Early Days of Perturbative QCD

Rick Field
University of Florida

Outline

- My Story of QCD.
- FF1: The Black Box Model.
- FF2: The QCD Improved Parton Model.
- My talk at ICHEP78 Tokyo and my lectures at Boulder79.
- The QCD Improved Parton Model: Drell-Yan Lepton-Pair Production.
- The QCD Improved Parton Model: Large Transverse Momentum Mesons in Hadron-Hadron Collisions.
- The QCD Improved Parton Model: Jet Production in Hadron-Hadron Collisions.
- From 7 GeV $\pi^0$’s to 2 TeV Jets.
My Story of QCD

The Parton Model

Before we knew the partons were quarks and gluons, non-interacting partons!

SU(3) Flavor Triplet

The Quark Model
The Eightfold Way
My Story of QCD

The Quark Model
The Eightfold Way

Before we knew the partons were quarks and gluons!
Non-interacting partons!

Partons = quarks, anti-quarks, and glue!
Still non-interacting partons!

The Parton Model

The Naïve Parton Model
My Story of QCD

The Quark Model
The Eightfold Way

SU(3)_{flavor} Triplet

The Parton Model

Before we knew the partons were quarks and gluons!
Non-interacting partons!

Interacting quarks, anti-quarks, and gluons!

Partons = quarks, anti-quarks, and glue!
Still non-interacting partons!

The QCD Improved Parton Model

Parton Model + Perturbative QCD!

The Naïve Parton Model
My Story of QCD

The Quark Model
The Eightfold Way

SU(3)_flavor Triplet

Before we knew the partons were quarks and gluons!
Non-interacting partons!

Parton Model + Perturbative QCD!
Interacting quarks, anti-quarks, and gluons!

The QCD Improved Parton Model

Partons = quarks, anti-quarks, and glue!
Still non-interacting partons!

QCD Theory!

LPCC Workshop
CERN, June 14, 2017

Rick Field – Florida
Page 5
From 7 GeV/c $\pi^0$'s to 2 TeV Jets. The early days of trying to understand and simulate hadron-hadron collisions.
From 7 GeV/c $\pi^0$'s to 2 TeV Jets. The early days of trying to understand and simulate hadron-hadron collisions.
The Feynman-Field Days 1973-1983

The Feynman-Field Days

1973-1983


My 1st graduate student!
**QCD The Feynman-Field Days**


My 1st graduate student!
**QCD The Feynman-Field Days**


My 1st graduate student!
Electron-Positron Annihilations:
\[ e^- + e^+ \rightarrow X \]

\[ e^- + e^+ \rightarrow \gamma^* \rightarrow q + \bar{q} \]

Born Term = \[ \frac{G_F}{2\sqrt{2}} q^2 \]

Virtual Photon

Inelastic Electron Proton Scattering:
\[ e^- + p \rightarrow e^- + X \]

\[ e^- + q \rightarrow e^- + q \]

Born Term = \[ \frac{G_F}{2\sqrt{2}} q^2 \]
**QCD**  The Naïve Parton Model

Electron-Positron Annihilations:

\[ e^- + e^+ \rightarrow X \]

\[ e^- + e^+ \rightarrow \gamma^* \rightarrow q + \bar{q} \]

Born Term =

- Quarks interacting with photons (electromagnetic interactions)

Inelastic Electron Proton Scattering:

\[ e^- + p \rightarrow e^- + X \]

\[ e^- + q \rightarrow e^- + q \]

Born Term =
Naïve Parton Model
Electron-Positron Annihilations

\[ R_{e^+e^-} = \frac{\sigma(e^+e^- \rightarrow \text{Hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_{i=1}^{n_f} e_{q_i}^2 \]

**Color Factor**

**Count the Number of Quark Flavors:** Measure the ratio \( R \) and verify that there are indeed three colors of quarks.

\[
3 \sum_{i=1}^{3} e_{q_i}^2 = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = 3 \left( \frac{6}{9} \right) = 2
\]
Naïve Parton Model
Electron-Positron Annihilations

Count the Number of Quark Flavors: Measure the ratio $R$ and verify that there are indeed three colors of quarks.

The data are from ORSAY, FEASCATI, NOVOSIBIRSK, SLAC-LBL, DASP, CLEO, DHHM, CELLO, JADE, MARK J, PLUTO, and TASSO. Compiled by B. Wiik (1978).

\[
5 \text{ Flavors: } 3 \sum_{i=1}^{5} e_{q_i}^2 = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} \right) = 3 \left( \frac{11}{9} \right) = \frac{11}{3}
\]
Naïve Parton Model
Electron-Positron Annihilations

Quark Fragmentation Functions:
Measure the probability of finding a hadron of type $h$ carrying the fraction $z$ of the parent quarks momentum.

$$e^+ + e^- \rightarrow h + X$$

$$D^{(0)}_{q \rightarrow h}(z)$$

Very unlikely that a parton will give all its momentum to a single hadron!
Quark Fragmentation Functions:

Measure the probability of finding a hadron of type $h$ carrying the fraction $z$ of the parent quarks momentum.

$$e^+ + e^- \rightarrow h + X$$

$$D_{q \rightarrow h}^{(0)} (z)$$

Very unlikely that a parton will give all its momentum to a single hadron!

$E_{CM} = 30$ GeV (1978).
Parton Model: Virtual photon with 4-momentum $q$ interacting with a parton of type $i$ with mass $m$ and electric charge $e_i$ carrying fraction $\xi$ of the proton’s 4-momentum $P$. ($M$ = proton mass)

$$m^2 = (\vec{p}')^2 = (\vec{p} + \vec{q})^2 = (\xi \vec{P} + \vec{q})^2 = \xi^2 \vec{P}^2 + \vec{q}^2 + 2\xi \vec{P} \cdot \vec{q} = \xi^2 M^2 - Q^2 + 2\xi M v$$

$$\frac{2\xi M v}{Q^2} = 1 + \frac{(m^2 - \xi^2 M^2)}{Q^2} \rightarrow 1$$

Provided $Q^2$ is large!

Deep Inelastic Scattering (DIS)

**A fast moving proton** is a collection of **partons** (constituents of the proton) each carrying a certain fraction $\xi = x$ of the proton momentum, $P$.

$G_A^{(0)}(x)$ is the number of partons of type $i$ within a fast moving hadron of type $A$ with fraction of momentum $\xi$ ($p_i = \xi P$) between $\xi$ and $\xi + d\xi$.

**Scaling:** Predict that the DIS structure functions are not a function of both $v$ and $Q^2$, but are simply a function of the parton scaling variable, $x$.

$$F_2^{ep}(x, Q^2) \rightarrow F_2^{ep}(x) = \sum_i e_i^2 x G_p^{(0)}(x)$$

Also predict: $$R^{DIS}(x, Q^2) \rightarrow 0 \quad Q^2 >> M^2$$

**Momentum Sum Rule:** The sum of the momentum of all the constituents must equal one.

$$\sum_{All \ Partons} \int_0^1 x G_A^{(0)}(x) dx = 1$$
In the early days of the Parton Model we did not know the partons were quarks, anti-quarks, and gluons! We learned this from DIS and e⁺e⁻ experiments!
A fast moving proton is a collection of partons (quarks, anti-quarks, and gluons) each carrying a certain fraction \( x \) of the proton momentum, \( P \).

