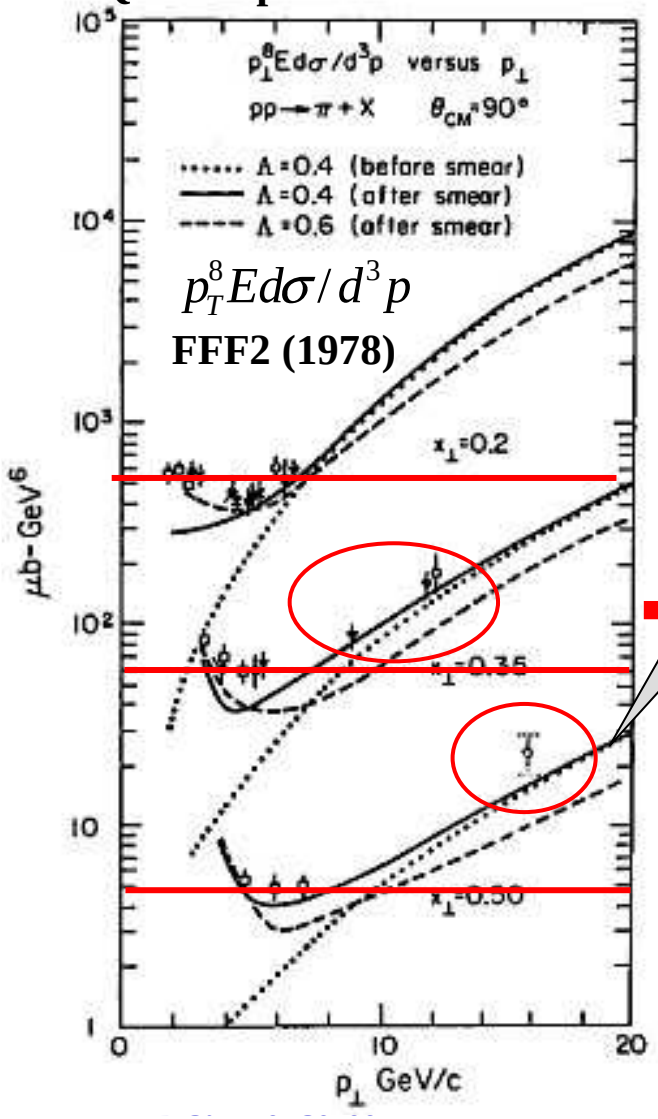
QCD Improved Parton Model




R. D. Field ICHEP78

➡ **Data on p_T^8 times $Ed\sigma/d^3p$ for large p_T pion production at θ_{cm} = 90° and <u>fixed</u> x_T = 0.2, 0.35, and 0.5 versus p_T compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{primordial}$ = 848 MeV, respectively, with Λ = 0.4 GeV/c and the dashed curves are the results of using Λ = 0.6 GeV/c (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.**

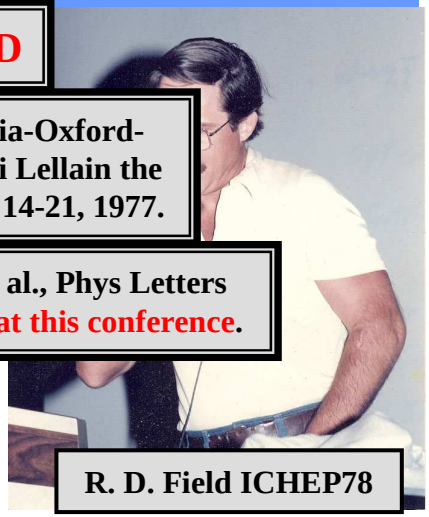
QCD Improved Parton Model



Early Experimental Test of QCD

The ISR data (triangles) are from CERN-Columbia-Oxford-Rockefeller Experiment (CCOR), reported by L. Di Lellain the Workshop on Future ISR Experiments, September 14-21, 1977.

The ISR data (open circles) are from A. G. Clark et al., Phys Letters 74B (1978) 267. Also, talk presented by A. G. Clark at this conference.

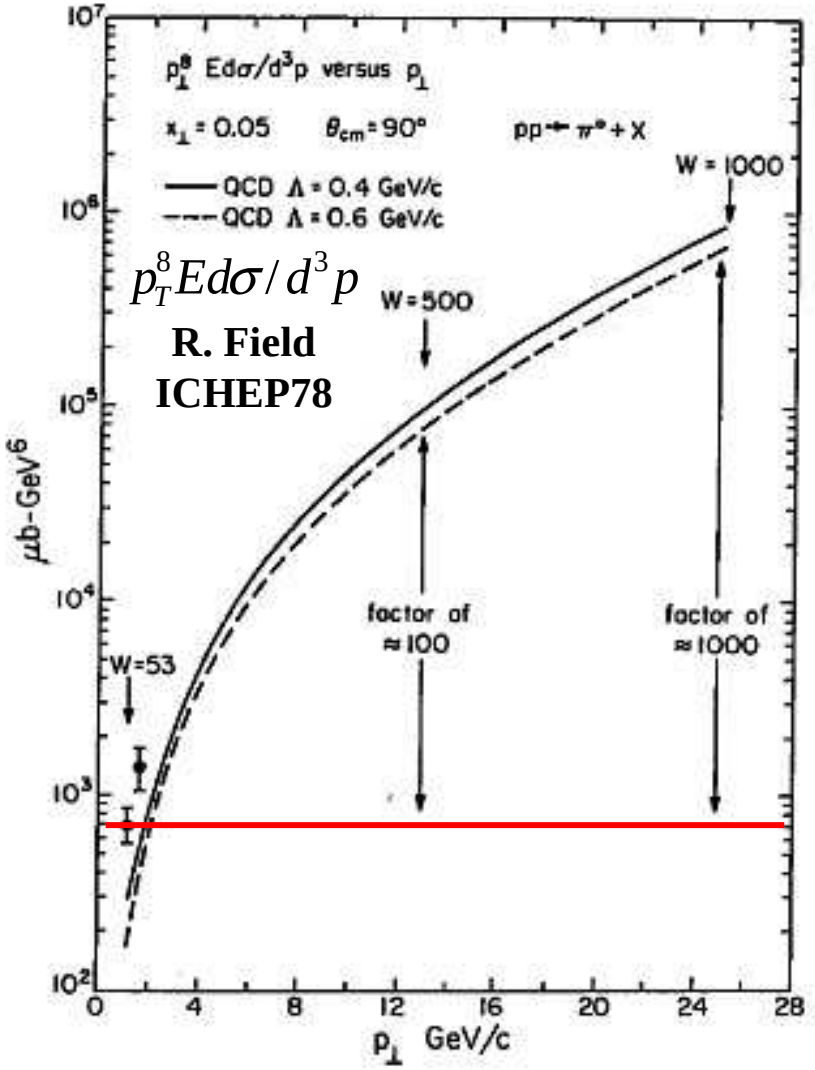


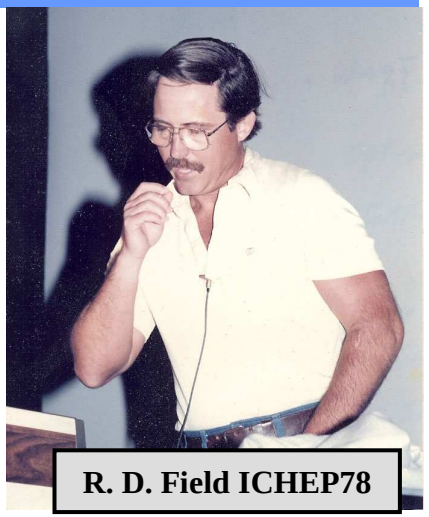
Approaches a p_T^4 behavior at large p_T!

Data on p_T^8 times $Ed\sigma/d^3p$ for large p_T pion production at θ_{cm} = 90° and fixed x_T = 0.2, 0.35, and 0.5 versus p_T compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{primordial}$ = 848 MeV, respectively, with Λ = 0.4 GeV/c and the dashed curves are the results of using Λ = 0.6 GeV/c (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.

