Surface Effects

U^{inf} = infinite crystal

U^{fin} = finite crystal

Ex. Cubic sym., inv. sym.

$$U^{inf} \simeq \sum_R v(r-R)$$

Wigner-Seitz

$$v(r) = -e \int_{\substack{C \\ \text{at } R}} d^3r' \rho(r') \frac{1}{4\pi\varepsilon_0 |r-r'|}$$

total charge density

Expand: $$v(r) = \frac{-eQ}{4\pi\varepsilon_0 r} - \frac{e\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

$$Q = \int_C d^3r' \rho(r')$$

$$\vec{P} = \int_C d^3r' \, \vec{r}' \rho(r')$$

$Q = 0$, since must be periodic

Inversion sym. $\rightarrow \vec{p} = 0$

Cubic sym. $\frac{1}{r^3}$ vanish

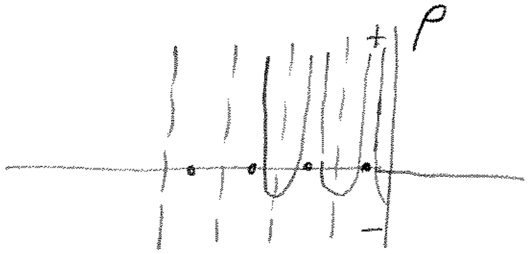
Inversion sym. $\frac{1}{r^4}$ vanish

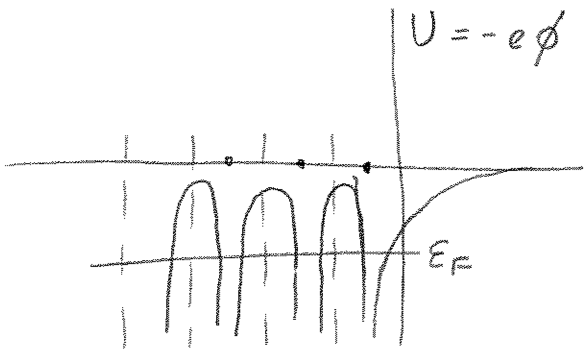
$\rightarrow \frac{1}{r^5}$ lowest order

<u>decays rapidly</u>

Work fnt.

1. No charge redistribution:

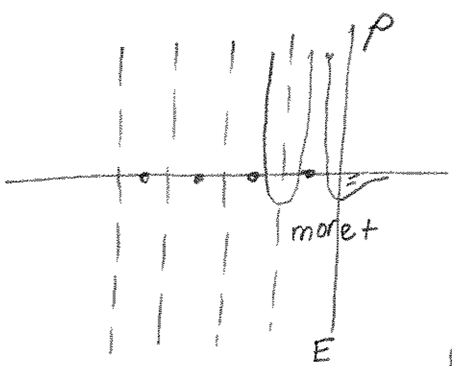




decays to zero
L, R symmetry

$W = 0 - \varepsilon_F = -\varepsilon_F \quad (\varepsilon_F < 0)$

2. Charge redistribution, no net charge

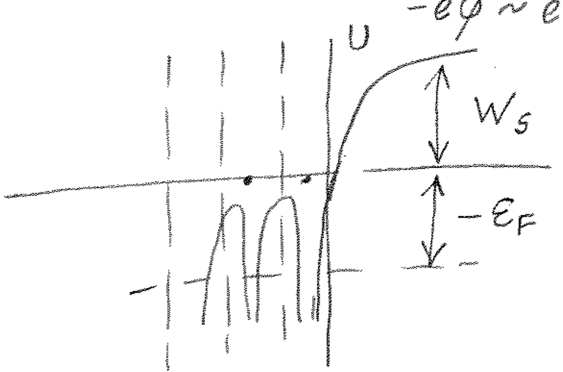


dipole layer

$\overrightarrow{E}$

$\phi \sim -Ex$

$-e\phi \sim eEx$



$W = W_s - \varepsilon_F$

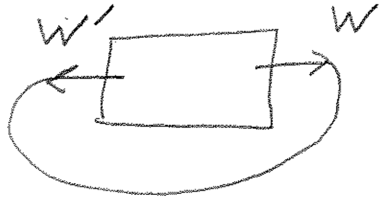
other side same

3. Charge redistribution and net charge

Inequivalent surfaces

(equilibrium)

