Landau levels: $\mathcal{E}(\nu, k_z) = \frac{\hbar^2}{2m} k_z^2 + \hbar\omega_c(\nu + \frac{1}{2})$

$$m\frac{v^2}{r} = qvB \rightarrow \omega_c = \frac{eB}{m}$$

Highly degenerate in x-y direction.

2D-DOS : $g = 2 \cdot \dfrac{\frac{2\pi \cancel{k\,dk}}{(2\pi)^2}}{\frac{\hbar^2}{m}\cancel{k\,dk}} = \frac{2}{2\pi}\frac{m}{\hbar^2} = \frac{2m}{\hbar h}$

per area (g), spin (2)

number of states in $\Delta E = \hbar\omega_c$ is

$$L^2 \cdot g \cdot \hbar\omega_c = L^2 \cdot \frac{2\cancel{m}}{\cancel{\hbar}h}\cancel{\hbar}\frac{eB}{\cancel{m}} = L^2 \cdot \frac{2e}{h} \cdot B$$

$\frac{h}{2e}$ has units of flux: Tesla$\cdot$m^2.

$h = 6.6 \times 10^{-34}\, m^2 kg/s$

$e = 1.6 \times 10^{-19} C$

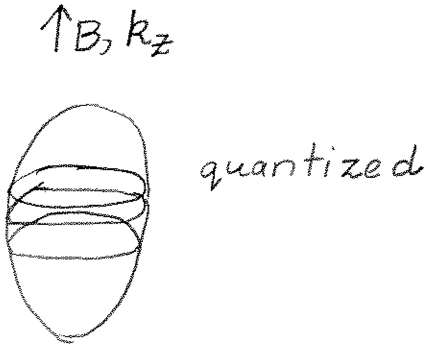
$$\left.\right\} \underset{\Phi_0}{\frac{h}{2e}} \approx 2 \times 10^{-15} \underbrace{T\cdot m^2}_{Wb}$$

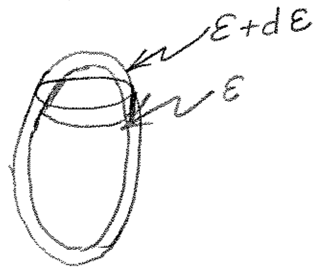
Tesla = 10^4 Gauss

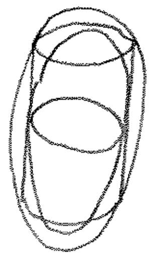
m = 10^2 cm

$$\rightarrow \frac{h}{2e} \approx 2 \times 10^{-7}\, G\cdot cm^2$$

At 1 kG for a sample 1cm × 1 cm, the degeneracy is $\sim \frac{1}{2} \times 10^{10}$.







Another way:

$$g(\varepsilon) = 2\int \frac{dk_z}{2\pi} \sum_\nu \delta\left(\varepsilon - \varepsilon_{k_z} - \hbar\omega_c\left(\nu + \tfrac{1}{2}\right)\right)$$

$$= 2\,\frac{1}{2\pi}\sum_\nu \left(\frac{\partial \varepsilon_{k_z}}{\partial k_z}\right)^{-1}$$ at extremal $\nabla_k \varepsilon_k$ is perp. to $\hat{z}$

Since $A = (\nu+\lambda)\Delta A = (\nu+\lambda)\dfrac{2\pi e B}{\hbar}$

$$(\nu+\lambda) = \frac{A}{\Delta A} = \frac{A\hbar}{2\pi e}\,\frac{1}{B}$$

$$\boxed{\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{A\hbar}}$$

Estimate $\hbar\omega_c$:

$$\frac{\hbar e B}{m} = \frac{1}{2\pi}\frac{(6.6\times10^{-34})(1.6\times10^{-19})\,1\,\text{Tesla}}{9.1\times10^{-31}\,\text{kg}} = 1.85\times10^{-23}\,\text{J}$$

$$1\,\text{eV} = 1.6\times10^{-19}\,\text{J} \longrightarrow \frac{\hbar e B}{m} \approx 1.16\times10^{-4}\,\text{eV} \text{ @ } 1\text{ Tesla}$$

$$1.16\times10^{-8}\,\text{eV} \text{ @ } 1\text{ Gauss}$$

$$\rightarrow \frac{\varepsilon_F}{\hbar\omega_c} \left.\begin{matrix}\sim 10^4 \text{ at } 1 \text{ Tesla} \\ \sim 10^8 \text{ at } 1 \text{ Gauss}\end{matrix}\right\} \rightarrow \text{many levels}$$

Bohr's correspondence principle:

$$\varepsilon_{\nu+1}(k_z) - \varepsilon_\nu(k_z) = \frac{h}{T(\varepsilon, k_z)} \quad \swarrow \text{ period}$$

However, we have seen that

$$T(\varepsilon, k_z) = \frac{\hbar^2}{eB}\frac{\partial A}{\partial \varepsilon}$$

$$\rightarrow (\varepsilon_{\nu+1} - \varepsilon_\nu)\frac{\partial A}{\partial \varepsilon} = \frac{eB}{\hbar^2} h = 2\pi\frac{eB}{\hbar}$$

$$\approx A(\varepsilon_{\nu+1}) - A(\varepsilon_\nu)$$

$$\hookrightarrow \Delta A = \frac{2\pi e B}{\hbar}$$

$$A = (\nu + \lambda)\Delta A$$

Temperature:

$$\hbar\omega_c \text{ @ } 1 \text{ Tesla} = 10^{-4}\,\text{eV}$$

$$1\,\text{eV} \sim k_B\, 10^4\,\text{K}$$

$$\frac{\hbar\omega_c}{k_B\, 1\text{K}} \approx 1$$

Scattering: $\omega_c \tau \gg 1$

Spin: $\mu_B = \dfrac{e\hbar}{2m_e}$ (in CGS $\dfrac{e\hbar}{2mc}$)

$$\pm g\mu_B \frac{1}{2} B \approx \mu_B B \quad \text{since } g \approx 2$$

$$\left.\begin{array}{l} \mu_B B \text{ @ } 1 \text{ Tesla} \approx 5.8 \times 10^{-5}\,\text{eV} \\ k_B\, 1\text{K} \approx 8.6 \times 10^{-5}\,\text{eV} \end{array}\right\} \text{comparable}$$

$$g(\varepsilon) = \frac{1}{2} g_0\left(\varepsilon + \frac{ge\hbar}{2 \cdot 2m_e}\right) + \frac{1}{2} g_0\left(\varepsilon - \frac{ge\hbar}{2 \cdot 2m_e}\right)$$