Chpt. 1 – Drude model = free els + relax. time

* Free el. demo

* Core els. & conduction els. Z = # cond. els. per atom

Example (periodic table) $\frac{1}{n} = \frac{4\pi}{3} r_s^3$ $a_0 = 0.529\,\mathring{A}$

"sep. els."

	Z	n	r_s	r_s/a_0
Li	1	$4.7 \times 10^{22}/cm^3$	$1.72\,\mathring{A}$	3.25
Ca	2	$4.6 \times 10^{22}/cm^3$	$1.73\,\mathring{A}$	3.27
Fe	2	$17.0 \times 10^{22}/cm^3$	$1.12\,\mathring{A}$	2.12
Al	3	$18.1 \times 10^{22}/cm^3$	$1.10\,\mathring{A}$	2.07
Sn (Tin)	4	$14.8 \times 10^{22}/cm^3$	$1.17\,\mathring{A}$	2.22
Bi	5	$14.1 \times 10^{22}/cm^3$	$1.19\,\mathring{A}$	2.25
Au	1	$5.9 \times 10^{22}/cm^3$	$1.59\,\mathring{A}$	3.01

DC conductivity

$j = \sigma E$ $\qquad \rho = \frac{1}{\sigma} \qquad E = \rho j$

conductivity $\qquad$ resistivity

Accelerates between collisions then starts over.

$\vec{F} = -eE \qquad p = -eE\tau \qquad v = -\frac{eE\tau}{m}$

$$j = -nev$$

$$\rightarrow j = \frac{ne^2\tau}{m} E$$

$$\boxed{\frac{1}{\rho} = \sigma = \frac{ne^2\tau}{m}}$$

Know $n, e, m, \rho \rightarrow$ find τ. $\quad \rho$ in $\mu\Omega\,cm$

	77K		273K		373K	τ (273K)
Li	1.04	8.2	8.55	1.45	12.4	0.88×10^{-14} sec
Ca			3.43	1.45	5.0	2.2×10^{-14} sec
Fe	0.66	13.5	8.9	1.65	14.7	0.24×10^{-14} sec
Al	0.3	8.2	2.45	1.45	3.55	0.8×10^{-14} sec
Sn	2.1	5.0	10.6	1.49	15.8	0.23×10^{-14} sec
Bi	35	3.1	107	1.45	156	0.023×10^{-14} sec
Au	0.5	4.1	2.04	1.39	2.84	3.0×10^{-14} sec
	decays faster			linear		

$273/77 = 3.5 \qquad 373/273 = 1.37$

Hall Effect & Magnetoresistance

$$\frac{dp}{dt} = -\frac{p}{\tau} + F(t) \qquad \text{(alternate derivation)}$$

$$\frac{dp}{dt} = -\frac{p}{\tau} - e\left(E + \frac{p}{m} \times B\right) \qquad \text{(MKS units)}$$

$$H = \frac{B}{\mu_0} - M$$

B in $\hat{z}$, steady state

$$0 = -\frac{p_x}{\tau} - eE_x - e\frac{p_y}{m}B \qquad \left[\frac{eB}{m}\right] = \frac{1}{\text{time}}$$

$$0 = -\frac{p_y}{\tau} - eE_y + e\frac{p_x}{m}B \qquad \frac{eB}{m} = \omega_c = \text{cyclotron frequency}$$

Multiply by $\frac{ne\tau}{m}$:

$$0 = -\frac{ne\,p_x}{m} - \frac{ne^2\tau}{m}E_x - \frac{ne\tau}{m}\omega_c B\,p_y$$

$$0 = -\frac{ne\,p_y}{m} - \frac{ne^2\tau}{m}E_y + \frac{ne\tau}{m}\omega_c B\,p_x$$

Use $\sigma_0 = \frac{ne^2\tau}{m}$, $j = -ne\frac{p}{m}$

$$\rightarrow 0 = j_x - \sigma_0 E_x + \omega_c \tau j_y$$

$$0 = j_y - \sigma_0 E_y - \omega_c \tau j_x$$

$$\sigma_0 E_x = j_x + \omega_c \tau j_y$$

$$\sigma_0 E_y = j_y - \omega_c \tau j_x$$

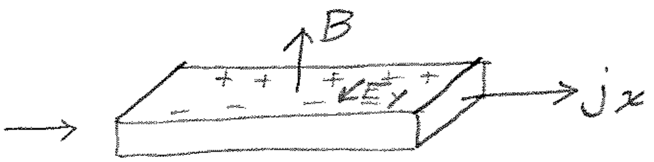
Resistivity matrix:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