QCD Improved Parton Model

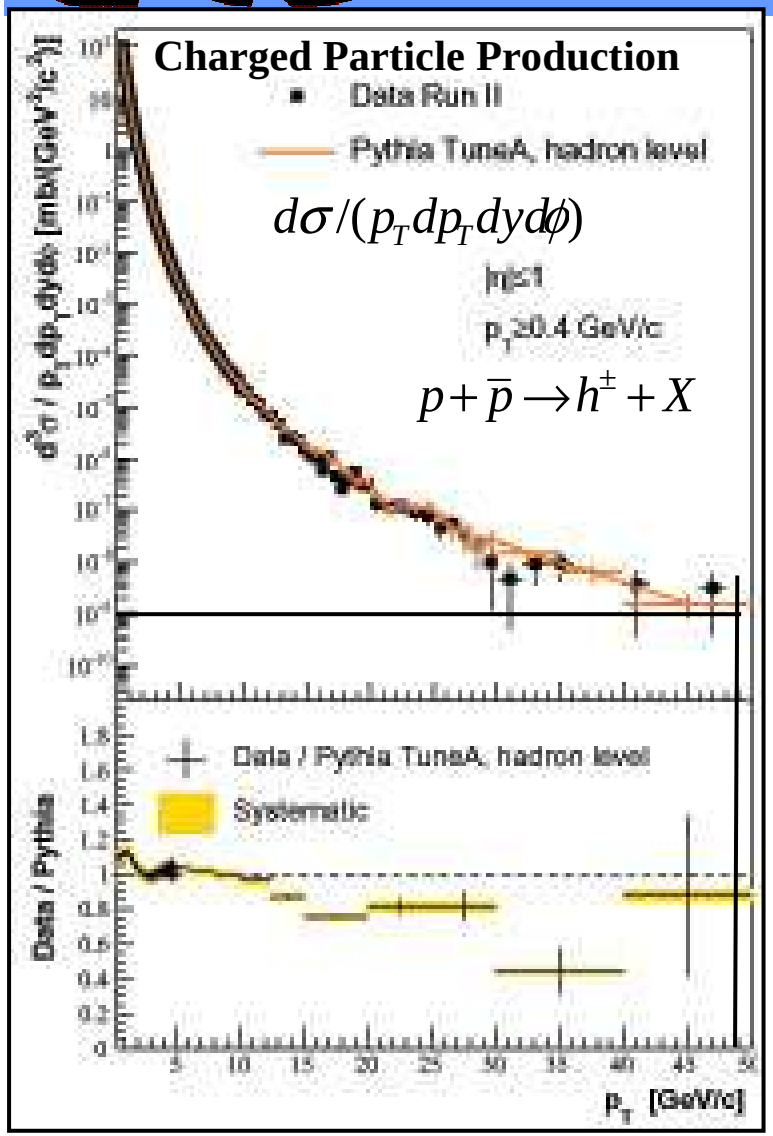
Early QCD Predictions





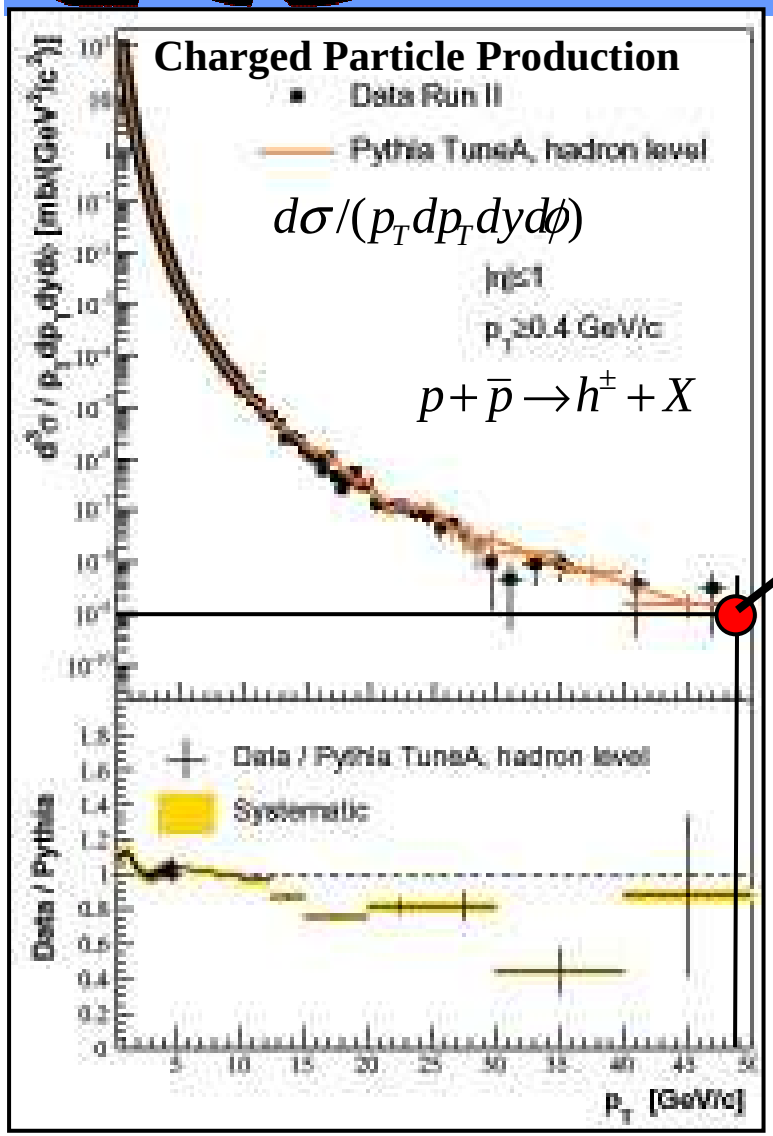
➡ **The behavior of p_T^8 times the 90° single π^0 cross section, Edσ/d^{3}p, at x_T = 0.05 versus p_T calculated from the QCD approach with Λ = 0.4 GeV/c (solid curves) and Λ = 0.6 GeV/c (dashed curves). The two low p_T data points are at 53 and 63 GeV. The predictions are a factor of 100 (1000) times larger than the flat horizontal ($1/p_T^8$) extrapolation to W = 500 GeV (1000 GeV).**

QCD CDF Run 2 Data



Measurement of Particle Production and Inclusive Differential Cross Sections in Proton-Antiproton Collisions at 1.96 TeV,
CDF Collaboration, Phys. Rev. D79, 112005 (2009)

QCD CDF Run 2 Data



Measurement of Particle Production and Inclusive Differential Cross Sections in Proton-Antiproton Collisions at 1.96 TeV,
CDF Collaboration, Phys. Rev. D79, 112005 (2009)

W = 1.96 TeV x_T = 0.05

$$p_T = \tfrac{1}{2}(0.05)(1960 GeV) = 49 GeV/c$$

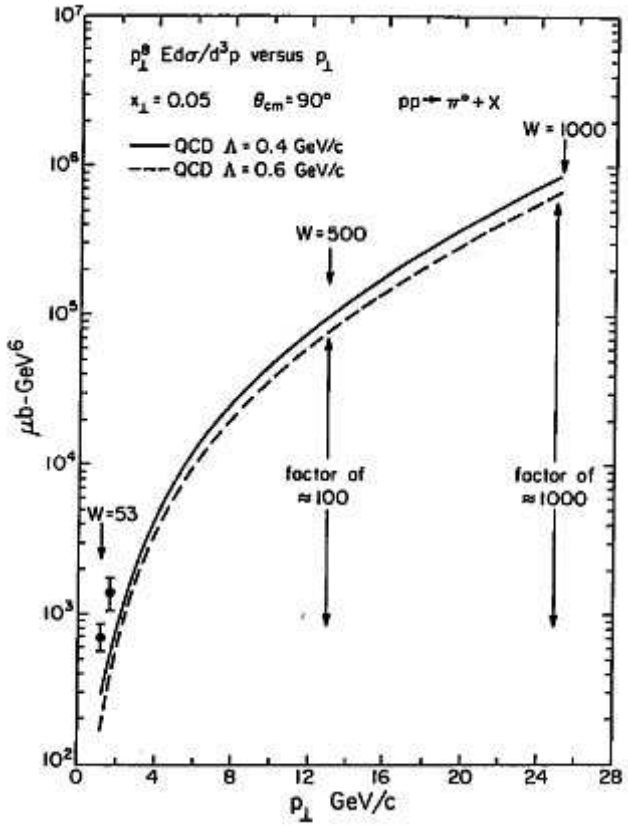
$$\frac{d\sigma^{h^{\pm}}(x_T = 0.05, p_T = 49 GeV/c)}{p_T dp_T dy d\phi} \approx 10^{-9}\, mb/(GeV/c)^2$$

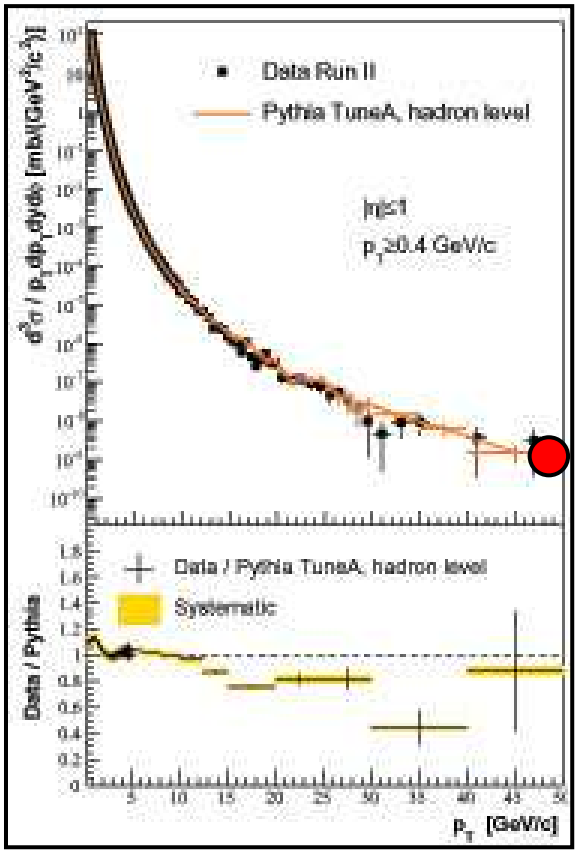
$$E\frac{d\sigma}{d^3p} = \frac{d\sigma}{y d^2 p_T} = \frac{d\sigma}{y p_T dp_T d\phi} \qquad \frac{\pi^0}{h^+ + h^-} \approx 0.5$$