**Inelastic Electron-Proton Scattering:**

\[
F_2^{ep}(x) = \frac{4}{9} x \left( G_{p\rightarrow u}^{(0)}(x) + G_{p\rightarrow \bar{u}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p\rightarrow d}^{(0)}(x) + G_{p\rightarrow \bar{d}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p\rightarrow s}^{(0)}(x) + G_{p\rightarrow \bar{s}}^{(0)}(x) \right) + \ldots
\]

**Net Number of Quarks:** The net number of \( u \) quarks in a proton is 2 and the net number of \( d \) quarks is 1.

\[
\int_0^1 (G_{p\rightarrow u}^{(0)}(x) - G_{p\rightarrow \pi}^{(0)}(x)) dx = 2 \quad \int_0^1 (G_{p\rightarrow d}^{(0)}(x) - G_{p\rightarrow \bar{d}}^{(0)}(x)) dx = 1 \quad \int_0^1 (G_{p\rightarrow s}^{(0)}(x) - G_{p\rightarrow \bar{s}}^{(0)}(x)) dx = 0
\]

**DIS Experiments (1972-1975):** Observe approximate scaling and measure quark distributions.

Find that only about one-half of the proton momentum is carried by the charged quarks:

\[
\sum_{i=1}^{n_f} \int_0^1 x \left( G_{p\rightarrow q_i}^{(0)}(x) + G_{p\rightarrow \bar{q}_i}^{(0)}(x) \right) dx \approx 0.5
\]

The remaining momentum must be carried by electrically neutral partons (i.e. gluons).
Quarks have fractional electric charge: By comparing deep inelastic electron-proton scattering with deep inelastic neutrino-proton scattering one can determine the electric charge of the quarks.

**Inelastic Electron-Proton Scattering:** \[ e^- + p \rightarrow e^- + X \]

\[
F_2^{ep}(x) = \frac{4}{9} x \left( G_{p \rightarrow u}^{(0)}(x) + G_{p \rightarrow \bar{u}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p \rightarrow d}^{(0)}(x) + G_{p \rightarrow \bar{d}}^{(0)}(x) \right) + \frac{1}{9} x \left( G_{p \rightarrow s}^{(0)}(x) + G_{p \rightarrow \bar{s}}^{(0)}(x) \right) + \ldots
\]

**Inelastic Neutrino-Proton Scattering:** \[ \nu_e + p \rightarrow e^- + X \]

\[
F_2^{vp}(x) = 2 x \left( G_{p \rightarrow u}^{(0)}(x) + G_{p \rightarrow \bar{u}}^{(0)}(x) \right) + 2 x \left( G_{p \rightarrow d}^{(0)}(x) + G_{p \rightarrow \bar{d}}^{(0)}(x) \right) + 2 x \left( G_{p \rightarrow s}^{(0)}(x) + G_{p \rightarrow \bar{s}}^{(0)}(x) \right) + \ldots
\]
**Naïve Parton Model: DIS**

**Quarks have fractional electric charge:** By comparing deep inelastic electron-proton scattering with deep inelastic neutrino-proton scattering one can determine the electric charge of the quarks.

**Inelastic Electron-Proton Scattering:**

\[ F_2^{ep}(x) = \frac{4}{9} x(G^{(0)}_u(x) + G^{(0)}_d(x)) \]

**Inelastic Neutrino-Proton Scattering:**

\[ F_2^{vp}(x) = 2x(G^{(0)}_u(x) + G^{(0)}_d(x)) \]

![Parton Distribution Functions (PDF’s)](image)
The Naïve Parton Model

Inelastic Neutrino Proton Scattering:
$$\nu_e + p \rightarrow e^- + X$$

Born Term =

Drell-Yan Muon-Pair Production:
$$p + p \rightarrow \mu^+ \mu^- + X$$

Born Term =

$$q + \bar{q} \rightarrow \mu^+ + \mu^-$$
**QCD**

The Naïve Parton Model

### Inelastic Neutrino Proton Scattering:
\[ \nu_e + p \rightarrow e^- + X \]

![Diagram](image)

**Quarks interacting with W & Z**
(weak interactions)

\[ \nu_e + q \rightarrow e^- + q' \]

**Quarks interacting with photons**
(electromagnetic interactions)

### Drell-Yan Muon-Pair Production:
\[ p + p \rightarrow \mu^+ \mu^- + X \]

![Diagram](image)

Born Term =

Quark
\[ q + \bar{q} \rightarrow \mu^+ + \mu^- \]

<table>
<thead>
<tr>
<th>Quark</th>
<th>Anti-quark</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td></td>
</tr>
</tbody>
</table>

LPCC Workshop
CERN, June 14, 2017

Rick Field – Florida
Page 24
Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Quark-Quark Elastic Scattering

\[ q + q \rightarrow q + q \]

2-to-2 Scattering

Born Term =

Quarks interacting with quarks

(strong interactions)

What do I do now??
Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Quark-Quark Elastic Scattering

\[ q + q \rightarrow q + q \]

2-to-2 Scattering

Born Term =

Quarks interacting with quarks (strong interactions)

What do I do now??

Field-Feynman Black-Box Model
What happens when two hadrons collide at high energy?

Most of the time the hadrons ooze through each other and fall apart (i.e. no hard scattering). The outgoing particles continue in roughly the same direction as initial proton and antiproton.

Occasionally there will be a large transverse momentum meson. Question: Where did it come from?

We assumed it came from quark-quark elastic scattering, but we did not know how to calculate it!
What happens when two hadrons collide at high energy?

Most of the time the hadrons ooze through each other (no hard scattering) and the outgoing particles continue in roughly the same direction as initial proton and antiproton.

Occasionally there is a large transverse momentum meson. Question: Where did it come from?

We assumed it came from quark-quark elastic scattering, but we did not know how to calculate it!

Feynman quote from FF1
“The model we shall choose is not a popular one, so that we will not duplicate too much of the work of others who are similarly analyzing various models (e.g. constituent interchange model, multiperipheral models, etc.). We shall assume that the high $P_T$ particles arise from direct hard collisions between constituent quarks in the incoming particles, which fragment or cascade down into several hadrons.”

“Black-Box Model”
Quark-Quark Black-Box Model

Quark Distribution Functions determined from deep-inelastic lepton-hadron collisions

FF1 1977

No gluons!

Quark Distribution Functions
Quark Black
Box Model

Quark-Fragmenation Functions
determined from e⁺e⁻ annihilations

Quark-Quark Cross-Section
Unknown! Determined from hadron-hadron collisions.
Quark-Quark Black-Box Model

Quark Distribution Functions
determined from deep-inelastic
lepton-hadron collisions

FF1 1977

Quark-Quark Cross-Section
Unknown! Determined from
hadron-hadron collisions.

Quark Fragmentation Functions
determined from e+e- annihilations

Feynman quote from FF1
“Because of the incomplete knowledge of
our functions some things can be predicted
with more certainty than others. Those
experimental results that are not well
predicted can be “used up” to determine
these functions in greater detail to permit
better predictions of further experiments.
Our papers will be a bit long because we
wish to discuss this interplay in detail.”

No gluons!
**Quark-Quark Black-Box Model**

**Predict**
- particle ratios

**FF1 1977**

- Predict
crushed
crushed

- Predict
increase with increasing
CM energy W

- 7 GeV/c \(\pi^+\)'s!

- "Beam-Beam Remnants"

- Predict
overall event topology
(FFF1 paper 1977)
The beginning of the “underlying event”!

