PHY2060 Review

Relativistic Kinetic Energy (derivation)

Relativistic Force:

The force is equal to the rate of chance of the (relativistic) momentum as follows:

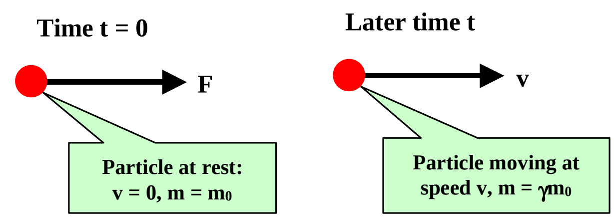
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\vec{v}$$

Classically this term is zero and F = ma

where $m = \gamma m_0$ **is the relativistic mass.**

Relativistic Kinetic Energy:

The kinetic energy of a particle is (as classical) the total work done in moving particle from rest to the speed v as follows:



$$KE = \int F dx = \int \frac{d(mv)}{dt} dx = \int v d(mv) = \int (mv dv + v^2 dm)$$

$$= \int_{m_0}^{m} ((c^2 - v^2)dm + v^2 dm) = c^2 \int_{m_0}^{m} dm = mc^2 - m_0 c^2 = E - RME$$

where I used

$$v^2 = c^2(1 - m_0^2/m^2) \quad \textbf{and} \quad mv dv = (c^2 - v^2)dm.$$

Energy Momentum Connection:

$$m^2 = m_0^2/(1 - v^2/c^2) \quad \textbf{and} \quad m^2(1 - v^2/c^2) = m_0^2$$

which implies that

$$m^2 - m^2v^2/c^2 = m_0^2 \quad \textbf{and} \quad m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

thus

$$E^2 = (cp)^2 + (m_0c^2)^2.$$

Speed β of a particle:

Since $p = mv$ **and** $m = E/c^2$ **we get** $p = Ev/c^2$ **and thus**

$$\beta = \frac{v}{c} = \frac{cp}{E} = \frac{cp}{\sqrt{(cp)^2 + (m_0c^2)^2}}.$$