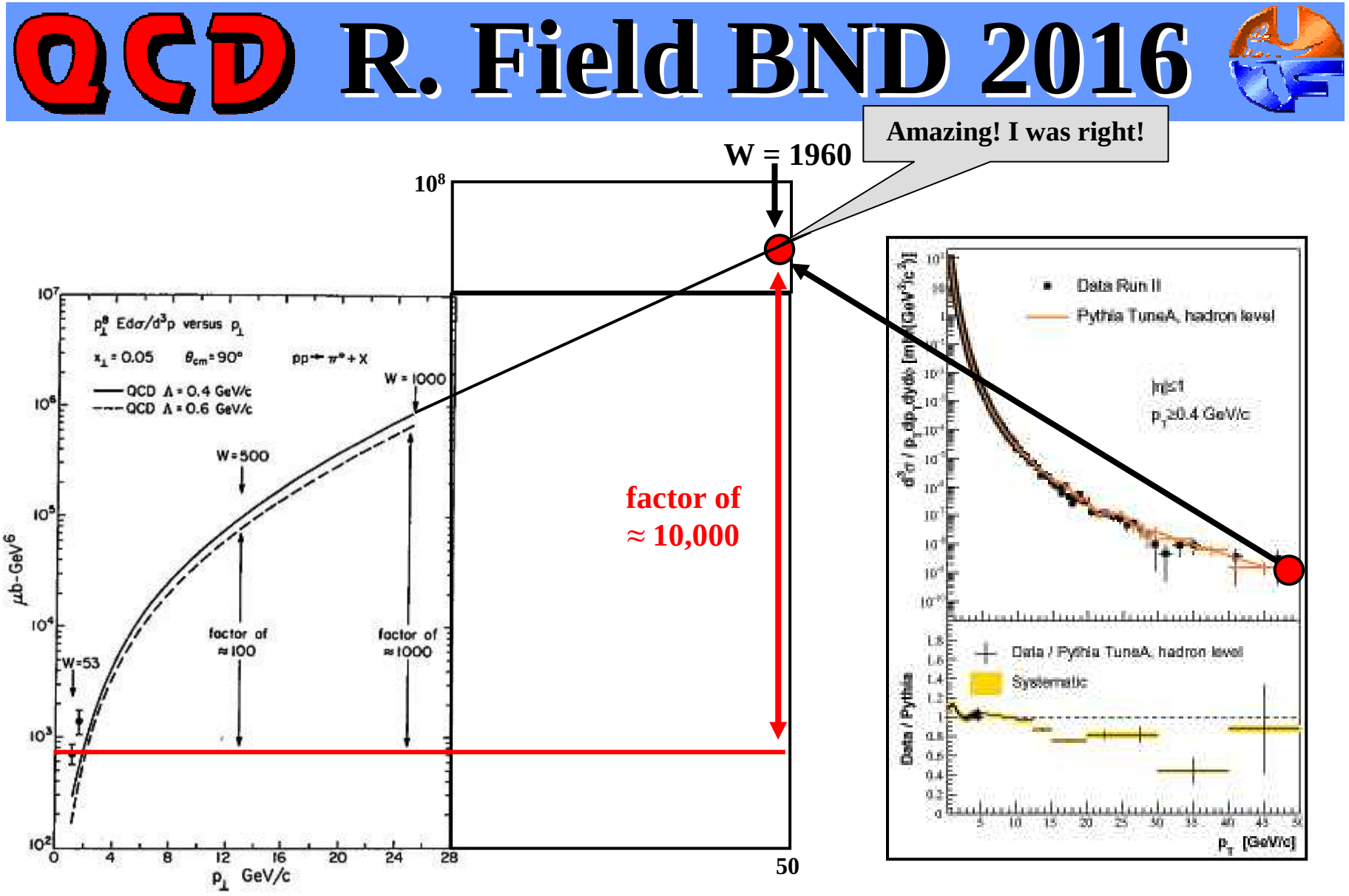
$$p_T^8 E \frac{d\sigma^{\pi^0}(x_T = 0.05, p_T = 49 GeV/c)}{d^3p}$$

$$\approx (0.5)(49 GeV/c)^8 \times 10^{-9}\, mb/(GeV/c)^2$$

$$\approx 1.66 \times 10^7\, \mu b \cdot (GeV/c)^6$$


$p_\perp^8 Ed\sigma/d^3p$ versus $p_\perp$
$x_\perp = 0.05$ $\theta_{cm} = 90°$ $pp \to \pi^0 + X$
QCD Λ = 0.4 GeV/c
QCD Λ = 0.6 GeV/c
W = 1000
W = 500
W = 53
factor of ≈ 100
factor of ≈ 1000
μb-GeV⁶
$p_\perp$ GeV/c


Data Run II
Pythia TuneA, hadron level
|η|<1
$p_T \geq 0.4$ GeV/c
Data / Pythia TuneA, hadron level
Systematic
Data / Pythia
p_T [GeV/c]


Amazing! I was right!
W = 1960
10^8
50
factor of
≈ 10,000
p_T^8 Edσ/d³p versus p_T
x_T = 0.05
θ_{cm} = 90°
pp → π° + X
QCD Λ = 0.4 GeV/c
QCD Λ = 0.6 GeV/c
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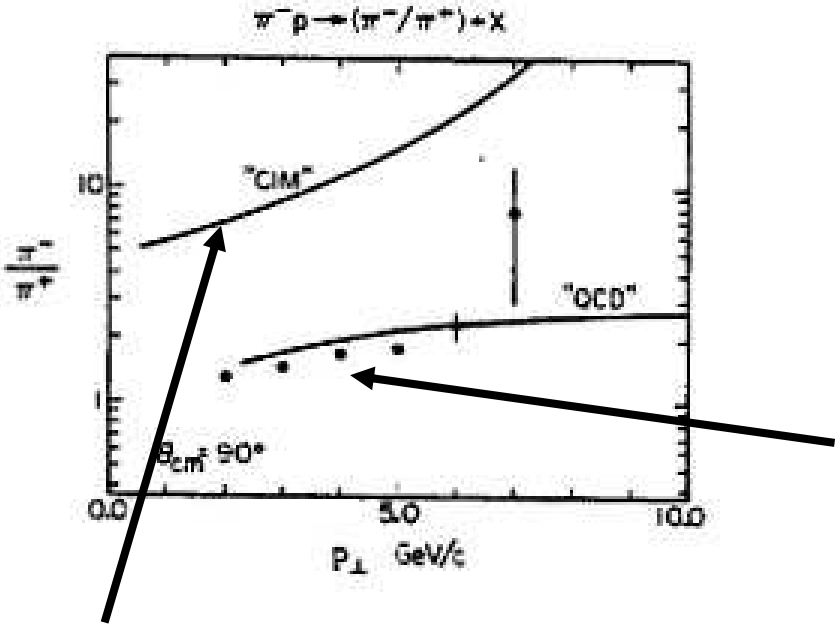
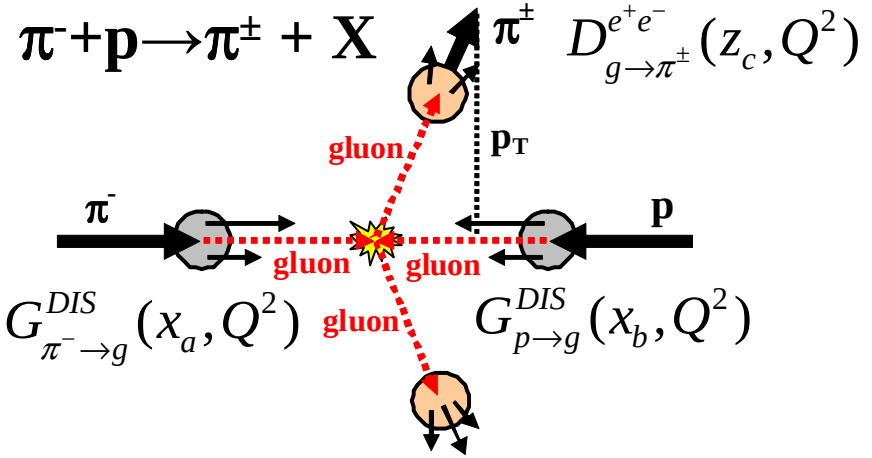
factor of ≈ 100
factor of ≈ 1000
μb-GeV⁶
p_T GeV/c
Data Run II
Pythia TuneA, hadron level
|η|<1
p_T≥0.4 GeV/c
Data / Pythia TuneA, hadron level
Systematic
Data / Pythia
p_T [GeV/c]

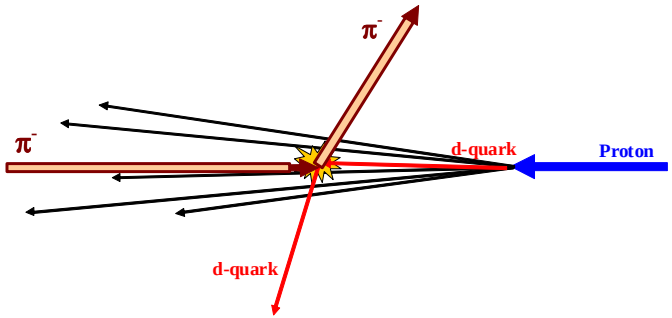
CIM versus the QCD Improved Parton Model

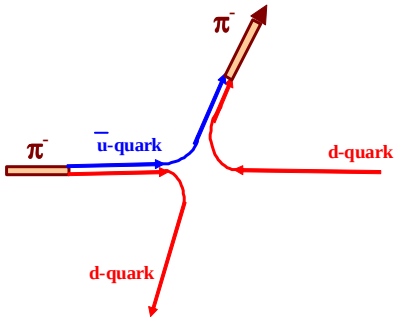




At low x_T the dominate QCD sub-process is gluon production which fragments equally into π^+ and π^-!

