Chpt. 24 Measuring Phonon Dispersion Relations

* Conservation energy
* Conservation crystal momentum

Zero-Phonon:

$$E' = \frac{p'^2}{2M_n} = E = \frac{p^2}{2M_n}$$

$$p' = p + \hbar K$$

One-Phonon:

$$E' = \frac{p'^2}{2M_n} = \frac{p^2}{2M_n} + \hbar\omega_s(k)$$

$$p' = p + \hbar k + \hbar K$$

Two-Phonons:

$$\frac{p'^2}{2M_n} = \frac{p^2}{2M_n} + \hbar\omega_s(k) + \hbar\omega_s(k')$$

$$p' = p + \hbar k + \hbar k' + \hbar K$$

Zero-Phonon:

$$p' = \hbar q', \quad p = \hbar q$$

Energy conservation $\rightarrow$ $q' = q$

$q' = q + K$, Laue formulation

One-Phonon:

Since $\hbar\omega_s(k \pm K) = \hbar\omega_s(k)$,

$$\frac{p'^2}{2M_n} = \frac{p^2}{2M_n} + \hbar\omega_s\left(\frac{p'-p}{\hbar}\right).$$

In experiment fix $\vec{p}$ (incoming $\vec{p}$ & E).
$\vec{p}'$ unknown. Measure outgoing $|\vec{p}'|$ and $\hat{p}'$.
Determine $\hbar\omega_s\left(\frac{p'-p}{\hbar}\right)$.

Provided peak or discrete $\vec{p}$.
3 unknowns, 1 condition $\rightarrow$ surface
for each ω_s branch. Fix angle $\hat{p}'$. Discrete points

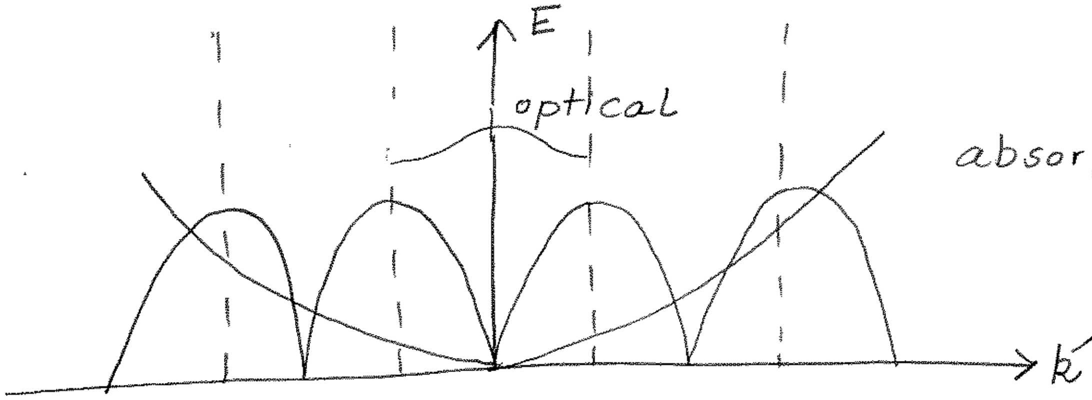
Two-Phonon:

$$E' = E + \hbar\omega_s(k) + \hbar\omega_{s'}\left(\frac{p'-p}{\hbar} - k\right)$$

Again point, but k is a continuum.
Smooth out points. $\rightarrow$ Background.
Distinguish 1 & 2 phonon processes.

