Drude con't.

Assume local: $\vec{j}(r,\omega) = \sigma(\omega)\vec{E}(\vec{r},\omega)$

Substitute into Maxwell's eqs.

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\left.\begin{aligned} \nabla \times E &= -\frac{1}{c}\frac{\partial H}{\partial t} \\ \nabla \times H &= \frac{4\pi}{c} j + \frac{1}{c}\frac{\partial E}{\partial t} \end{aligned}\right\} \begin{aligned} \nabla \times E &= \frac{i\omega}{c} H \\ \nabla \times H &= \frac{4\pi}{c}\sigma E - \frac{i\omega}{c} E \end{aligned}$$

$$\nabla \times (\nabla \times E) = -\nabla^2 E = \frac{i\omega}{c}\nabla \times H = \frac{i\omega}{c}\left(\frac{4\pi\sigma}{c}E - \frac{i\omega}{c}E\right)$$

$$-\nabla^2 E = \frac{\omega^2}{c^2}\underbrace{\left(1 + \frac{4\pi i\sigma}{\omega}\right)}_{\varepsilon(\omega)} E$$

$$\varepsilon(\omega) = 1 + \frac{4\pi i\sigma}{\omega}$$

If $\omega\tau \gg 1$, $\sigma(\omega) = \dfrac{\sigma_0}{1 - i\omega\tau} \rightarrow \dfrac{i n e^2}{m\omega}$,

$$\varepsilon(\omega) \rightarrow 1 - \frac{4\pi n e^2}{m}\frac{1}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

ω_p = <u>plasma frequency</u>, $\omega_p^2 = \dfrac{4\pi n e^2}{m}$

$\varepsilon < 0$ for $\omega < \omega_p \rightarrow$ exp. decay soln.

$\varepsilon > 0$ for $\omega > \omega_p \rightarrow$ propagate

Observations Table 1.5 in Alkali

Charge density oscillations:

$$\nabla \cdot j = -\frac{\partial \rho}{\partial t} \qquad \nabla \cdot j(\omega) = i\omega\rho(\omega)$$

$$\nabla \cdot E(\omega) = 4\pi\rho(\omega)$$

$$\nabla \cdot E(\omega) = \frac{i\omega\rho(\omega)}{\sigma(\omega)}$$

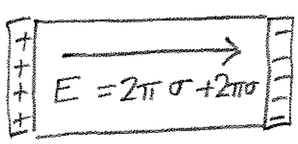
Consistent iff $4\pi = \dfrac{i\omega}{\sigma(\omega)}$

or $\dfrac{4\pi\sigma}{i\omega} = 1$

or $1 + \dfrac{4\pi i \sigma}{\omega} = 0$

Onset of propagation

Simple model for plasmons:



$\sigma = nde \qquad \sigma = -nde$

move uniformly

$$N m \ddot{d} = -Ne\,|4\pi\sigma|$$
$$= -Ne\,4\pi ned$$
$$= -4\pi ne^2 Nd$$
$$\omega_p^2 = \frac{4\pi ne^2}{m}$$

Thermal properties:

Wiedemann Franz Law:

(emperical)

Thermal conductivity

$$\frac{K}{\sigma T} \sim 2 \times 10^{-8} \ W \cdot \Omega / K^2$$

(Table 1.6)

$$j^q = -K \nabla T$$

1D example: $j^q = -K \dfrac{dT}{dx}$

high T low T

$$\text{1D:} \quad jq = \frac{n}{2} v \left[\mathcal{E}(T(x - v\tau)) - \mathcal{E}(T(x + v\tau)) \right]$$

$$jq = \frac{n}{2} v (-2v\tau) \frac{d\mathcal{E}}{dT} \left(\frac{dT}{dx} \right)$$

$$= n v^2 \tau \frac{d\mathcal{E}}{dT} \left(\frac{-dT}{dx} \right)$$

$$n \frac{d\mathcal{E}}{dT} = \frac{N}{V} \frac{d\mathcal{E}}{dT} = \frac{1}{V} \frac{dE}{dT} = c_v = \text{electronic specific heat}$$

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$jq = \frac{1}{3} v^2 \tau c_v \left(-\frac{dT}{dx} \right)$$

$$K = \frac{1}{3} v^2 \tau c_v = \frac{1}{3} \ell v c_v$$

$$\frac{K}{\sigma} = \frac{\frac{1}{3} v^2 \cancel{\tau} c_v}{\frac{ne^2 \cancel{\tau}}{m}} = \frac{\frac{1}{3} c_v m v^2}{n e^2}$$

$$\frac{1}{2} m v^2 \rightarrow \frac{3}{2} k_B T \quad , \quad c_v \rightarrow \frac{3}{2} n k_B$$

$$\frac{K}{\sigma} = \frac{\frac{1}{3}\left(\frac{3}{2} n k_B\right)\left(3 k_B T\right)}{n e^2} = \underbrace{\frac{3}{2}\left(\frac{k_B}{e}\right)^2}_{1 \times 10^{-8}\, W\Omega/K^2} T$$

Wow, about half exp.l value.

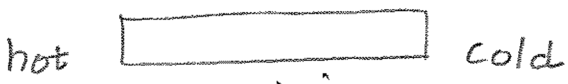
Careful: see size of c_v and mv^2 are off by 100 in Drude.

too big

too small

(related to fermi function)

Seebeck effect:



$\longrightarrow \dot{j}$ heat

$\longrightarrow$ electrons

$\longrightarrow E$ to oppose & stop current flow

$$E = Q \nabla T \qquad Q = \text{thermopower} < 0 \text{ here}$$

$$v_Q = \frac{1}{2}\left[v(x - v\tau) - v(x + v\tau)\right] = -\tau v \frac{dv}{dx}$$

$$= -\tau \frac{d}{dx}\left(\frac{v^2}{2}\right)$$

Again $v = v_x$.

$$v_Q = -\frac{\tau}{6}\frac{dv^2}{dT}\nabla T$$

$$v_E = \frac{-eE\tau}{m}$$

$$v_E + v_Q = 0 \rightarrow -\frac{eE\tau}{m} - \frac{\tau}{6}\frac{dv^2}{dT}\nabla T = 0$$

$$E = -\frac{1}{6e}\frac{d}{dT}mv^2 \nabla T = \underbrace{\frac{-1}{3e}\frac{1}{n}c_v}_{Q} \nabla T$$

If $c_v = \frac{3}{2}nk_B$, then $Q = -\frac{k_B}{2e} = -0.43\times10^{-4}\,V/K$

Observed Q are 100 times smaller.