de Haas – van Alphen for free electrons

$$\mathcal{E}(\nu, k_z) = \frac{\hbar^2}{2m} k_z^2 + \hbar\omega_c(\nu + \tfrac{1}{2})$$

$$m\frac{v^2}{r} = qvB \longrightarrow \omega_c = \frac{eB}{m}$$

2D-DOS: $$g = \underset{\substack{\uparrow \\ \text{spin}}}{2}\ \frac{\frac{2\pi k\,dk}{(2\pi)^2}}{\frac{\hbar^2}{m} k\,dk} = \frac{2}{2\pi}\frac{m}{\hbar^2} = \frac{2m}{\hbar h}$$

$$\Delta E = \hbar\omega_c$$

$$\Delta E \cdot g = \hbar\omega_c \frac{2m}{\hbar h} = \frac{eB}{\cancel{m}} \cdot \frac{2\cancel{m}}{h} = \frac{2eB}{h} \quad \text{(per unit area)}$$

$$g_{3D}(E) = \sum_\nu \int \frac{dk_z}{2\pi} \delta\left(E - \frac{\hbar^2}{2m}k_z^2 - \hbar\omega_c(\nu + \tfrac{1}{2})\right) \cdot \frac{2eB}{h} \qquad \checkmark \text{ units}$$

$$= \sum_{\nu=0}^{\nu_{max}} \left. \frac{2}{2\pi}\frac{1}{v_z\hbar} \cdot \frac{2eB}{h}\frac{m}{m} \right\} \leftarrow \frac{2m}{\pi h}\frac{\omega_c}{v_z\hbar}$$

where ν_{max} is the largest ν so that

$$\hbar\omega_c(\nu + \tfrac{1}{2}) < E$$

$$\nu < \frac{E}{\hbar\omega_c} - \frac{1}{2}$$

$$\nu_{max} = \text{floor}\left(\frac{E}{\hbar\omega_c} - \frac{1}{2}\right),$$

and $$\frac{\hbar^2 k_z^2}{2m} = E - \hbar\omega_c(\nu + \tfrac{1}{2})$$

$$p_z = \sqrt{2m(E - \hbar\omega_c(\nu + \tfrac{1}{2}))}$$

$$v_z = \frac{p_z}{m} = \sqrt{\frac{2}{m}}\sqrt{E - \hbar\omega_c(\nu + \tfrac{1}{2})}$$

$$\rightarrow g_{3D}(E) = \sum_{\nu=0}^{\nu_{max}} \frac{\sqrt{2}\,m^{3/2}}{\pi h \hbar} \frac{\omega_c}{\sqrt{E - \hbar\omega_c(\nu + \tfrac{1}{2})}}$$

$$\boxed{g_{3D}(E) = \sum_{\nu=0}^{\nu_{max}} \frac{\sqrt{2}\, m^{3/2}}{2\pi^2\hbar^3} \frac{\hbar\omega_c}{\sqrt{E-\hbar\omega_c(\nu+\frac{1}{2})}}}$$

Check: $\hbar\omega_c \to 0$, $\varepsilon = \hbar\omega_c(\nu+\frac{1}{2})$.

$$g_{3D}(E) = \int_0^E d\varepsilon \frac{(2m)^{3/2}}{(2\pi)^2\hbar^3} \frac{1}{\sqrt{E-\varepsilon}}$$

$$= \frac{(2m)^{3/2}}{(2\pi)^2\hbar^3} 2\sqrt{E} = \frac{\sqrt{2}\, m^{3/2}}{\pi^2\hbar^3}\sqrt{E}$$

What do we expect in 3D?

$$g_{3D} = 2\frac{\frac{4\pi k^2 dk}{(2\pi)^2}}{\frac{\hbar^2 k\, dk}{m}} = \boxed{\frac{1}{\pi^2}\frac{mk}{\hbar^2}} \quad (2.64 \text{ A\&M})$$

$$\frac{\hbar^2k^2}{2m} = E \longrightarrow k^2 = 2mE/\hbar^2, \quad k = \sqrt{2mE/\hbar^2}$$

$$g_{3D} = \frac{1}{\pi^2}\frac{m}{\hbar^2}\sqrt{2mE/\hbar^2} = \frac{\sqrt{2}}{\pi^2}\frac{m^{3/2}}{\hbar^3}\sqrt{E}$$