More on 1 Phonon:

$p \approx 0 \qquad \frac{p'^2}{2M_n} = \hbar\omega_s\left(\frac{p'}{\hbar}\right) \qquad p' = \hbar k'$



absorption

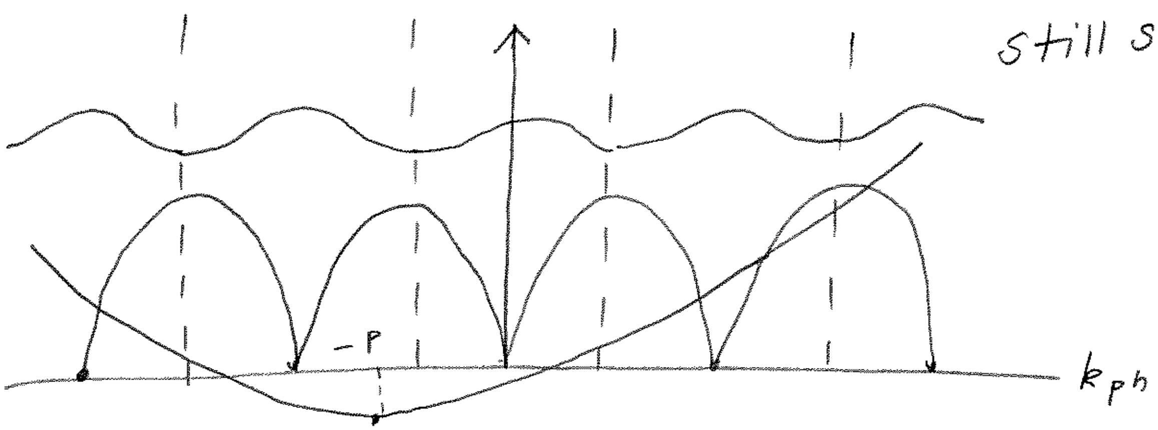
more generally,

$$\frac{p'^2}{2M_n} = \frac{p^2}{2M_n} + \hbar\omega_s\Big(\overbrace{\frac{p'-p}{\hbar}}^{k_{ph}}\Big) \qquad p'-p = \hbar k_{ph}$$

$$\frac{p'^2}{2M_n} - \frac{p^2}{2M_n} = \hbar\omega_s(k_{ph})$$

$$\frac{(\hbar k_{ph}+p)^2}{2M_n} - \frac{p^2}{2M_n} = \hbar\omega_s(k_{ph})$$

still solutions



Numbers:

$\Theta_D = 100$ to $1000\,K$

$k_B \Theta_D$ Cold-neutron scattering

$E_N = 2.1\, q^2$ meV $\quad$ q in $Å^{-1}$

$\frac{E_N}{k_B} = 24\, q^2\, K$ } small even at zone boundary

X-Ray:

X-ray several keV (10^3 eV)

phonon meV (10^{-3} eV) $< \sim 0.1$ eV at Θ_D

Tough small changes energy.

Optical (visible):

energy eV, but can measure energy shifts

$k_{ph} \sim 10^5\, cm^{-1} \ll k_{BZ} \sim 10^8\, cm^{-1} \rightarrow k = 0$

Brillouin: acoustic

Raman: optical

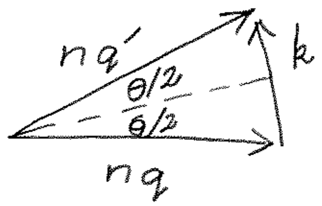
$\hbar\omega' = \hbar\omega \pm \hbar\omega_s(k)$ $\quad$ (anti-Stokes) + absorbed

$\hbar n q' = \hbar n q \pm \hbar k + \hbar K$ $\quad$ − emitted (Stokes)

n index refraction $\quad$ $K \to 0$ k_{ph} small

$\left\{ v = f\lambda = \frac{c}{n} \rightarrow \lambda \rightarrow \frac{\lambda}{n} \right\}$

$\hbar\omega_0 \approx 10^{-2}\,eV, \quad \omega' \approx \omega$



$$k = 2nq\sin\frac{\theta}{2} = \frac{2\omega n}{c}\sin\left(\frac{\theta}{2}\right)$$

phonon

Brillouin: $\omega_s(k) = c_s(\hat{k})\,k = \Delta\omega$

$$c_s(\hat{k}) = \frac{\Delta\omega}{k} = \frac{\Delta\omega}{2\omega n/c}\csc\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{2}\frac{\Delta\omega}{\omega}\frac{c}{n}\csc\left(\frac{\theta}{2}\right)$$