Homework 3
(due Monday, September, 28)

1. Consider the one dimensional periodic Hamiltonian

$$H = \frac{p^2}{2m} + U_K \cos(Kx), \tag{1}$$

where U_K is small and K is fixed, representing a single reciprocal lattice vector.

(a) Apply perturbation theory to find ϵ_k for this Hamiltonian.

(b) Sketch your result for ϵ_k in the extended zone scheme.

(c) Next consider the one dimensional periodic Hamiltonian:

$$H = \frac{p^2}{2m} + U_K \cos(Kx) + U_{K/2} \cos(Kx/2). \tag{2}$$

Again apply perturbation theory to find ϵ_k.

(d) Sketch your result for ϵ_k with the Hamiltonian in Eq. (2) on the same graph as for Eq. (1).

(e) The inclusion of the $U_{K/2}$ term means that the periodicity has doubled in real space. For an unperturbed Fermi wave vector of $K/4$, adding the new periodicity term $U_{K/2}$ will lower the energy. Why? Hint: Think of the filling the energy levels up to $K/4$. This is the basis for the Peierls instability.

2. Consider a two dimensional square lattice with lattice spacing a.

(a) Draw the first 4 Brillouin zones for this lattice. I would encourage doing this in color for clarity.

(b) On a graph for the Brillouin zones draw a Fermi circle of radius $k_F = 0.8\pi/a$. Sketch how you think the Fermi surface (curve in 2D) will deform in the presence of a weakly periodic potential.

(c) Repeat (b) for $k_F = 0.4\pi/a$.

(d) Repeat (b) for $k_F = 1.2\pi/a$.

(e) Bonus: Compute the Fermi surface numerically and plot it for cases (b)-(d).