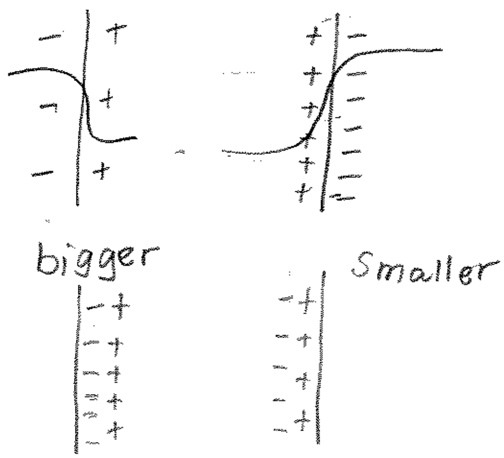
$$0 = W - e(\phi' - \phi) - W'$$

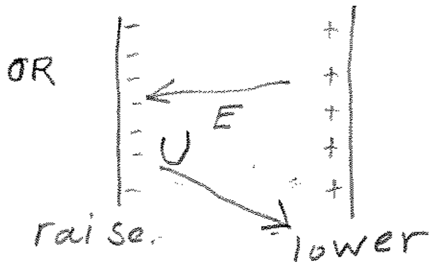


$\vec{E}$ outside metal.

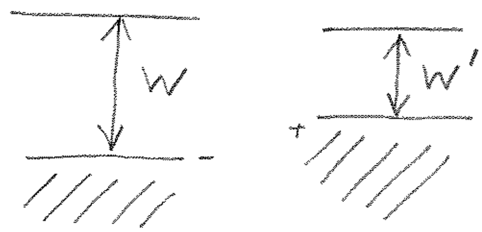
$$-e(\phi - \phi') = W - W'$$

includes only work in double layer

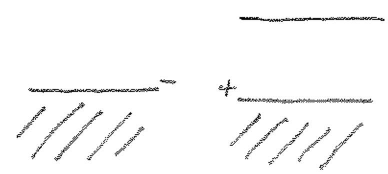




Measuring Contact Potential:



if contact or joined by wire



edges same as before
current momentary

$\phi = Ex$

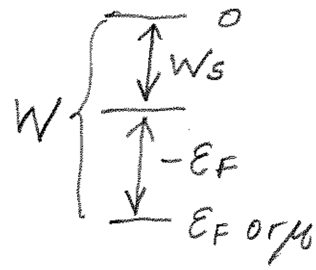
$U = -eEx$

$$E = \frac{\sigma}{\varepsilon_0} \rightarrow \sigma = \varepsilon_0 E = \frac{\varepsilon_0 V}{d}$$

Change d, σ flows.

or add voltage so that no charge flows.

Thermionic Emission:



$$j = e \int_{\substack{k_x > 0 \\ v_x > v_{min}}} \frac{d^3k}{4\pi^3}\, v_x \, \frac{1}{e^{\beta(\varepsilon_k - e\phi - \mu)} + 1}$$

$$\approx e \int_{v_x > v_{min}} \frac{d^3k}{4\pi^3}\, v_x \, e^{-\beta(\underbrace{\varepsilon_k}_{\frac{\hbar^2 k^2}{2m}} - e\phi - \mu)}$$

$$= \frac{e \cdot e^{\beta(e\phi + \mu)}}{4\pi^3} \left.\int_{v_x > v_{min}} dk_x \, v_x \, e^{-\frac{\beta\hbar^2}{2m} k_x^2} \right\} \frac{1}{\hbar} \int d\varepsilon_{k_x} e^{-\beta \varepsilon_{k_x}} = \frac{1}{\hbar\beta} e^{-\beta \varepsilon_{k_x min}}$$

$$\underbrace{\int dk_y \, e^{-\frac{\beta\hbar^2}{2m} k_y^2}}_{\frac{\sqrt{\pi}}{\sqrt{\beta\hbar^2/2m}}} \; \underbrace{\int dk_z \, e^{-\frac{\beta\hbar^2}{2m} k_z^2}}_{\frac{\sqrt{\pi}}{\sqrt{\beta\hbar^2/2m}}}$$

$$d\,\frac{\hbar^2 k_x^2}{2m} = \frac{\hbar^2 k_x}{m} dk_x = \hbar \, v_x \, dk_x$$

$$= \frac{e}{4\pi^3} \quad \frac{\pi}{\beta\hbar^2/2m} \frac{1}{\beta\hbar} \, e^{-\beta\underbrace{(\underbrace{\varepsilon_{k_x min} - e\phi}_{E_{min}} - \mu)}_{W}}$$

$$\boxed{j = \frac{em}{2\pi^2\hbar^3} (k_B T)^2 e^{-\beta W}}$$

(Richardson's Law)