Predict overall event topology (FFF1 paper 1977)

“Beam-Beam Remnants”

7 GeV/c π⁺’s!

Predict increase with increasing CM energy W

Predict particle ratios

Quark-Quark Black-Box Model

The beginning of the “underlying event”!
QCD Approach:
Quarks & Gluons

Parton Distribution Functions
Q^2 dependence predicted from QCD

Quark & Gluon Cross-Sections
Calculated from QCD

FFF2 1978
QCD Approach: Quarks & Gluons

- **Quark & Gluon Fragmentation Functions**
  - Q^2 dependence predicted from QCD

- **Parton Distribution Functions**
  - Q^2 dependence predicted from QCD

- **Quark & Gluon Cross-Sections**
  - Calculated from QCD
QCD Approach: Quarks & Gluons

Quark & Gluon Fragmentation Functions
Q² dependence predicted from QCD

Parton Distribution Functions
Q² dependence predicted from QCD

Feynman quote from FFF2
“We investigate whether the present experimental behavior of mesons with large transverse momentum in hadron-hadron collisions is consistent with the theory of quantum-chromodynamics (QCD) with asymptotic freedom, at least as the theory is now partially understood.”

Quark & Gluon Cross-Sections
Calculated from QCD
Dynamics of High Energy Reactions

Rick Field
California Institute of Technology

(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)

The QCD Parton Model Approach - Outline of Talk

- The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale $\Lambda$.
- Quark and Gluon Distributions within Hadrons: Scale Breaking.
  - Analysis of ep and $\mu p$ Data.
  - Analysis of $F_2$ and $xF_3$ in Neutrino Processes.
- Muon-Pair Production in pp Collisions.
  - QCD Factorization – “Constant” Pieces.
  - Large $p_T$ Muon-Pair Production.
  - “Scaling” in $pp \to \mu^+\mu^- + X$.
- Quark and Gluon Fragmentation Functions: Scale Breaking.
  - Gluon Jets.
- Large $p_T$ Production of Mesons and Jets in pp Collisions.
  - Scale Breaking Effects: $E d\sigma/d^3p$.
  - Correlations – Evidence for Gluons.
  - The Jet Cross Section..
- A Look to the Future.
  - $pp \to \pi^0 + X$ at $W = 500$ GeV.
  - Large $p_T$ Charm Production.
  - $\gamma\gamma \to$ Jet + Jet and $e^+e^- \to e^+e^- +$Jet+Jet.
  - Three Jets: $e^+e^- \to q+\bar{q}+\text{Jet}+\text{Jet}+X$.
Dynamics of High Energy Reactions

Rick Field
California Institute of Technology
(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)

The QCD Parton Model Approach - Outline of Talk

- The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale $\Lambda$.
- Quark and Gluon Distributions within Hadrons: Scale Breaking.
  - Analysis of ep and $\mu p$ Data.
  - Analysis of $F_2$ and $xF_3$ in Neutrino Processes.
- Muon-Pair Production in pp Collisions.
  - QCD Factorization – “Constant” Pieces.
  - Large $p_T$ Muon-Pair Production.
  - “Scaling” in $pp \rightarrow \mu^+\mu^- + X$.
- Large $p_T$ Production of Mesons and Jets in pp Collisions.
  - Scale Breaking Effects: $Ed\sigma/d^3p$.
  - Correlations – Evidence for Gluons.
  - The Jet Cross Section.
- Quark and Gluon Fragmentation Functions: Scale Breaking.
  - Gluon Jets.
- A Look to the Future.
  - $pp \rightarrow \pi^0 + X$ at $W = 500$ GeV.
  - Large $p_T$ Charm Production.
  - $\gamma\gamma \rightarrow$ Jet + Jet and $e^+e^- \rightarrow e^+e^- +$Jet+Jet.
  - Three Jets: $e^+e^- \rightarrow q+\bar{q}+$Jet+Jet and $pp \rightarrow$Jet+Jet+Jet+X.
The Effective Strong Interaction Coupling Constant, $\alpha_s(Q^2)$, and the Mass Scale $\Lambda$.

Quark and Gluon Distributions within Hadrons: Scale Breaking.

Analysis of $e^p$ and $\mu p$ Data.

Analysis of $F_2$ and $x F_3$ in Neutrino Processes.

Muon-Pair Production in $pp$ Collisions.

QCD Factorization – "Constant" Pieces.

Large $p_T$Muon-Pair Production.

"Scaling" in $pp \rightarrow \mu^+\mu^- + X$.

Dynamics of High Energy Reactions

Rick Field

California Institute of Technology

(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)

The QCD Parton Model Approach - Outline of Talk

1. The Effective strong interaction coupling $\alpha_s(Q^2)$ and the Mass Scale $\Lambda$

2. Quark and Gluon Distributions within Hadrons: Scale Breaking
   a. Analysis of $e^p$, $\mu p$ data
   b. Analysis of $F_2$, $x F_3$ in neutrino processes

3. Muon pair production in $pp$ collisions
   a. QCD factorization: "constant" pieces
   b. Large $p_T$ muon pair production
   c. "Scaling" in $pp \rightarrow \mu^+\mu^- + X$

4. Quark and Gluon Fragmentation Functions - Scale Breaking
   a. Gluon jets

5. Large $p_T$ production of mesons and jets in $pp$ collisions
   a. Scale Breaking Effects: Eddy Drift
   b. Correlations - Evidence for Gluons
   c. The Jet Cross Section...

6. A Look to the Future
   a. $pp \rightarrow 3 \mu^+\mu^- + X$
   b. Large $p_T$ production
   c. The Jet Cross Section
   d. $\gamma\gamma \rightarrow \text{Jet} + \text{Jet}$
   e. $e^+e^- \rightarrow \text{Jet} + \text{Jet}$
   f. $3 \text{Jets} + \text{etc.}$ and $pp \rightarrow 4 \text{Jets} + \text{etc.}$

7. Final Remarks
QCD Improved Parton Model

Drell-Yan Muon-Pair Production

\[ p + p \rightarrow \mu^+ \mu^- + X \]

Born Term = \( \hat{t} \rightarrow \gamma^* \)

Born Amplitude = \( A_0 = \gamma^* \rightarrow \gamma \)

Include Leading Order QCD Corrections (Order \( \alpha_s \))

“Annihilation” \( q + \bar{q} \rightarrow \gamma^* + g \)

\[ A_R = \hat{t} \rightarrow g_s + \hat{t} \rightarrow g_s \]

“Compton” \( q + g \rightarrow \gamma^* + q \)

\[ C_R = \hat{t} \rightarrow g_s + \hat{t} \rightarrow g_s \]

LPCC Workshop  
CERN, June 14, 2017

Rick Field – Florida  
Page 39
QCD Improved Parton Model

Drell-Yan Muon-Pair Production

\[ p + p \rightarrow \mu^+ \mu^- + X \]

Born Term = \( \hat{t} \rightarrow \gamma \)

Born Amplitude = \( A_0 = \)

Include Leading Order QCD Corrections (Order \( \alpha_s \))

“Annihilation” \( q + \bar{q} \rightarrow \gamma^* + g \)

“Compton” \( q + g \rightarrow \gamma^* + q \)

\[ q + \bar{q} \rightarrow \mu^+ + \mu^- \]

Virtual Corrections

\[ A_V = \]

\[ A_R = \]

\[ C_R = \]
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative predictions. The “Compton” and “annihilation” contributions are given by the dashed and dotted curves, respectively.
QCD Improved Parton Model