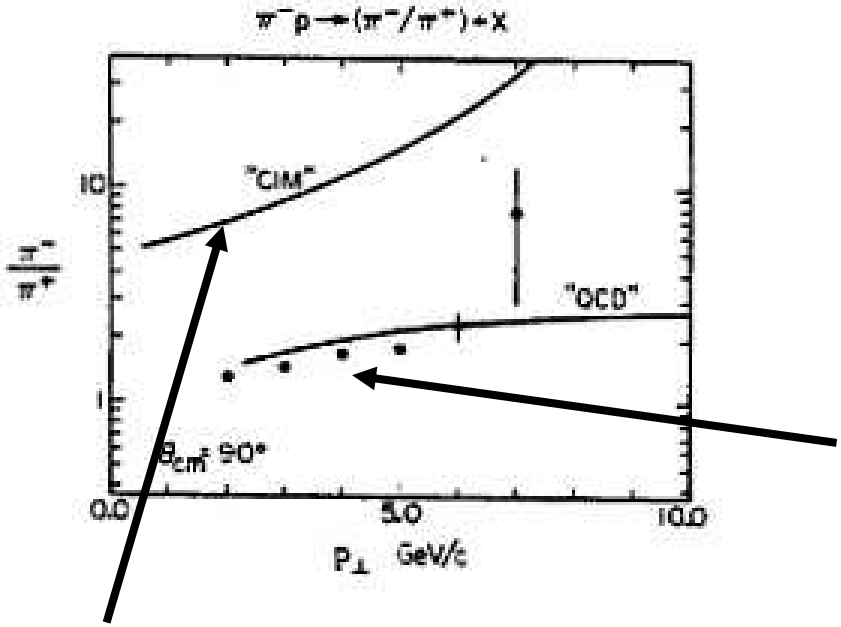
Direct CIM Contribution π^- + d-quark → π^- + d-quark



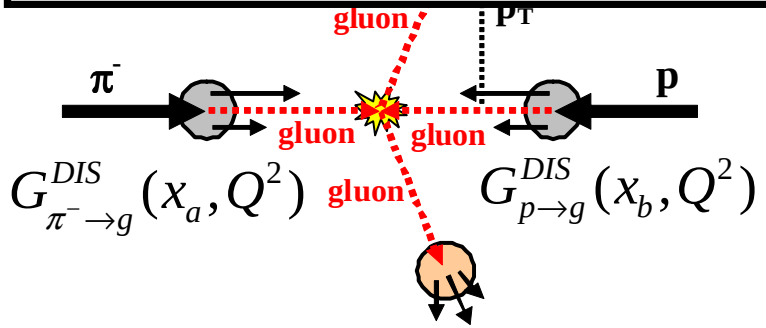


To produce an outgoing π^+ in the CIM approach one must find a π^+ within the incoming π^- or the incoming proton (with small probability)!

CIM versus the QCD Improved Parton Model



The data are from H. J. Frisch, *Precise Measurement of the High p_T Single Particle Spectra in πp Collisions at Fermilab (E258)*, talk presented at the XIV Moriond Conference (1979)..



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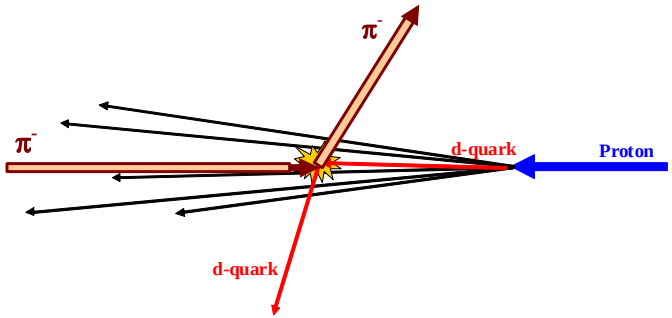

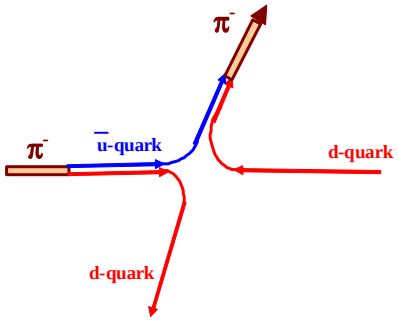

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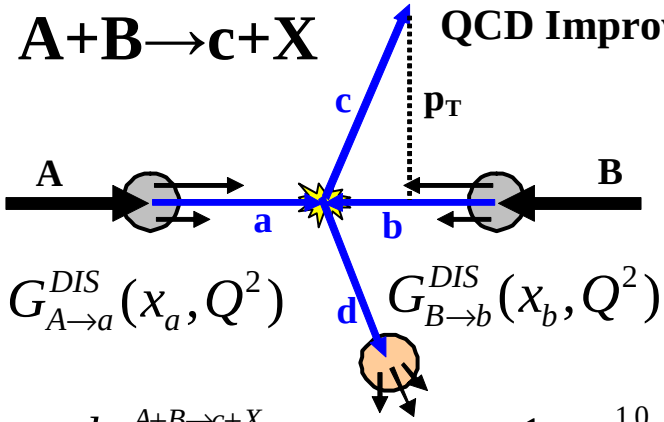
QCD "Jet" Production



QCD Improved Parton Model

Large Transverse Momentum Parton Production

$$D^{e^+e^-}_{c\to h}(z_c,Q^2)=\delta(1-z_c)$$

$$x_a=\frac{x_b x_1}{z_c x_b - x_2} \qquad x_b=\frac{x_a x_2}{z_c x_a - x_1} \qquad z_c=\frac{x_1}{x_a}+\frac{x_2}{x_b} \qquad \frac{dz_c}{dx_b}=-\frac{x_2}{x_b^2}$$

$$E\frac{d\sigma^{A+B\to c+X}}{d^3p}(s,p_T,\theta_{cm})=\frac{1}{\pi}\sum_{a,b}\int_{x_a^{\min}}^{1.0}dx_a\int_{x_b^{\min}}^{1.0}dx_b G^{DIS}_{A\to a}(x_a,Q^2)G^{DIS}_{B\to b}(x_b,Q^2)\delta(1-z_c)\frac{1}{z_c}\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t})$$

$$E\frac{d\sigma^{A+B\to c+X}}{d^3p}(s,p_T,\theta_{cm})=\frac{1}{\pi}\sum_{a,b}\int_{x_a^{\min}}^{1.0}dx_a\int_{z_c^{\min}}^{1.0}dz_c G^{DIS}_{A\to a}(x_a,Q^2)G^{DIS}_{B\to b}(x_b,Q^2)\delta(1-z_c)\frac{x_b^2}{x_2 z_c}\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{t}}(\hat{s},\hat{t})$$

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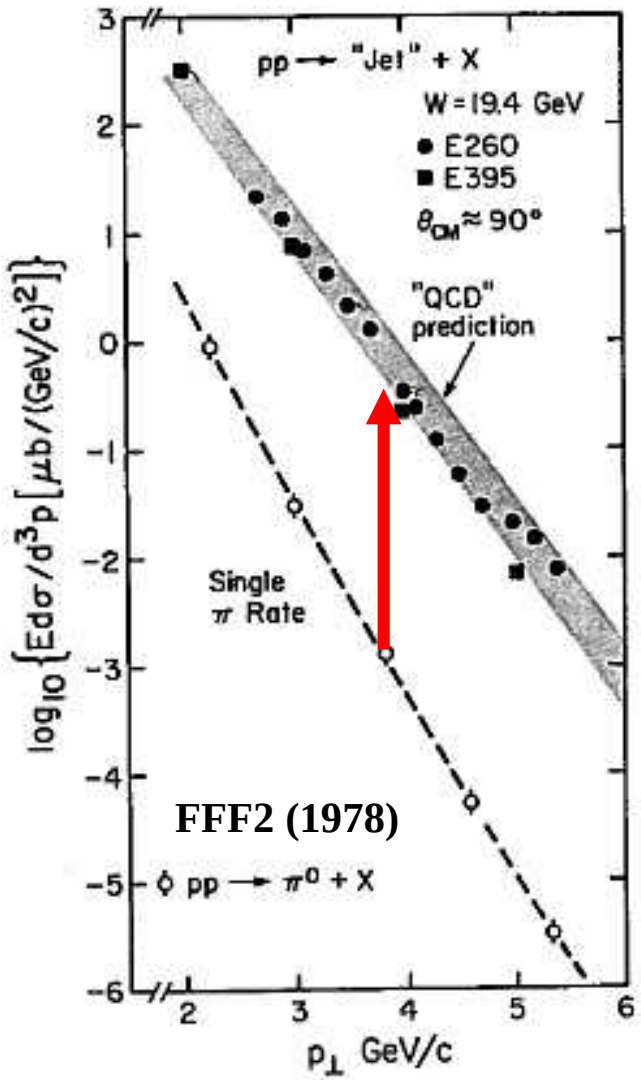
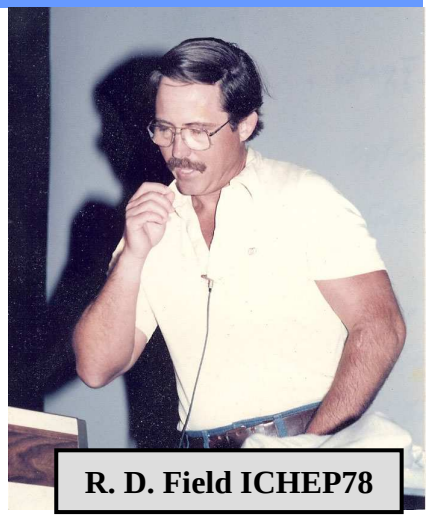
$$x_a^{\min}=\frac{x_1}{1-x_2} \qquad x_b=\frac{x_a x_2}{x_a - x_1}$$

QCD Improved Parton Model



Early Experimental Test of QCD



R. D. Field ICHEP78

The FNAL data (E260) are from C. Bromberg et al., Phys Rev Letters 38 (1977); C. Bromberg et al., CALT-68-613 (*to be published in Nucl. Phys.*).