Conductivity matrix:

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Hall effect:



$$j_y = 0 \implies \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 \\ -\omega_c \tau \end{pmatrix} j_x$$

$$\rho_{xx} = \frac{E_x}{j_x} = \frac{1}{\sigma_0}$$

$$\boxed{R_H = \frac{E_y}{j_x B}} = -\frac{\omega_c \tau}{\sigma_0 B} = -\frac{\frac{\cancel{e}\cancel{B}}{\cancel{m}}\cancel{\tau}}{\frac{ne^{\cancel{2}}\cancel{\tau}}{\cancel{m}}\cancel{B}} = \boxed{-\frac{1}{ne}}$$

Hall Coefficient

$$\boxed{n = \frac{-1}{R_H e}}$$

or $$-\frac{1}{R_H n e} = 1$$

Examples:

	Valence	$-1/R_H e$
Li	1	0.8
Ca	—	
Fe	—	
Al	3	−0.3 !
Sn	—	
Bi	—	
Au	1	1.5

AC conductivity:

uniform in space, really $\lambda \gg \ell$

$$E(t) = \mathrm{Re}\left(E(\omega)e^{-i\omega t}\right)$$

$$\frac{dp}{dt} = -\frac{p}{\tau} - eE$$

$$p(t) = \mathrm{Re}\left(p(\omega)e^{-i\omega t}\right)$$

$$-i\omega p(\omega) = -\frac{p(\omega)}{\tau} - eE(\omega) \Rightarrow p(\omega) = \frac{-eE(\omega)}{\frac{1}{\tau} - i\omega}$$

$$j(\omega) = \frac{-nep(\omega)}{m} = +\frac{\frac{ne^2}{m}}{\frac{1}{\tau} - i\omega}E(\omega)$$

$$\boxed{j(\omega) = \frac{ne^2\tau}{m}\frac{1}{1-i\omega\tau}E(\omega)}$$

Plasma oscillations:

$$\nabla \cdot E = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times B = \mu j + \overset{\frac{1}{c^2}}{\mu \varepsilon} \frac{\partial E}{\partial t}$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

$$\nabla \times (\nabla \times E) = -\nabla^2 E = i\omega \nabla \times B$$

$$= i\omega \left(\mu j + \mu \varepsilon (-i\omega E) \right)$$

$$= i\omega \left(\mu \overset{\sigma(\omega)}{\sigma} E - i\mu \varepsilon \omega E \right)$$

$$= \frac{\omega^2}{c^2} \left(1 + \frac{\sigma i}{\varepsilon \omega} \right) E$$

At high frequency, $\omega\tau \gg 1$, $\sigma(\omega) \approx \dfrac{\sigma_0}{-i\omega\tau}$

and $$-\nabla^2 E = \frac{\omega^2}{c^2} \left(1 - \underbrace{\frac{\sigma_0/\tau}{\varepsilon \omega^2}}_{\frac{ne^2/m}{\varepsilon \omega^2}} \right) E$$

$$\omega_p^2 \equiv \frac{ne^2}{\varepsilon m}$$

$$\boxed{\omega_p = \sqrt{\frac{ne^2}{\varepsilon m}}} \quad \text{(MKS)}$$

$$-\nabla^2 E = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) E$$

$\omega < \omega_p$: decay exp.ly

$\omega > \omega_p$: oscillate

Alkali metals transparent in ultraviolet.

Li : $\nu_p = \frac{\omega_p}{2\pi} = 2 \times 10^{15}\,Hz$

$$\lambda_p = \frac{c}{\nu_p} = \frac{3 \times 10^8}{2 \times 10^{15}} = 1.5 \times 10^{-7}\,m = 1500\,\mathring{A}$$

observed $1000\,\mathring{A}$

Charge density oscillation: (should go back to p.6)

$$\nabla \cdot j = -\frac{\partial \rho}{\partial t} \qquad \nabla \cdot j = i\omega\rho$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon}$$

$$j = \sigma E$$

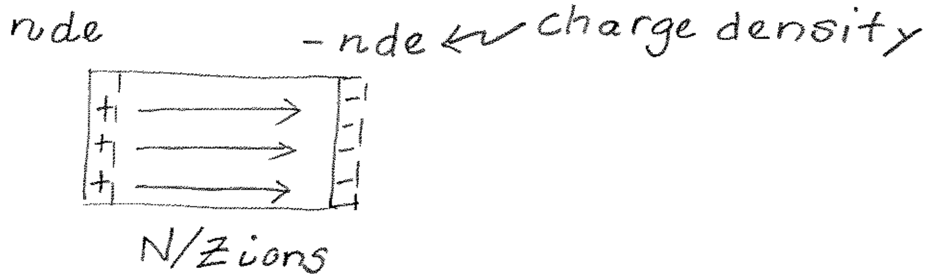
$$\Rightarrow i\omega\rho = \frac{\sigma(\omega)}{\varepsilon}\rho$$