Real Gluon Emissions Order $\alpha_s^2$
\[
\sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) \equiv \frac{d\sigma_{A}^{A+B\rightarrow \gamma^* + X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T) + \frac{d\sigma_{C}^{A+B\rightarrow \gamma^* + X}}{dM_{\mu\mu}^2 dy_{\mu\mu} dp_T^2}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T)
\]

Born Term Plus Virtual Gluon Emissions Order $\alpha_s^2$
\[
\sigma_V^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}) \equiv \frac{d\sigma_{B+V}^{A+B\rightarrow \gamma^* + X}}{dM_{\mu\mu}^2 dy_{\mu\mu}}(s, M_{\mu\mu}^2, y_{\mu\mu})
\]

Total Cross-Section Order $\alpha_s^2$
\[
\sigma_{tot}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}) \equiv \frac{d\sigma_{B+V}^{A+B\rightarrow \gamma^* + X}}{dM_{\mu\mu}^2 dy_{\mu\mu}}(s, M_{\mu\mu}^2, y_{\mu\mu}) + \int \sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T^2) dp_T^2
\]

Finite $=$ Infinite $+$ Infinite

Total Cross-Section Leading Order
\[
\sigma_{tot-LO}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{d\sigma_{tot-LO}^{DY}}{dM_{\mu\mu}^2 dy_{\mu\mu}}(s, M_{\mu\mu}^2, y_{\mu\mu}) = \frac{4\pi\alpha_{em}^2}{9M_{\mu\mu}^2s} P_{qq}^{DIS}(x_a, x_b, M_{\mu\mu}^2)
\]
\[
P_{qq}^{DIS}(x_a, x_b, M_{\mu\mu}^2) = \sum_{i=1}^{nf} e_i^2 [G_{A\rightarrow q_i}^{DIS}(x_a, M_{\mu\mu}^2)G_{B\rightarrow q_i}^{DIS}(x_b, M_{\mu\mu}^2) + G_{A\rightarrow \bar{q}_i}^{DIS}(x_a, M_{\mu\mu}^2)G_{B\rightarrow \bar{q}_i}^{DIS}(x_b, M_{\mu\mu}^2)]
\]
Parton Intrinsic (Primordial) Transverse Momentum:

From the uncertainty principle one expects that the partons will have some intrinsic (primordial) transverse momentum.

\[ c \Delta p_x \approx \frac{\hbar c}{\Delta x} = \frac{197 \, .3 \, MeV \cdot fm}{1 \, fm} \approx 200 \, MeV \]

Expect intrinsic \( k_T(P\rightarrow q) \approx 200 – 300 \, MeV/c! \)

Jet Size: Also expect that the hadrons within a jet will have some intrinsic transverse momentom.

Two Jet Final State: \( e^+ e^- \rightarrow \text{Jet} + \text{Jet} \)

Expect intrinsic \( k_T(q\rightarrow h) \approx 200 – 300 \, MeV/c! \)
QCD Primordial $k_T$ Smearing

$$f(k_T^2) = \frac{1}{4\pi \sigma_{\text{primordial}}^2} e^{-k_T^2}$$

$$<k_T>_{\text{primordial}} = 2\sigma_{\text{primordial}}^2$$

$$\sigma_{\text{smear}}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T, \sigma_{\text{primordial}}) = \int f(k_T^2) \left[ \sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, q_T^2) + \sigma_V^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}) \delta(q_T^2) \right] d^2k_T$$

$$\sigma_{\text{smear}}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T, \sigma_{\text{primordial}}) = \int \sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, q_T^2) \left[ f(k_T^2) - f(p_T^2) \right] d^2k_T + f(p_T^2) \int \left[ \sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, q_T^2) + \sigma_V^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}) \delta(q_T^2) \right] d^2q_T$$

$$\sigma_{\text{smear}}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, p_T, \sigma) = \int \sigma_R^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu}, q_T^2) \left[ f(k_T^2) - f(p_T^2) \right] d^2k_T + f(p_T^2) \sigma_{\text{tot}}^{DY}(s, M_{\mu\mu}^2, y_{\mu\mu})$$

Finite

Finite
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $<k_T> = 600$ MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.
The distribution in transverse momentum, $p_T$, of the $\mu^+\mu^-$ pair produced in pp collisions at $W = 27.4$ GeV together with the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle = 600$ MeV (solid curve). The dashed curve results from the parton primordial motion only with no perturbative QCD terms.
Energy dependence of the large $p_T$ tail expected for $pp \rightarrow \mu^+\mu^- + X$ from the QCD perturbative prediction folded with a Gaussian primordial parton momentum spectrum with $\langle k_T \rangle = 600$ MeV.

Data on the energy dependence of the $\langle p_T \rangle$ of the $\mu^+\mu^-$ pair in proton-nucleon collisions which indicate that the $p_T$ distribution is becoming broader as the energy increases.

Naive Parton Model

External Variables (neglect the mass of A, B, and h)

\[ s = (\vec{P}_A + \vec{P}_B)^2 = 2\vec{P}_A \cdot \vec{P}_B \]
\[ t = (\vec{P}_h - \vec{P}_A)^2 = -2\vec{P}_h \cdot \vec{P}_A \]
\[ u = (\vec{P}_h - \vec{P}_B)^2 = -2\vec{P}_h \cdot \vec{P}_B \]
\[ x_1 = -\frac{u}{s} \]
\[ x_2 = -\frac{t}{s} \]
\[ x_T = 2p_T / W \]

Internal-External Connection

\[ \tilde{p}_a = x_a \vec{P}_A \]
\[ \tilde{p}_b = x_b \vec{P}_B \]
\[ \tilde{p}_h = z_c \vec{P}_c \]

Internal Variables (neglect masses)

\[ \hat{s} = (\tilde{p}_a + \tilde{p}_b)^2 = 2\tilde{p}_a \cdot \tilde{p}_b = 2x_a x_b (\tilde{P}_A \cdot \tilde{P}_B) = x_a x_b s \]
\[ \hat{t} = (\tilde{p}_c - \tilde{p}_a)^2 = -2\tilde{p}_c \cdot \tilde{p}_a = -2x_a (\tilde{P}_h \cdot \tilde{P}_A) / z_c = x_a t / z_c \]
\[ \hat{u} = (\tilde{p}_c - \tilde{p}_b)^2 = -2\tilde{p}_c \cdot \tilde{p}_b = -2x_b (\tilde{P}_h \cdot \tilde{P}_B) / z_c = x_b u / z_c \]

\[ \hat{s} + \hat{t} + \hat{u} = 0 \quad \rightarrow \quad x_a x_b \frac{x_a x_2}{z_c x_2 - x_2} - \frac{x_b x_1}{z_c x_1 - x_1} = 0 \]

\[ x_a = \frac{x_b x_1}{z_c x_2 - x_2} \]
\[ x_b = \frac{x_a x_2}{z_c x_1 - x_1} \]
\[ z_c = \frac{x_1 + x_2}{x_a + x_b} \]
\[ A + B \rightarrow h + X \]

\[ D_{c \rightarrow h}^{(0)}(z_c) \]