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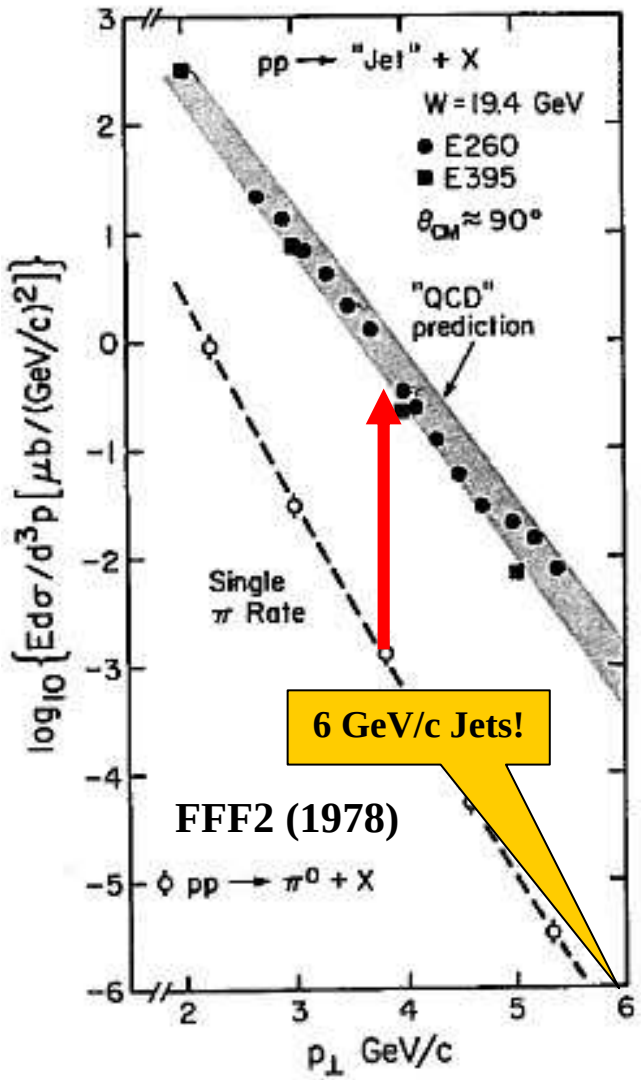
➡ **Comparison of the jet and single π^0 cross sections measured at 200 GeV (W = 19.4 GeV) and $\theta_{cm} = 90^o$. The jet data are from two FNAL experiments, E260 and E395, where a jet is defined as the sum of all the particles into their respective detectors. Also shown is the QCD prediction for the cross section of producing a parton (quark, antiquark, or gluon) at W = 19.4 GeV and $\theta_{cm} = 90^o$.**

QCD Improved Parton Model



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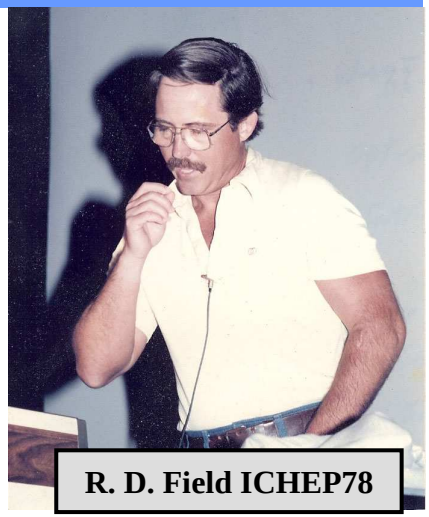

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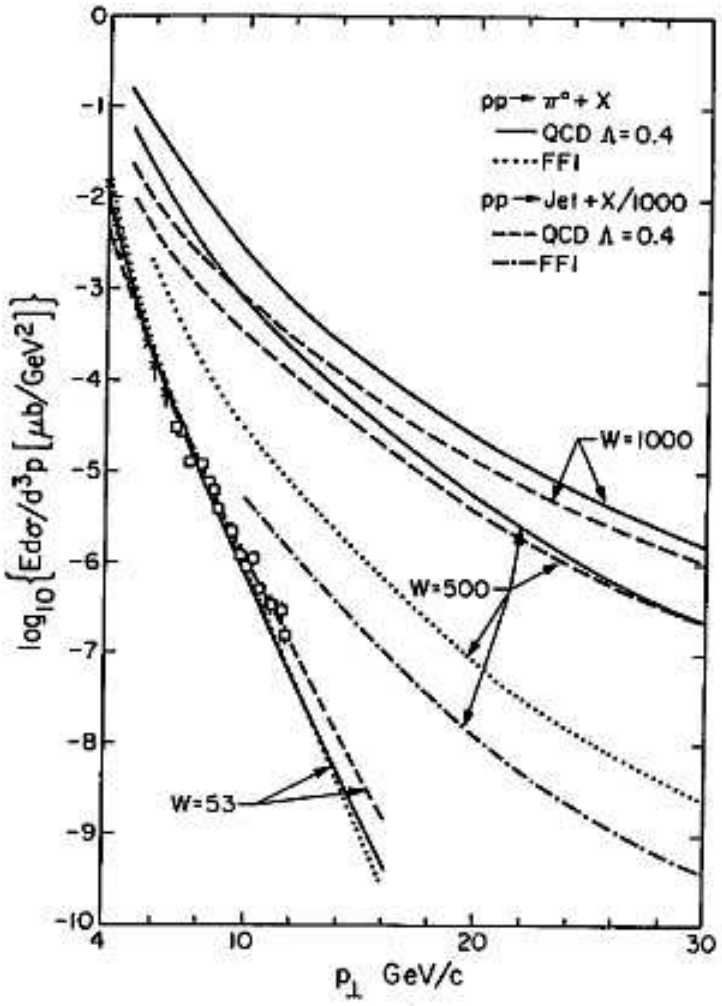
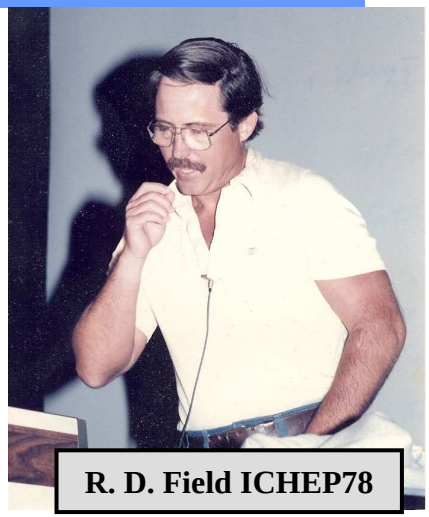
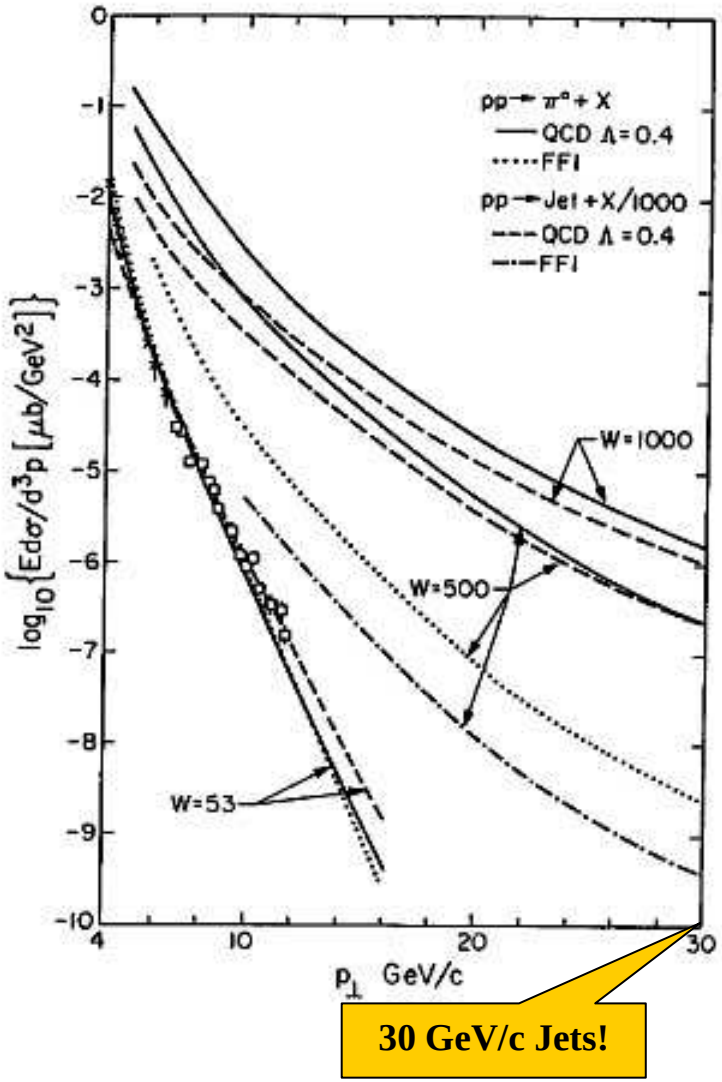


Early QCD Predictions


R. D. Field ICHEP78

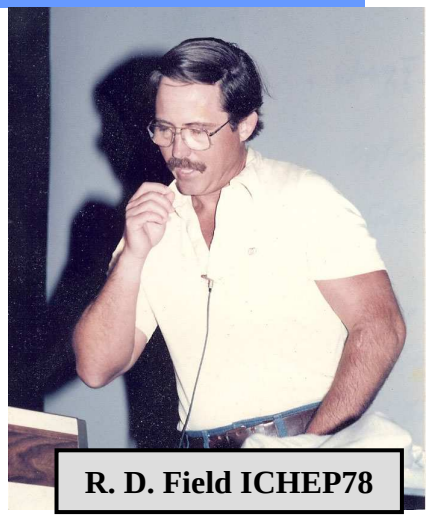
➡ **Comparison of the results on the 90° π^0 cross section, $Ed\sigma/d^3p$, from the QCD approach with $\Lambda = 0.4$ GeV/c (solid curves) and the quark-quark "black-box" parton model of FF1 (dotted curves). Both models agree with the data at W = 53 GeV. The QCD approach results in much larger cross sections than the FF1 model at W = 500 and 1000 GeV. The FF1 results at 1000 GeV (not shown) are only slightly larger than at 500 GeV. Also shown are the cross sections for producing a jet at 90° (divided by 1000) as predicted by the QCD approach (dashed curves) and the FF1 model (dot-dashed curve)**