$$1 - \frac{\sigma(\omega)}{i\omega\varepsilon} = 0$$

Again high frequency: $\sigma(\omega) \approx \frac{\sigma_0}{-i\omega\tau}$

$$\Rightarrow 1 - \frac{ne^2}{m\varepsilon\omega^2} = 0 \Rightarrow \omega = \omega_p$$

$\boxed{\text{Simple}}$ model of Plasma oscillations:



$$Nm\ddot{d} = -NeE = -\frac{Ne\,nde}{\varepsilon}$$

$$\rightarrow \ddot{d} = -\frac{ne^2}{m\varepsilon}\,d \quad (\text{Again } \omega_p)$$

Thermal Conductivity of a Metal

K = thermal conductivity

$j^q = -K \nabla T$ (hot to cold)

Wiedemann-Franz law:

$$\frac{K}{\sigma T} \approx 2 \text{ to } 3 \times 10^{-8} \ W\text{-}\Omega/K^2 \quad \text{(Table 1.6)}$$

1D model:

$$j^q = \frac{1}{2} n v \left[\mathcal{E}(T(x - v\tau)) - \mathcal{E}(T(x + v\tau)) \right]$$

(left to right)

$$j^q = n v^2 \tau \frac{d\mathcal{E}}{dT} \left(-\frac{dT}{dx} \right)$$

In 3D:

$$j^q = \left(\frac{1}{3}\right) v^2 \tau \, c_v (-\nabla T)$$

$$\rightarrow K = \frac{1}{3} v^2 \tau c_v = \frac{1}{3} \ell v c_v$$

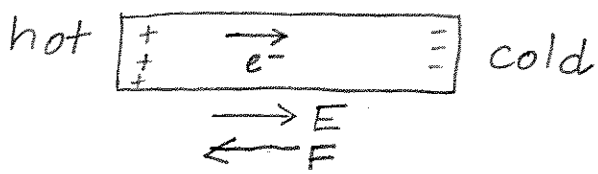
$$\frac{K}{\sigma} = \frac{\frac{1}{3} v^2 \cancel{\tau} c_v}{n e^2 \cancel{\tau}/m} = \frac{1}{3} \frac{\frac{1}{2} m v^2 c_v}{n e^2}$$

$$\left.\begin{aligned} c_v &= \frac{3}{2} n k_B \\ \frac{1}{2} m v^2 &= \frac{3}{2} k_B T \end{aligned}\right\} \rightarrow \frac{K}{\sigma} = \frac{1}{3} \frac{3 k_B T \frac{3}{2} n k_B}{n e^2}$$

$$\rightarrow \frac{K}{\sigma T} = \frac{3}{2} \left(\frac{k_B}{e} \right)^2 = 1.11 \times 10^{-8} \ W\text{-}\Omega/K^2$$

Agreement fortuitous. $c_v \neq \frac{3}{2} n k_B$ (much less),
c_v 100× smaller, v^2 100× larger (Fermi distribution)

Thermopower (Seebeck effect)



$$E = Q \nabla T \qquad (Q < 0 \text{ above})$$

1D model again:

$$v_Q = \frac{1}{2}\left[v(x - v\tau) - v(x + v\tau)\right] = -\tau v \frac{dv}{dx}$$

$$= -\tau \frac{d}{dx}\left(\frac{1}{2} v^2\right)$$

$$v^2 \rightarrow v_x^2 = \frac{1}{3}\langle v^2 \rangle$$

$$v_Q = -\frac{\tau}{6} \frac{dv^2}{dT} \frac{dT}{dx}$$

$$v_E = -\frac{eE\tau}{m}$$

$$v_Q + v_E = 0 \longrightarrow \frac{eE}{\cancel{m}} + \frac{1}{3\cancel{m}} \frac{d\,\frac{1}{2}mv^2}{dT} \frac{dT}{dx} = 0$$

$$\text{thermo-power} = Q = -\frac{1}{3e} \frac{d}{dT} \frac{1}{2} m v^2 = -\frac{c_v}{3ne}$$

If $c_v = -\frac{3}{2} n k_B$, then $Q = -\frac{k_B}{2e} = -0.43 \times 10^{-4}$ V/K

Observed 100 times smaller, as for thermal conductivity.