**External Variables**

\[ W = E_{cm} = \sqrt{s} \quad x_T = 2p_T / W \]

\[ t = -x_2 s = -\frac{1}{2} sx_1 e^{-y} \]

\[ u = -x_1 s = -\frac{1}{2} sx_1 e^{+y} \]

\[ x_1 \equiv -u / s = \frac{1}{2} (x_E + x_L) = \frac{1}{2} x_T e^{+y} \]

\[ x_2 \equiv -t / s = \frac{1}{2} (x_E - x_L) = \frac{1}{2} x_T e^{-y} \]

**Internal Variables**

\[ \hat{s} = x_a x_b s \]

\[ \hat{t} = x_a t / z_c = -x_a x_2 s / z_c \]

\[ \hat{u} = x_b u / z_c = -x_b x_1 s / z_c \]

\[ \hat{s} + \hat{t} + \hat{u} = 0 \]

\[ \hat{p}_T = p_T / z_c \]

\[ z_c = \frac{x_1}{x_a} + \frac{x_2}{x_b} \]

\[ d\hat{z}_c d\hat{t} = \frac{1}{\pi c E} \]

\[ x_a = \frac{x_b x_1}{z_c x_b - x_2} \quad x_b = \frac{x_a x_2}{z_c x_a - x_1} \]

\[ x_a = \frac{x_1}{1 - x_2} \quad x_b = \frac{x_1}{z_c x_a - x_1} \quad z_c = 1 \]
QCD Parton-Parton Scattering

Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Include All The QCD 2-to-2 Parton-Parton Processes (Order \( \alpha_s^2 \))

Parton = quark, anti-quark, & gluon

\[
\frac{d\hat{\sigma}^a+b\rightarrow c+d}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{\pi \alpha_s^2(Q^2)}{\hat{s}^2} |A_{a+b\rightarrow c+d}(\hat{s}, \hat{t})|^2
\]

(1) \( q_i + q_j \rightarrow q_i + q_j \)
(2) \( q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j \)
(3) \( q_i + q_i \rightarrow q_i + q_i \)
(4) \( q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i \)
(5) \( q_i + \bar{q}_i \rightarrow g + g \)
(6) \( gg \rightarrow q_i + \bar{q}_i \)
(7) \( q_i + g \rightarrow q_i + g \)
(8) \( \bar{q}_i + g \rightarrow \bar{q}_i + g \)
(9) \( g + g \rightarrow g + g \)

---

**TABLE I.** Cross sections for the various constituent quark-quark, quark-gluon, and gluon-gluon subprocesses. The differential cross section is given by \( d\hat{\sigma}/d\hat{t} = \pi \alpha_s^2(Q^2)|A|^2/\hat{s}^2 \), where \( \alpha_s(Q^2) \) is the effective coupling given by Eq. (3.1).

| Subprocess | \(|A|^2| \) |
|------------|----------|
| 1. \( q_iq_i \rightarrow q_iq_i \) | \[ \frac{4}{9} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
| 2. \( q_iq_i \rightarrow q_iq_j \) | \( (i \neq j) \) |
| 3. \( q_i\bar{q}_i \rightarrow q_i\bar{q}_i \) | \[ \frac{4}{9} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
| 4. \( q_i\bar{q}_i \rightarrow gg \) | \[ \frac{32}{27} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
| 5. \( gg \rightarrow q_i\bar{q}_i \) | \[ \frac{1}{6} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
| 6. \( gg \rightarrow q_iq_i \) | \[ \frac{4}{9} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
| 7. \( gg \rightarrow gg \) | \[ \frac{9}{2} \left( \frac{\hat{s}^2 + \hat{t}^2}{\hat{u} \hat{t}} \right) \] |
**QCD Parton Model Scaling**

\[ \hat{s} + \hat{t} + \hat{u} = 0 \]

\[ \hat{p}_T^2 = \frac{\hat{t} \hat{u}}{\hat{s}} \]

\[ \hat{x}_T = 2 \hat{p}_T / \sqrt{\hat{s}} \]

\[ \hat{s} = 4 \hat{p}_T^2 / \hat{x}_T^2 \]

\[ \hat{t} = -\frac{1}{2} \hat{s} \left( 1 - \sqrt{1 - \hat{x}_T^2} \right) \]

\[ \hat{u} = -\frac{1}{2} \hat{s} \left( 1 + \sqrt{1 - \hat{x}_T^2} \right) \]

\[ \frac{d\hat{\sigma}}{d\hat{t}} (\hat{s}, \hat{t}) = \frac{4}{9} \frac{\pi \alpha_s^2}{\hat{s}^2} \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) = \frac{\pi \alpha_s^2}{\hat{p}_T^4} \left( \frac{\hat{x}_T^2}{9} \right) \left( \frac{\hat{t} \hat{u}}{\hat{s}^2} \right) \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \]

\[ \left( \frac{\hat{x}_T^2}{9} \right) \left( \frac{\hat{t} \hat{u}}{\hat{s}^2} \right) \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) = \left( \frac{\hat{x}_T^2}{9} \right) \left( \frac{\hat{t} \hat{u}}{\hat{s}^2} \right) \left( 1 + \left( \frac{\hat{u}}{\hat{s}} \right)^2 \right) = f_1(\hat{x}_T) \]

\[ P_1^{(0)}(x_a, x_b, z_c) = \sum_{i \neq j} G_{A \rightarrow q_i}(x_a) G_{B \rightarrow q_j}(x_b) D_{q_i \rightarrow h}(z_c) \]

\[ \frac{d\sigma_{A+B ightarrow h+X}^{A+B \rightarrow h+X}}{d^3 p} (s, p_T, \theta_{cm}) = \int_{x_a}^{1.0} dx_a \int_{x_b}^{1.0} dx_b P_1^{(0)}(x_a, x_b, z_c) \frac{\alpha_s^2}{z_c} f_1(x_a, x_b, x_T) = F_1(x_T, \theta_{cm}) \]

**Quark-Quark Elastic Scattering**

\[ A_{q_i + q_j \rightarrow q_i + q_j} (\hat{s}, \hat{t}) = \]

\[ \left| \overline{A}(q_i + q_j \rightarrow q_i + q_j) \right|^2 = \left( \frac{4}{9} \right) \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \]

Sum over final-state spins and color.
Average over initial-state spins and color.

**Parton Model Scaling**

Constant at fixed \( \theta_{cm} \) and \( x_T \)! (provided the PDF’s and the fragmentation functions scale and \( \alpha_s \) is constant)
Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)

1. $q_i + q_j \rightarrow q_i + q_j$
2. $\bar{q}_i + \bar{q}_j \rightarrow \bar{q}_i + \bar{q}_j$
3. $q_i + q_i \rightarrow q_i + q_i$
4. $q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i$
5. $q_i + \bar{q}_i \rightarrow g + g$
6. $gg \rightarrow q_i + \bar{q}_i$
7. $q_i + g \rightarrow q_i + g$
8. $\bar{q}_i + g \rightarrow \bar{q}_i + g$
9. $g + g \rightarrow g + g$

$$\frac{d\hat{\sigma}_k}{dt}(\hat{s}, \hat{t}) = \frac{\pi \alpha_s^2}{\hat{p}_T^4} f_k(\hat{x}_T) \quad k = 1 \text{ to } 9$$

$$p_T^4 \frac{d\sigma^{A+B \rightarrow h+X}_{all}}{d^3 p}(s, p_T, \theta_{cm}) = \sum_{r=1}^{9} F_k(x_T, \theta_{cm})$$

$\hat{p}_T = \frac{x_1}{2} x_T / \tan(\theta_{cm}/2)$
$x_2 = \frac{1}{2} x_T \tan(\theta_{cm}/2)$
$x_T = 2 p_T / \sqrt{s}$

$\min x_a = \frac{x_1}{1-x_2}$
$\min x_b = \frac{x_a x_2}{x_a - x_1}$
$z_c = \frac{x_1 + x_2}{x_a x_b}$

Parton Model Scaling

(provided the PDF’s and the fragmentation functions scale and $\alpha_s$ is constant)
High $p_T \pi$-Meson Production


The FNAL data (open circles) are from D. C. Carey et al., Fermilab Report FNAL-PUB-75120-EXP (1975).

