Drude Model

Free electron + Scattering

See table 1.1 with $Z, r_0, r_s, r_s/a_0$.

$$\frac{1}{n} = \frac{4\pi r_s^3}{3}$$

a_0 = Bohr radius ≈ 0.529 Å

$$\frac{d\vec{p}}{dt} = \vec{F} - \frac{\vec{p}}{\tau}$$

1. DC conductivity & resistivity

$(v = v_{avg})$

$$\frac{d\vec{p}}{dt} = 0 = -e\vec{E} - \frac{\vec{p}}{\tau} \rightarrow p = mv = -eE\tau$$

$$j = -nev = -ne\left(\frac{-eE\tau}{m}\right)$$

$$= \frac{ne^2\tau}{m} E$$

$$\sigma = \frac{ne^2\tau}{m}, \quad \rho = \frac{m}{ne^2\tau}$$

See table 1.2 with resistivities in $\mu\Omega\cdot$cm.

Changes ~linear at higher temp incl. room temp.

See table 1.3 lifetimes in 10^{-14} sec.

2. $\vec{E}$ & $\vec{B}$ no time dependence.

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}\right) - \frac{\vec{p}}{\tau} \qquad \text{... note units}$$

$\vec{H} = H\hat{z}$:

$$0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau}$$

$$0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau}$$

$$\omega_c = \frac{eH}{mc} \quad \text{cyclotron frequency}$$

$$\frac{mv^2}{r} = \frac{eHv}{c}$$

$$\rightarrow \frac{v}{r} = \frac{eE}{mc} = \omega$$

$\sigma_0 = \frac{ne^2\tau}{m}$

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$

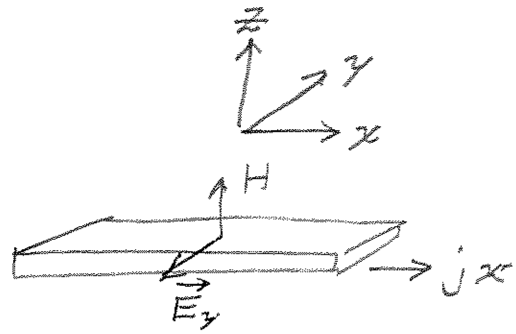
$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \underbrace{\frac{1}{\sigma_0}\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}}_{\text{resistivity matrix}} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \underbrace{\frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}}_{\text{conductivity matrix}} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Implications of not diagonal.

Hall geometry



$$j_y = 0: \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 \\ -\omega_c \tau \end{pmatrix} j_x$$

$$\frac{E_x}{j_x} = \frac{1}{\sigma_0}$$ magnetoresistance (field indep. here)

$$\frac{E_y}{j_x} = \frac{1}{\sigma_0} (-\omega_c \tau)$$

$$\sigma_0 = \frac{ne^2\tau}{m}, \quad \omega_c\tau = \frac{eH}{mc}\tau$$

$$\frac{E_y}{j_x} = -\frac{1}{enc} H$$

$$\boxed{R_H = \frac{E_y}{j_x H} = -\frac{1}{enc}}$$ Hall coefficient. measures n

See Table 1.4 with measured n compared to Drude n. Order mag. OK, but sign! Look at n_0/n vs. T for Al in Fig. 1.4. Puzzle.

3. AC conductivity

$$\frac{dp}{dt} = -eE(t) - \frac{p}{\tau}$$

$$-i\omega p = -eE(\omega) - \frac{p}{\tau}$$

$$p = \frac{-eE(\omega)}{\frac{1}{\tau} - i\omega} = \frac{-eE(\omega)\tau}{1 - i\omega\tau}$$

$$j = -nev = -\frac{nep}{m} = \underbrace{\frac{ne^2\tau}{m}\frac{1}{1-i\omega\tau}}_{\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}} E(\omega)$$