QCD Improved Parton Model



30 GeV/c Jets!

Early QCD Predictions



R. D. Field ICHEP78

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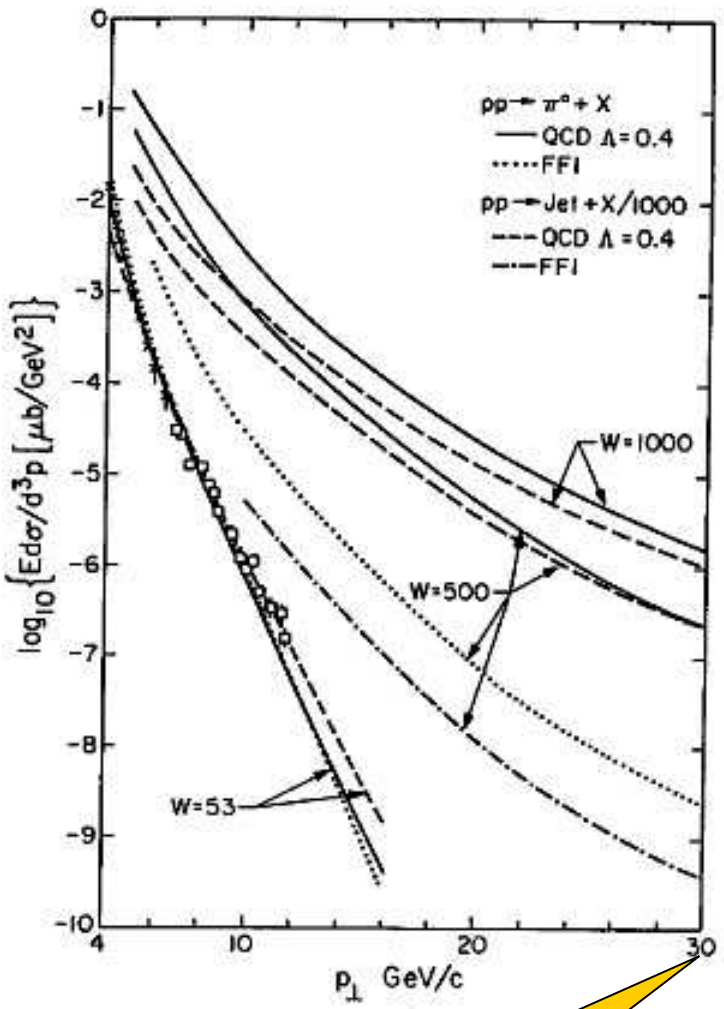
QCD Boulder 1979



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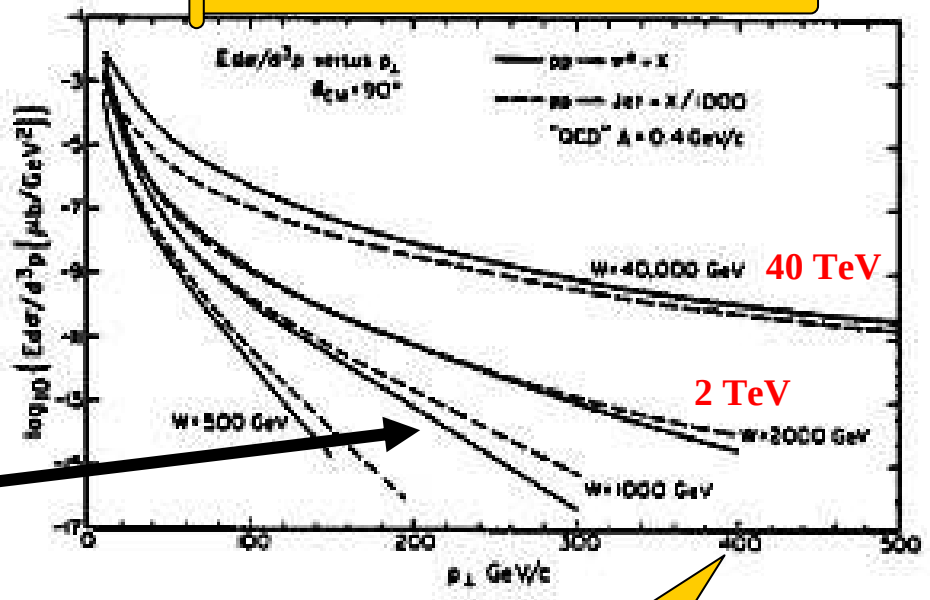
R. Field Boulder 1979



40 TeV

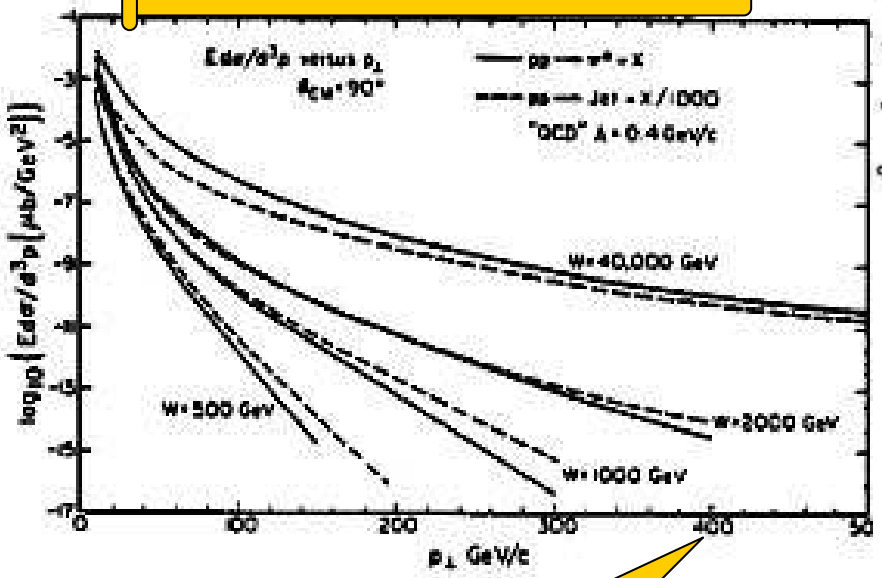
2 TeV

400 GeV/c Jets!

➡ **The NATO Advanced Institute on Quantum Flavordynamics, Quantum Chromodynamics, and Unified Theories, held at the University of Colorado, Boulder, Colorado, July 9 – 27, 1979.**