The ISR data (solid dots) are from B. Alper et al. (BS Collaboration), Nucl. Phys. B100 (1975).

The ISR data (crosses) are from F. W. Busser et al., Nucl. Phys. B106 (1976).

$$W = \sqrt{s} \quad x_T = 2 p_T / \sqrt{s} \quad p_T = \frac{1}{2} x_T W$$

$W = 19.4$ GeV $x_T = 0.2$

$$p_T = \frac{1}{2} (0.2)(19.2 \text{GeV}) = 1.92 \text{GeV}/c$$

$W = 53$ GeV $x_T = 0.2$

$$p_T = \frac{1}{2} (0.2)(53 \text{GeV}) = 5.3 \text{GeV}/c$$
High $p_T \pi$-Meson Production

$Ed\sigma/d^3p$

High $p_T$ Data 1977

$\theta=90^\circ$


The FNAL data (open circles) are from D. C. Carey et al., Fermilab Report FNAL-PUB-75120-EXP (1975).

The ISR data (solid dots) are from B. Alper et al., Nucl. Phys. B100 (1975).

The ISR data (crosses) are from F. W. Busser et al., Nucl. Phys. B106 (1976).

Yikes!! The data is behaving like $1/p_T^8$ not $1/p_T^4$ at fixed $x_T$.

$W = 53$ GeV $x_T = 0.2$

$p_T = \frac{1}{2} (0.2)(53\text{GeV}) = 5.3\text{GeV}/c$

$LPC workshop
CERN, June 14, 2017

Rick Field – Florida

Page 54
QCD

FF1: Black-Box Model

\[ E \frac{d\sigma^{A+B \rightarrow h+X}}{d^3 p}(s, p_T, \Theta_{cm}) = \]

\[ \frac{1}{\pi} \sum_{a,b}^{1.0} x_a \int dx_b G^{(0)}_{A \rightarrow a}(x_a) G^{(0)}_{B \rightarrow b}(x_b) D^{(0)}_{c \rightarrow h}(z_c) \frac{1}{z_c} \frac{d\hat{\sigma}^{\text{Black-Box}}}{d\hat{t}}(\hat{s}, \hat{t}) \]

\[ a, b = u, d, s, \bar{u}, \bar{d}, \bar{s} \]

Naïve Parton-Parton Black-Box Model

\[ d\hat{\sigma}_{FF1}^{\text{Black-Box}}(\hat{s}, \hat{t}) = \frac{A}{\hat{s}^4} \left( \frac{\hat{s}}{-\hat{t}} \right)^3 = \frac{A}{-\hat{s} \hat{t}} \]

\[ A_{FF1} = 2.3 \times 10^6 \mu b \cdot GeV^6 \]

\[ E \frac{d\sigma_{FF1}^{A+B \rightarrow h+X}}{d^3 p}(s, p_T, \Theta_{cm}) = (2.3 \times 10^6 \mu b \cdot GeV^6) \frac{8}{p_T^8} F(x_T, \Theta_{cm}) \]
Constituent Interchange Model (CIM)

Meson within the proton!


\[ E \frac{d\sigma_{p+\overline{p} \rightarrow \pi+X}^{\text{CIM}}}{d^3 p} (s, p_T, \theta_{cm}) = \frac{1}{p_T^8} F_{\text{CIM}}(x_T, \theta_{cm}) \]
QCD Improved Parton Model

Large Transverse Meson Production in Hadron-Hadron Collisions

\[ A + B \rightarrow h + X \]

Parton = quark, anti-quark, & gluon

\[
\begin{align*}
q + q & \rightarrow q + q \\
g + q & \rightarrow g + q \\
g + g & \rightarrow q + \bar{q} \\
\bar{q} + \bar{q} & \rightarrow \bar{q} + \bar{q} \\
g + \bar{q} & \rightarrow g + \bar{q} \\
g + g & \rightarrow g + \bar{g} \\
q + \bar{q} & \rightarrow q + \bar{q} \\
q + \bar{q} & \rightarrow g + g
\end{align*}
\]

Quark-Quark Elastic Scattering

Include Leading Order QCD 2-to-2 Parton-Parton Processes (Order \(\alpha_s^2\))

\[
\frac{d\hat{\sigma}_{a+b\rightarrow c+d}}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{\pi \alpha_s^2}{\hat{s}^2} |\hat{A}(a+b \rightarrow c+d)|^2
\]

\[
|\hat{A}(q_i + q_j \rightarrow q_i + q_j)|^2 = \left(\frac{4}{9}\right) \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}
\]

Sum over final-state spins and color.
Average over initial-state spins and color.
QCD Improved Parton Model

\[ A + B \rightarrow h + X \]

Use the “renormalization group improved” PDF’s from DIS and “renormalization group improved” fragmentation functions from e^+e^- annihilations and the running QCD coupling!

\[ W = E_{cm} = \sqrt{s} \]
\[ x_1 = \frac{1}{2} x_T / \tan(\theta_{cm} / 2) \]
\[ x_2 = \frac{1}{2} x_T \tan(\theta_{cm} / 2) \]

QCD effective coupling

\[ \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{dt}(\hat{s}, \hat{t}) = \frac{\pi \alpha_s^2(Q^2)}{\hat{s}^2} \left| A^{a+b\rightarrow c+d}(\hat{s}, \hat{t}) \right|^2 \]

\[ E \frac{d\sigma^{A+B\rightarrow h+X}}{d^3p}(s, p_T, \theta_{cm}) \]

\[ = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{min}}^{1} dx_a \int_{x_b^{min}}^{1} dx_b G^{DIS}_{A\rightarrow a}(x_a, Q^2) G^{DIS}_{B\rightarrow b}(x_b, Q^2) D_{c\rightarrow h}^{e^+e^-}(z_c, Q^2) \left( \frac{1}{z_c} \right) \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{dt}(x_a x_b s, x_b t / z_c) \]

Q^2 dependent PDF’s!
Q^2 dependent fragmentation!
QCD Improved Parton Model

Renormalization Group Improved Fragmentation!

Use Renormalization Group Improved PDF’s!

QCD effective coupling

Use the "renormalization group improved" PDF’s from DIS and "renormalization group improved" fragmentation functions from $e^+e^-$ annihilations and the running QCD coupling!

$Q^2$ dependent PDF’s!