Units: $[\hbar] = M \cdot L^2/T$

$$\rightarrow [g_{3D}] = \frac{M^{3/2}}{M^3 L^6/T^3} M^{1/2} L/T = \frac{1}{M L^5/T^2} = \frac{1}{E}\cdot\frac{1}{L^3}$$

$$[F] = [evB] \rightarrow [eB] = \frac{[F]}{[v]} = \frac{M}{T}$$

$$\frac{[eB]}{[\hbar]} = \frac{M/T}{ML^2/T} = \frac{1}{L^2}$$

$$\frac{[m][\omega_c]}{[\hbar][v_z]} = \frac{M/T}{(ML^2/T)(L/T)} = \frac{1}{L^3}\frac{1}{1/T}$$

$$\boxed{\frac{g_{3D}(E,B)}{g_{3D}(E,B=0)} = \sum_{\nu=0}^{\nu_{max}} \frac{1}{2}\frac{1}{\sqrt{E}}\frac{\hbar\omega_c}{\sqrt{E-\hbar\omega_c(\nu+1/2)}}}$$

check again:

$$\int_0^E d\varepsilon \frac{1}{2}\frac{1}{\sqrt{E}}\frac{1}{\sqrt{E-\varepsilon}} = -\frac{1}{\sqrt{E}}\sqrt{E-\varepsilon}\Big|_0^E = 1 \checkmark$$

Fix and plot as a function of $\hbar\omega_c$.

$\nu_{max} = \text{floor}\left(\frac{E}{\hbar\omega_c} - \frac{1}{2}\right)$.

Take E to be large compared to $\hbar\omega_c$, say 1000.

We expect this to be periodic in

$$\Delta\left(\frac{E}{\hbar\omega_c}\right) = 1 = \Delta\left(\frac{Em}{\hbar eB}\right) \propto \Delta\left(\frac{1}{B}\right) \text{ or } \Delta\left(\frac{1}{\hbar\omega_c}\right).$$

$$\boxed{\Delta\left(\frac{1}{B}\right) = \frac{e}{m}\frac{\hbar}{E}} = \frac{e}{m}\frac{1}{\frac{\hbar^2k^2}{2m}} = \frac{2e}{\hbar k^2} = \boxed{\frac{2\pi e}{\hbar}\frac{1}{\pi k^2} = \frac{2\pi e}{\hbar}\frac{1}{A(E)}} \checkmark$$

Also examine

$$\boxed{\Delta A = \frac{2\pi e B}{\hbar}} \quad (14.12 \text{ in A \& M})$$

$$= \frac{2\pi}{\hbar} m\omega_c = \Delta(\pi k^2) \quad \Big\langle \frac{1}{\pi}\frac{\hbar^2}{2m}$$

$$\rightarrow \boxed{\hbar\omega_c = \Delta\left(\frac{\hbar^2k^2}{2m}\right)}$$

This is consistent with $E = \frac{\hbar^2k^2}{2m} = \hbar\omega_c(\nu+\frac{1}{2})$.

Continue on $\boxed{\overset{\text{spin}}{2}\dfrac{\Delta A}{(2\pi)^2} = \text{degeneracy} = \dfrac{2eB}{h}}$

$$\Delta A = \frac{2\pi e B}{\hbar} \text{ from prev. page}$$

$$\frac{2}{(2\pi)^2} \cdot \frac{2\pi e B}{\hbar} = \frac{2eB}{h} \checkmark$$

Finally look at the semiclassical quantization condition:

$$k \cdot 2\pi r = \frac{k \cdot 2\pi V}{\omega_c} = \frac{2\pi\, p \cdot v}{\hbar\omega_c} = 2\pi \cdot \frac{2E}{\hbar\omega_c} = 2\pi\, 2(\nu + \tfrac{1}{2}) \quad X$$

$\underline{\varepsilon_{\nu+1} - \varepsilon_\nu = \hbar\omega_c \text{ from prev. page is better.}}$ $\checkmark$