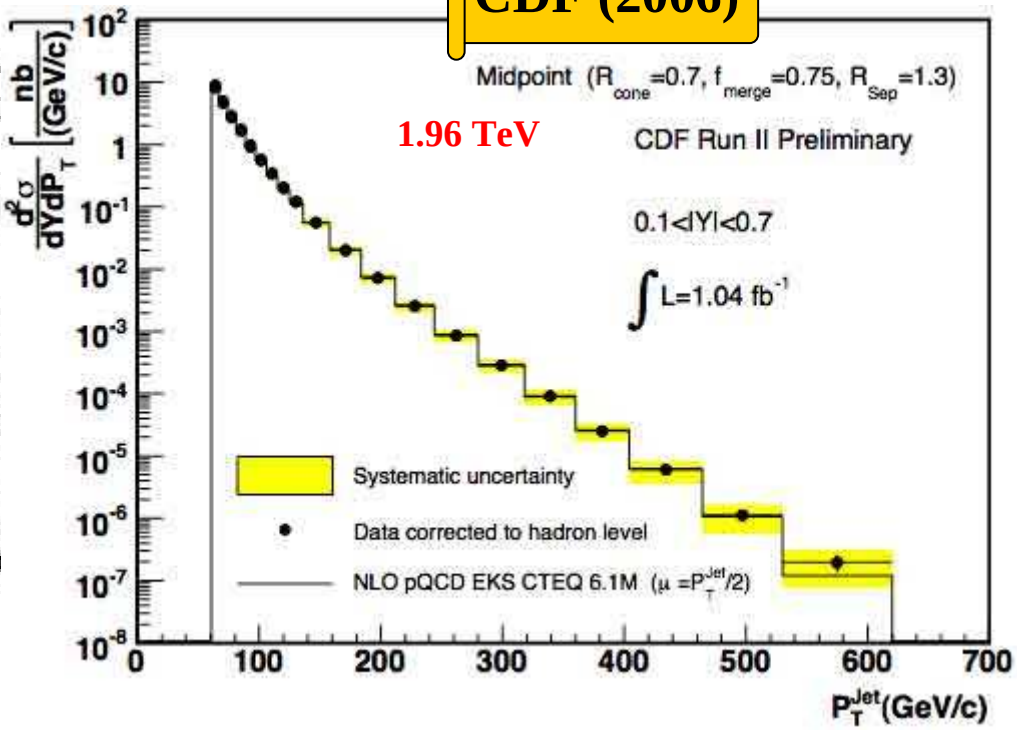
QCD R. Field BND 2016

R. Field Boulder 1979

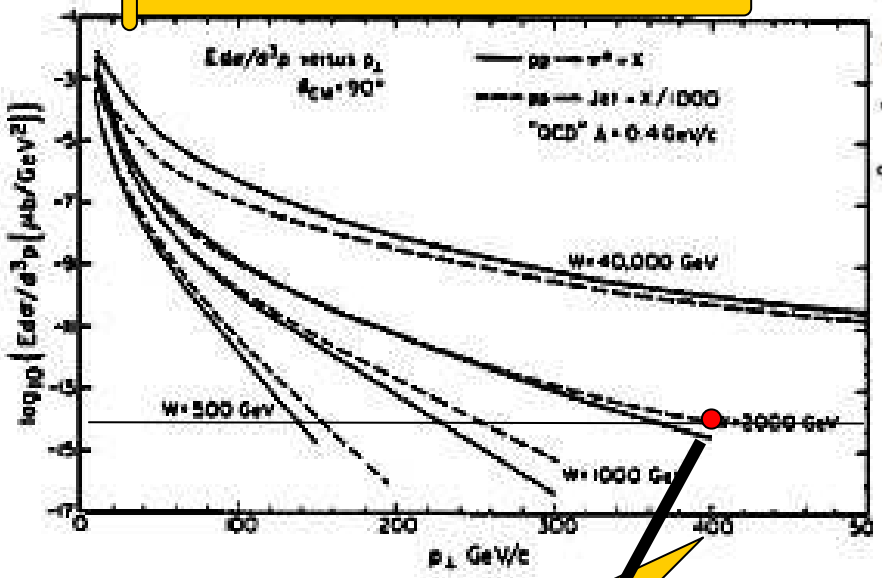


400 GeV/c Jets!

CDF (2006)



R. Field Boulder 1979



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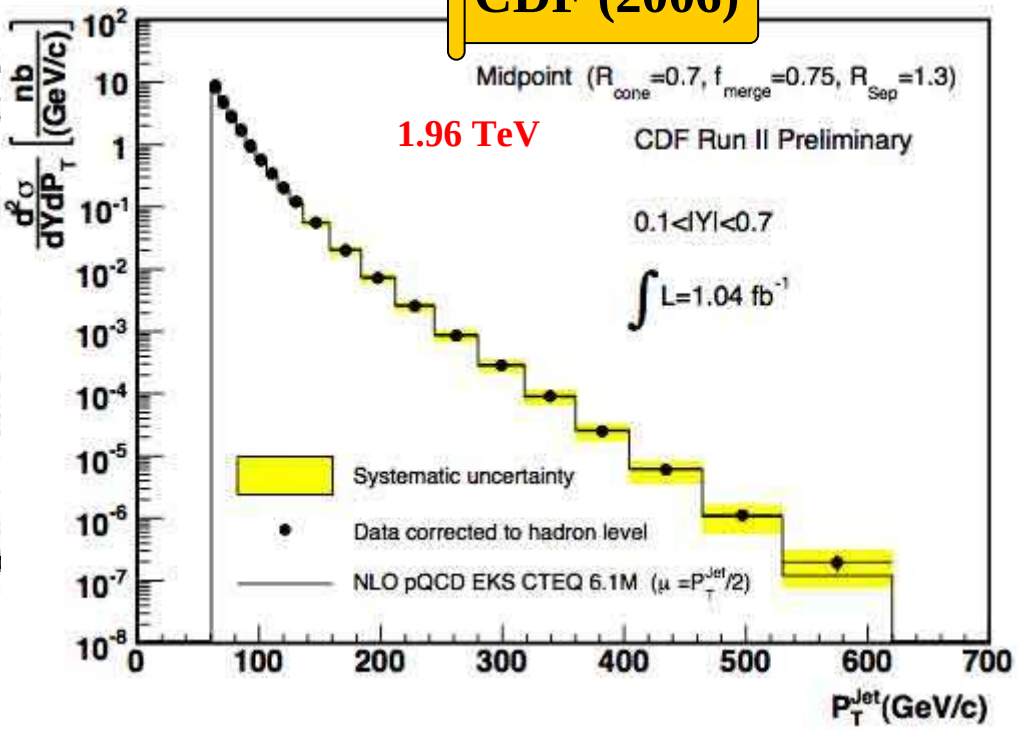
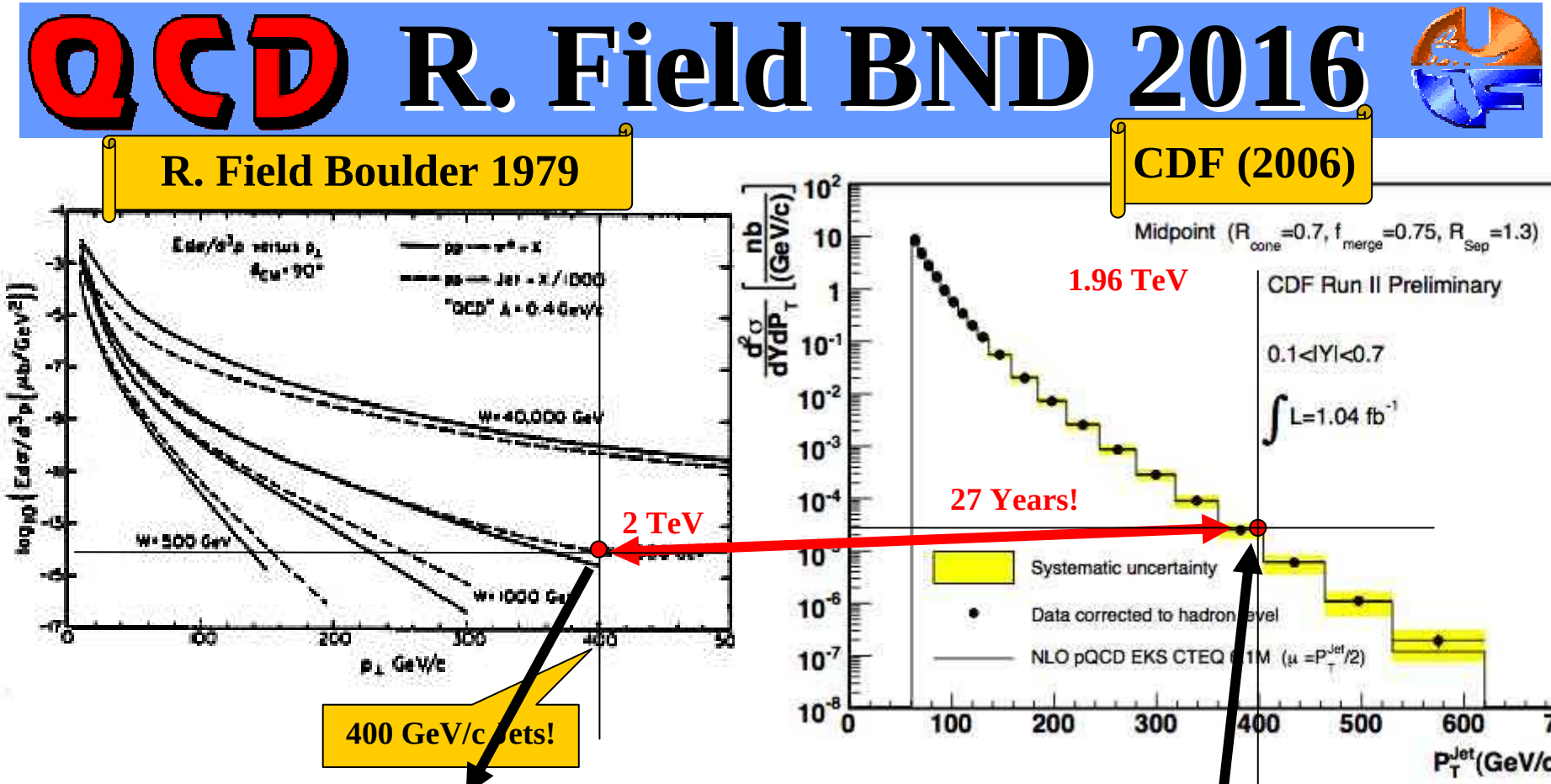