$Q^2$ dependent fragmentation!
QCD Improved Parton Model

Large Transverse Meson Production in Hadron-Hadron Collisions

Include All The QCD 2-to-2 Parton-Parton Processes (Order $\alpha_s^2$)

(1) $q_i + q_j \rightarrow q_i + q_j$
(2) $q_i + \bar{q}_j \rightarrow q_i + \bar{q}_j$
(3) $q_i + q_i \rightarrow q_i + q_i$
(4) $q_i + \bar{q}_i \rightarrow q_i + \bar{q}_i$
(5) $q_i + \bar{q}_i \rightarrow g + g$
(6) $g g \rightarrow q_i + \bar{q}_i$
(7) $q_i + g \rightarrow q_i + g$
(8) $\bar{q}_i + g \rightarrow \bar{q}_i + g$
(9) $g + g \rightarrow g + g$

$\frac{d\hat{\sigma}_k}{d\hat{t}} (\hat{s}, \hat{t}) = \frac{\pi\alpha_s^2 (Q^2)}{\hat{p}_T^4} f_k (\hat{x}_T) \quad k = 1 \text{ to } 9$

Use the “renormalization group improved” PDF’s from DIS and “renormalization group improved” fragmentation functions from $e^+e^-$ annihilations and the running QCD coupling!

Naïve parton model scaling is now broken!

Possible Choices for the Scale

$Q^2 = 4\hat{p}_T^2$

$Q^2 = \frac{2\hat{s}\hat{t}u}{\hat{s}^2 + \hat{t}^2 + \hat{u}^2}$
Comparison of a QCD model (normalized absolutely) with data on large $p_T$ pion production in proton-proton collisions at $W = 19.4$ and 53 GeV with $\theta_{cm} = 90^\circ$. The dot-dashed and solid curves are the results before and after smearing, respectively, with $\Lambda = 0.4$ GeV/c and $<k_T>_{primordial} = 848$ MeV/c and the dashed curves are with $\Lambda = 0.6$ GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark ($i.e.$ no gluons) is shown by the dotted curves.
Comparison of a QCD model (normalized absolutely) with data on large pion production in proton-proton collisions at $W = 19.4$ and $53$ GeV with $\theta_{cm} = 90^\circ$. The dot-dashed and solid curves are the results before and after smearing, respectively, with $\Lambda = 0.4$ GeV/c and $<k_T>_{primordial} = 848$ MeV/c and the dashed curves are with $\Lambda = 0.6$ GeV/c. The contributions from quark-quark, quark-antiquark, and antiquark-antiquark (i.e. no gluons) is shown by the dotted curves.

> Primordial $k_T$ has a big effect at low $p_T$ and low $W$ where the cross section is very steep!

Large primordial $k_T$!
Data on $p_T^4$ times $E d\sigma/d^3p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$. 
Data on $p_T^4$ times $E d\sigma / d^3 p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $\langle k_T \rangle_{\text{primordial}} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). The dotted curve is $p_T^4(1/p_T^8)$. 

R. D. Field ICHEP78
Data on $p_T^8$ times $Ed\sigma/d^3p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2, 0.35$, and $0.5$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.
Data on $p_T^8 E d\sigma/d^3p$ for large $p_T$ pion production at $\theta_{cm} = 90^\circ$ and fixed $x_T = 0.2, 0.35,$ and $0.5$ versus $p_T$ compared with the predictions (with absolute normalization) of a model that incorporates all the features expected from QCD. The dot-dashed and solid curves are the results before and after smearing with $<k_T>_{primordial} = 848$ MeV, respectively, with $\Lambda = 0.4$ GeV/c and the dashed curves are the results of using $\Lambda = 0.6$ GeV/c (after smearing). Recent data from ISR (triangles, open circles) show a deviation from a horizontal straight line ($1/p_T^8$) behavior as expected from QCD.

The ISR data (open circles) are from A. G. Clark et al., Phys Letters 74B (1978) 267. Also, talk presented by A. G. Clark at this conference.
The behavior of $p_T^8 E d\sigma/d^3p$ times the $90^\circ$ single $\pi^0$ cross section, $E d\sigma/d^3p$, at $x_T = 0.05$ versus $p_T$ calculated from the QCD approach with $\Lambda = 0.4$ GeV/c (solid curves) and $\Lambda = 0.6$ GeV/c (dashed curves). The two low $p_T$ data points are at 53 and 63 GeV. The predictions are a factor of 100 (1000) times larger than the flat horizontal ($1/p_T^8$) extrapolation to $W = 500$ GeV (1000 GeV).

\[ W = 1.96 \text{ TeV} \quad x_T = 0.05 \]

\[ p_T = \frac{1}{2} (0.05)(1960 \text{ GeV}) = 49 \text{ GeV/}c \]

\[ \frac{d\sigma^{\pi^0}(x_T = 0.05, p_T = 49 \text{ GeV/c})}{p_T dp_T dy d\phi} \approx 10^{-9} \text{mb/(GeV/c)}^2 \]

\[ E \frac{d\sigma}{d^3p} = \frac{d\sigma}{y d^2p_T} = \frac{d\sigma}{y p_T dp_T d\phi} \quad \frac{\pi^0}{h^+ + h^-} \approx 0.5 \]

\[ p_T^8 E \frac{d\sigma^{\pi^0}(x_T = 0.05, p_T = 49 \text{ GeV/c})}{d^3p} \approx (0.5)(49 \text{ GeV/c})^8 \times 10^{-9} \text{mb/(GeV/c)}^2 \]

\[ \approx 1.66 \times 10^7 \mu \text{b} \cdot (\text{GeV/c})^6 \]
Amazing! I was right!

factor of 
\[ \approx 10,000 \]
**QCD “Jet” Production**

\[ A + B \rightarrow c + X \]

**QCD Improved Parton Model**

Large Transverse Momentum Parton Production

\[ D^e_\rightarrow h (z_c, Q^2) = \delta(1 - z_c) \]

**Expressions for Parton Production**

\[
E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3p} (s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\text{min}}}^{1.0} dx_a \int_{x_b^{\text{min}}}^{1.0} dx_b G_{A\rightarrow a}^{\text{DIS}}(x_a, Q^2) G_{B\rightarrow b}^{\text{DIS}}(x_b, Q^2) \delta(1 - z_c) \frac{1}{z_c} \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}} (\hat{s}, \hat{t})
\]

\[
E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3p} (s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\text{min}}}^{1.0} dx_a \int_{x_b^{\text{min}}}^{1.0} dz_c G_{A\rightarrow a}^{\text{DIS}}(x_a, Q^2) G_{B\rightarrow b}^{\text{DIS}}(x_b, Q^2) \delta(1 - z_c) \frac{x_b^2}{x_2 z_c} \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}} (\hat{s}, \hat{t})
\]

\[
E \frac{d\sigma^{A+B\rightarrow c+X}}{d^3p} (s, p_T, \theta_{cm}) = \frac{1}{\pi} \sum_{a,b} \int_{x_a^{\text{min}}}^{1.0} dx_a \frac{x_b^2}{x_2} G_{A\rightarrow a}^{\text{DIS}}(x_a, Q^2) G_{B\rightarrow b}^{\text{DIS}}(x_b, Q^2) \frac{d\hat{\sigma}^{a+b\rightarrow c+d}}{d\hat{t}} (x_a x_b s, x_b t)
\]

**Expressions for Parton Variables**

\[
x_a^\text{min} = \frac{x_1}{1 - x_2}
\]

\[
x_b = \frac{x_a x_2}{x_a - x_1}
\]
Comparison of the results on the $90^\circ \pi^0$ cross section, $E d\sigma/ d^3 p$, from the QCD approach with $\Lambda = 0.4$ GeV/c (solid curves) and the quark-quark “black-box” parton model of FF1 (dotted curves). Both models agree with the data at $W = 53$ GeV. The QCD approach results in much larger cross sections than the FF1 model at $W = 500$ and 1000 GeV. The FF1 results at 1000 GeV (not shown) are only slightly larger than at 500 GeV. Also shown are the cross sections for producing a jet at $90^\circ$ (divided by 1000) as predicted by the QCD approach (dashed curves) and the FF1 model (dot-dashed curve).
Comparison of the jet and single $\pi^0$ cross sections measured at 200 GeV ($W = 19.4$ GeV) and $\theta_{cm} = 90^\circ$. The jet data are from two FNAL experiments, E260 and E395, where a jet is defined as the sum of all the particles into their respective detectors. Also shown is the QCD prediction for the cross section of producing a parton (quark, antiquark, or gluon) at $W = 19.4$ GeV and $\theta_{cm} = 90^\circ$.