$$E\frac{d\sigma}{d^3p}(pp \to Jet + X) \approx 10^{-14}(\mu b / GeV^2)$$

$$\frac{d^2\sigma}{dp_T dy} = 2\pi p_T\left(E\frac{d\sigma}{d^3p}\right)$$

QCD R. Field BND 2016



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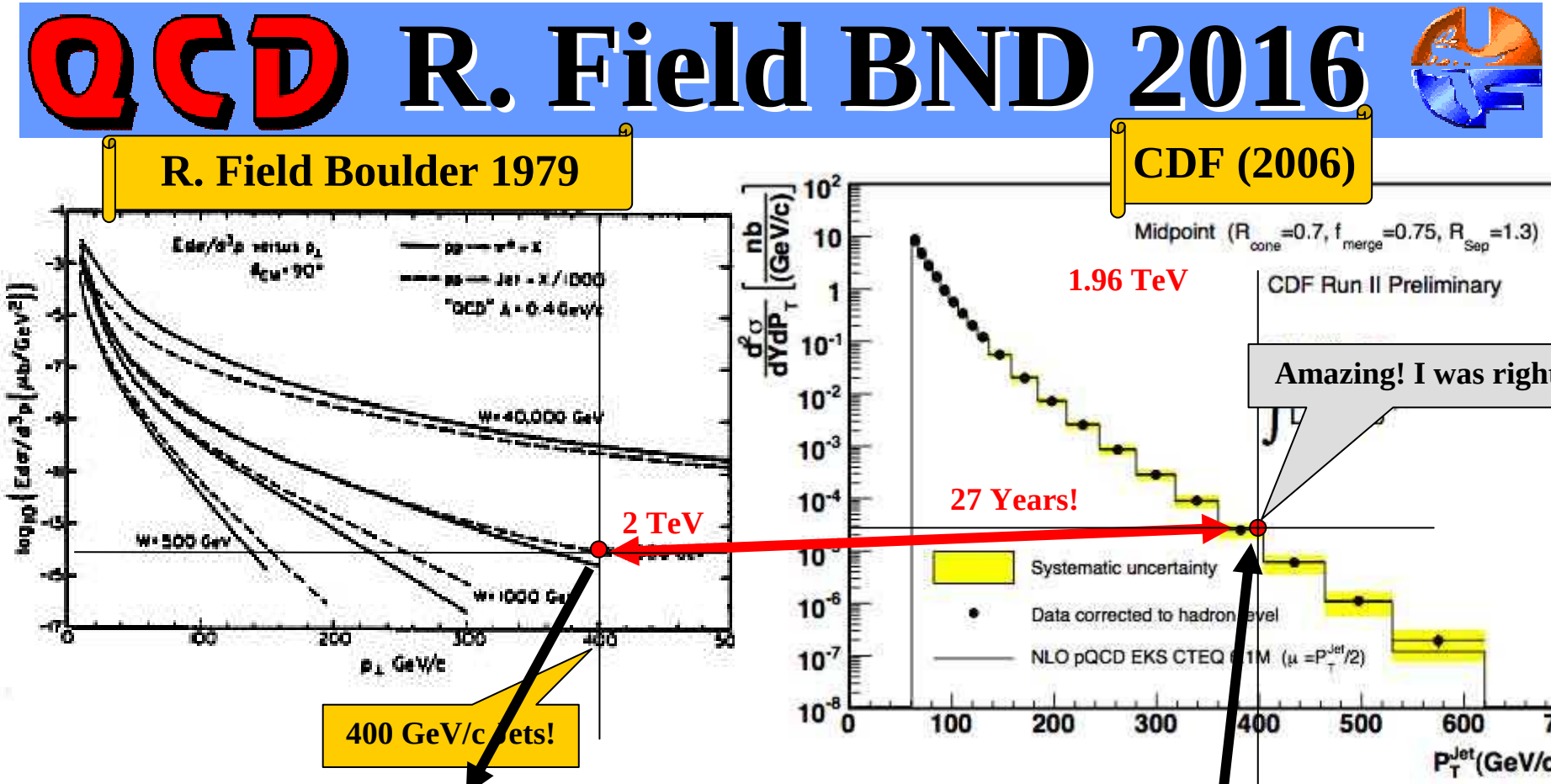

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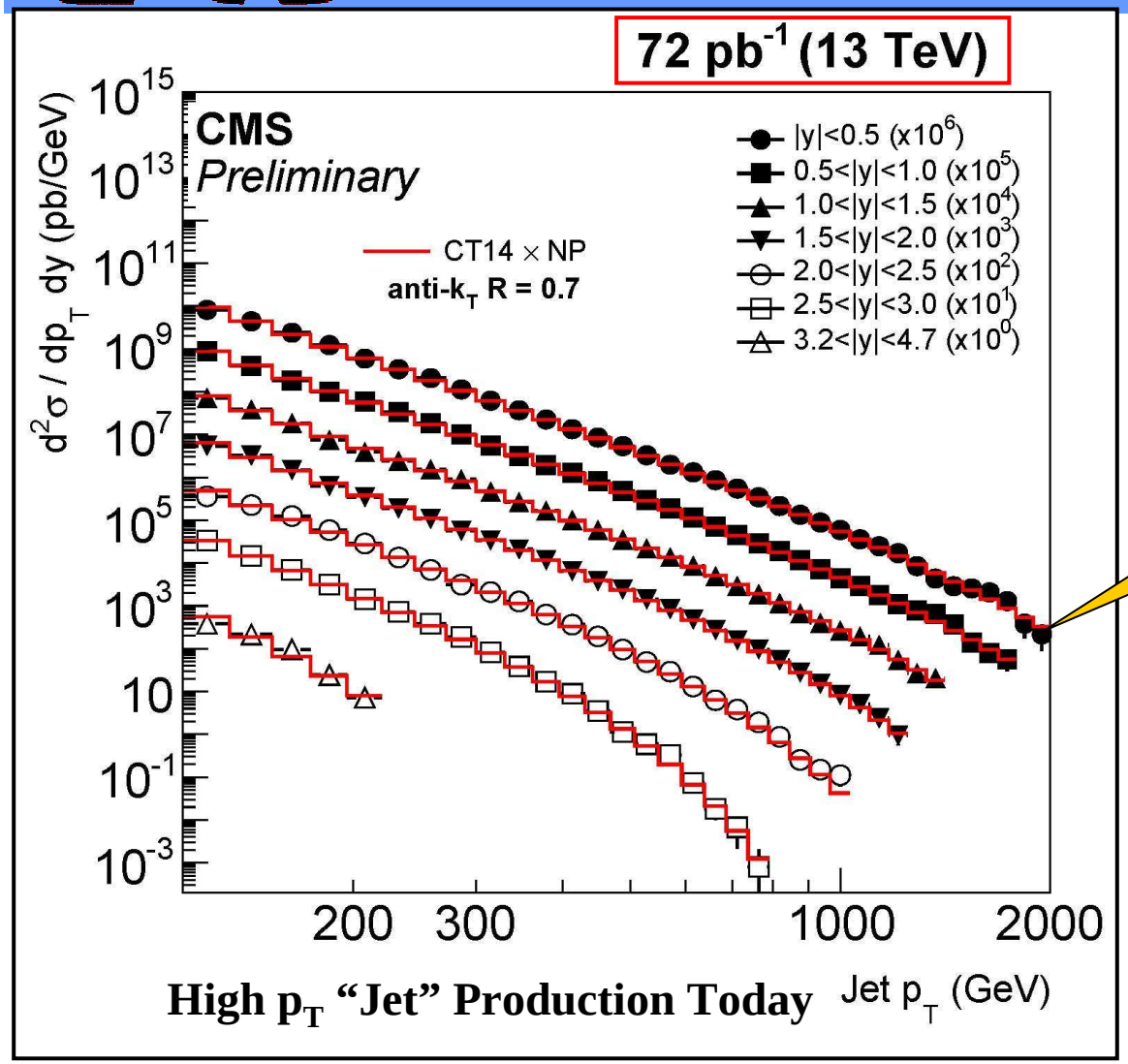
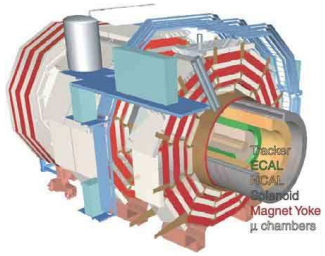
QCD "Jet" Production



High p_T "Jet" Production Today



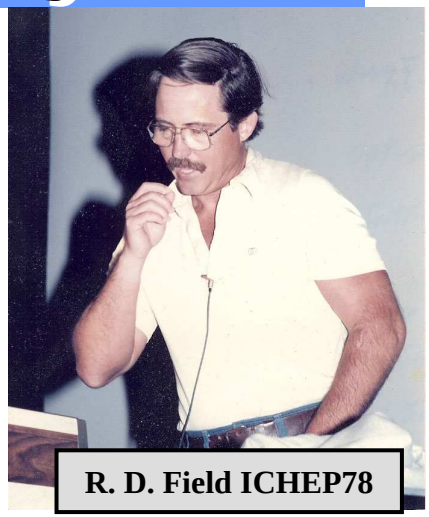
CMS at the LHC

2,000 GeV/c Jets!

QCD ICHEP 1978 Tokyo

Rick Field

(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)



R. D. Field ICHEP78

The QCD Parton Model Approach – Summary & Conclusions

- **No evidence against the (perturbative) QCD parton model approach.**
- **Lots of qualitative support for the approach:**
  - $ep \to e + X$, $\mu p \to \mu + X$, $\nu p \to \mu^- + X$.
  - $pp \to \mu^+\mu^- + X$, $pp \to \pi + X$, $pp \to Jet + X$.
  - $e^+e^- \to Jet + Jet + X$, $\nu p \to \mu^- + Jet + X$.
- **The applications of the theory are still quite crude and should be considered as qualitative first quesses.**
- **Many of the most dramatic and definitive predictions of the theory have not yet been observed experimentally.**
- **Our knoweledge of what the theory predicts is improving and soon (two years) we should have a more quantative picture (*i.e.* predictions to say 20% level).**
- **Care must be taken in using the (perturbative) aspects of the theory only where they apply (*i.e.* large Q^2). There are in many cases large corrections to the asymptotic forms at low Q^2 (*i.e.* $1/Q^2$ effects, non-perturbative effects) which cannot at present be calculated.**
- **This is a very exciting and fun time. We have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong interactions.**

QCD ICHEP 1978 Tokyo

Rick Field

(Plenary talk presented at the XIX International Conference on High Energy Ph… Tokyo, Japan)

The QC… Parton Model Appro… – Summar… onclusions

- No evidence … the (perturbati… parton … roach.
- Lots of qualitati… …r the …
  - ep→e + X, μp→μ …
  - pp→μ+μ- + X, pp…

R. Field ICHEP78

QCD is not just "another theory".
If it is not the correct description of nature,
then it will be quite some time before another
candidate theory emerges.

- R. Field, ICHEP78

- The appli… qualitative firs…
- Many of the most… …bserved experimenta…
- Our k… …ge of what th… …rs) we should have a more quantative picture (…
- Care must be taken in u… …e (per… …) aspe… …theory only … …e they apply (*i.e.* large Q^2). There are in many cases large c… …ns to the … …totic forms at lo… Q^2 (*i.e.* $1/Q^2$ effects, non-perturbative effects) which cann… …resent be ca… …ted.
- This is a very exciting and fun time. … have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong int… actions.