The FNAL data (E260) are from C. Bromberg et al., Phys Rev Letters 38 (1977); C. Bromberg et al., CALT-68-613 (to be published in Nucl. Phys.).

The FNAL data (E395) are from the Fermilab-Lehigh-Pennsylvania-Wisconsin Collaboration, talk given by W. Selove at this conference.
Feynman, Field, & Fox (1978)

High $P_T$ Jets

Feynman quote from FFF

“At the time of this writing, there is still no sharp quantitative test of QCD. An important test will come in connection with the phenomena of high $P_T$ discussed here.”

Lecturers
- G. Altarelli
- A. J. Buras
- C. De Tar
- S. Ellis
- R. D. Field
- D. Gross
- C. H. Llewellyn Smith
- J. Wess

Rick Field Boulder 1979

Jimmie Field Boulder 1979
At low $x_T$ the dominate QCD sub-process is gluon production which fragments equally into $\pi^+$ and $\pi^-$!

To produce an outgoing $\pi^+$ in the CIM approach one must find a $\pi^+$ within the incoming $\pi^-$ or the incoming proton (with small probability)!
At low $x_T$ the dominate QCD sub-process is gluon production which fragments equally into $\pi^+$ and $\pi^-$!

The data are from H. J. Frisch, Precise Measurement of the High $p_T$ Single Particle Spectra in $\pi p$ Collisions at Fermilab (E258), talk presented at the XIV Moriond Conference (1979).

To produce an outgoing $\pi^+$ in the CIM approach one must find a $\pi^+$ within the incoming $\pi$ or the incoming proton (with small probability)!
The NATO Advanced Institute on Quantum Flavor dynamics, Quantum Chromodynamics, and Unified Theories, held at the University of Colorado, Boulder, Colorado, July 9 – 27, 1979.
\[
E \frac{d\sigma}{d^3 p} (pp \rightarrow Jet + X) \approx 10^{-14} (\mu b / GeV^2)
\]

\[
\frac{d^2\sigma}{dp_T dy} = 2\pi p_T \left( E \frac{d\sigma}{d^3 p} \right)
\]

\[
\frac{d^2\sigma}{dp_T dy} (pp \rightarrow Jet + X) \approx 2.51 \times 10^{-5} (nb / GeV)
\]
$$E \frac{d\sigma}{d^3 p} \left( pp \rightarrow Jet + X \right) \approx 10^{-14} (\mu b / GeV^2)$$

$$\frac{d^2 \sigma}{dp_T dy} = 2\pi p_T \left( E \frac{d\sigma}{d^3 p} \right)$$

$$\frac{d^2 \sigma}{dp_T dy} \left( pp \rightarrow Jet + X \right) \approx 2.51 \times 10^{-5} (nb / GeV)$$
Rick Field

(Plenary talk presented at the XIX International Conference on High Energy Physics, Tokyo, Japan)

The QCD Parton Model Approach – Summary & Conclusions

 ➤ No evidence against the (perturbative) QCD parton model approach.

 ➤ Lots of qualitative support for the approach:
  - ep→e + X, μp→μ + X, νp→μ− + X.
  - pp→μ+μ− + X, pp→π + X, pp→Jet + X.
  - e+e−→Jet+Jet + X, νp→μ−+Jet + X.

 ➤ The applications of the theory are still quite crude and should be considered as qualitative first guesses.

 ➤ Many of the most dramatic and definitive predictions of the theory have not yet been observed experimentally.

 ➤ Our knowledge of what the theory predicts is improving and soon (two years) we should have a more quantitative picture (i.e. predictions to say 20% level).

 ➤ Care must be taken in using the (perturbative) aspects of the theory only where they apply (i.e. large \( Q^2 \)). There are in many cases large corrections to the asymptotic forms at low \( Q^2 \) (i.e. \( 1/Q^2 \) effects, non-perturbative effects) which cannot at present be calculated.

 ➤ This is a very exciting and fun time. We have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong interactions.
The QCD Parton Model Approach – Summary & Conclusions

- No evidence against the (perturbative) QCD parton model approach.
- Lots of qualitative support for the approach:
  - \( ep \rightarrow e + X, \mu p \rightarrow \mu + X \)
  - \( pp \rightarrow \mu^{+} + \mu^{-} + X, pp \rightarrow \pi^{+} + X, pp \rightarrow \text{Jet} + X \)
  - \( e^{+}e^{-} \rightarrow \text{Jet} + \text{Jet} + X, \nu p \rightarrow \mu^{-} + \text{Jet} + X \)

The applications of the theory are still quite crude and should be considered as qualitative first estimates.

- Many of the most dramatic and definitive predictions of the theory have not yet been observed experimentally.
- Our knowledge of what the theory predicts is improving and soon (two years) we should have a more quantitative picture (i.e., predictions to say 20% level).
- Care must be taken in using the (perturbative) aspects of the theory only where they apply (i.e., large \( Q^2 \)). There are many cases where corrections to the asymptotic forms at low \( Q^2 \) (i.e., 1/Q^2 effects, non-perturbative effects) cannot yet be calculated.
- This is a very exciting and fun time. We have a theory and it is up to all of us to confirm that it is indeed the correct theory of strong interactions.

- R. Field, ICHEP78
Three-Jet Final State:

Not observed until after Boulder 1979!

Observed by the JADE detector at PETRA at $E_{CM} = 30$ GeV (1980?).
**QCD** 7 GeV π⁰’s → 2 TeV Jets

**QCD Improved Parton Model**

- **CMS Preliminary**
- 72 pb⁻¹ (13 TeV)
- y < 0.5 (x 10⁴)
- 0.5 < y < 1.0 (x 10⁵)
- 1.0 < y < 1.5 (x 10⁵)
- 1.5 < y < 2.0 (x 10⁵)
- 2.0 < y < 2.5 (x 10⁵)
- 2.5 < y < 3.0 (x 10⁵)
- 3.2 < y < 4.7 (x 10⁵)

**CMS**

- 6 GeV/c Jets!
- FFF2 (1978)
- 2 TeV Jets!
QCD Improved Parton Model

Rick & Jimmie Wedding November 24, 1966
Little Chapel of the West, Las Vegas

Richard Field – Florida
QCD Improved Parton Model

Rick & Jimmie Wedding November 24, 1966
Little Chapel of the West, Las Vegas

7 GeV $\pi^0$'s $\rightarrow$ 2 TeV Jets

CMS Preliminary

Rick & Jimmie Celebration November 24, 2016
Little Chapel of the West, Las Vegas

Family, Children, & Grandchildren November 24, 2016
Little Chapel of the West, Las Vegas