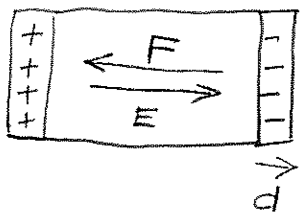
# 26 Phonons in Metals

## Plasmons (electrons)




$\sigma = nde$

$$Nm\ddot{d} = -\frac{nde^2}{\varepsilon_0}N$$ force on N electrons

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m}$$

also found $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$.

If do for ions, $\Omega_p^2 = \frac{n_i(Ze)^2}{\varepsilon_0 M} = \frac{n_i}{n_{el}}\frac{Z^2 m}{M} = \frac{Zm}{M}\omega_p^2$

Not $\Omega \propto k$ for small $k$.

Need to screen, $\boxed{\omega^2(k) = \frac{\Omega_p^2}{\varepsilon(k)}}$ ①, $\varepsilon(k) = 1 + \frac{k_0^2}{k^2}$

$$\rightarrow \omega^2(k) \overset{①'}{\sim} \frac{\Omega_p^2}{k_0^2}k^2 = \underbrace{\frac{Zm}{M}\frac{\omega_p^2}{k_0^2}}_{c^2}k^2$$

Use $\frac{e^2}{\varepsilon_0 k_0^2} = \frac{\hbar^2\pi^2}{mk_F}$ and $n_e = \frac{k_F^3}{3\pi^2}$,

$$\rightarrow c^2 = \frac{1}{3}Z\underbrace{\frac{m}{M}}_{10^{-4}\text{ or }10^{-5}}v_F^2$$ Bohm – Staver relation

$\frac{1}{100\text{th}}v_F \rightarrow 10^6\ \text{cm/s}$

## Dielectric constant:

$$\varepsilon \phi^{total} = \phi^{ext}$$

$\varepsilon^{el}$ - electrons alone

$\varepsilon^{ion}_{bare}$ - ions alone

$\varepsilon^{ion}_{dressed}$ - ions screened

$$\varepsilon^{el} \phi^{tot} = \phi^{ext} + \phi^{ion}$$

$$\varepsilon^{ion}_{bare} \phi^{tot} = \phi^{ext} + \phi^{el}$$

exact

$$\underbrace{(\varepsilon^{el} + \varepsilon^{ion}_{bare} - \varepsilon)}_{1} \phi^{total} = \phi^{ext} + \phi^{el} + \phi^{ion} = \phi^{total}$$

$$\boxed{\varepsilon = \varepsilon^{el} + \varepsilon^{ion}_{bare} - 1} \quad (2)$$

$$\phi^{total} = \frac{1}{\varepsilon^{ion}_{dressed}} \frac{1}{\varepsilon^{el}} \phi^{ext}$$

$$\frac{1}{\varepsilon} = \frac{1}{\varepsilon^{ion}_{dressed}} \frac{1}{\varepsilon^{el}}$$

$$\varepsilon^{ion}_{dressed} = 1 + \frac{1}{\varepsilon}(\varepsilon^{ion}_{bare} - 1)$$

$$\boxed{\varepsilon^{ion}_{bare} = 1 - \frac{\Omega_p^2}{\omega^2}} \quad , \quad \boxed{\varepsilon = 1 + \frac{k_0^2}{q^2} - \frac{\Omega_p^2}{\omega^2}} \quad ③$$

$$\varepsilon^{ion}_{dressed} = 1 - \frac{\Omega_p^2/\varepsilon_{el}}{\omega^2} = 1 - \frac{\omega(q)^2}{\omega^2} \quad , \quad \omega(q) = \frac{\Omega_p^2/\varepsilon^{el}}{\omega^2}$$

$$\boxed{\frac{1}{\varepsilon} = \left(\frac{1}{1 + k_0^2/q^2}\right)\left(\frac{\omega^2}{\omega^2 - \omega^2(q)}\right)} \quad ④$$

el-el interaction:

$$\frac{e^2}{\varepsilon_0 k^2} \rightarrow \frac{e^2}{\varepsilon_0 \varepsilon^{el} k^2} = \frac{e^2}{\varepsilon_0 (k^2 + k_0^2)}$$

$$\rightarrow \frac{e^2}{\varepsilon_0 (k^2 + k_0^2)} \left(\frac{\omega^2}{\omega^2 - \omega^2(k)}\right)$$

$$\boxed{= \frac{e^2}{\varepsilon_0 (k^2 + k_0^2)} \left(1 + \frac{\omega^2(k)}{\omega^2 - \omega^2(k)}\right)} \quad ⑤$$

$$q = k - k'$$

$$\omega = \frac{\varepsilon_k - \varepsilon_{k'}}{\hbar}$$

1. $\omega(q) \lesssim \omega_D \longrightarrow \omega \gg \omega_D$, phonons not important much of time since $\varepsilon_F \gg \omega_D$

2. $\hbar\omega < \hbar\omega_D$, sign change $\longrightarrow$ overscreening

Phonon contribution to $\mathcal{E}_k$:

$$\Delta\mathcal{E}_k = -\int \frac{d^3k'}{(2\pi)^3} \frac{4\pi e^2}{|k-k'|^2 + k_0^2} f(k') \times$$

$$\times \left\{ 1 + \frac{\omega(k-k')^2}{[(\mathcal{E}_k - \mathcal{E}_{k'})/\hbar]^2 - \omega(k-k')^2} \right\}$$

(Prob. 3) (1) Fermi surface unaffected by new term.

(2) $\mathcal{E}_k - \mathcal{E}_F \approx \dfrac{\mathcal{E}_k^{TF} - \mathcal{E}_F}{1+\lambda}$

$$\left.\begin{aligned} \lambda &= \int \frac{dS'}{8\pi^3 \hbar v(k')} \frac{4\pi e^2}{(k-k')^2 + k_0^2} \\ v(k) &= \frac{1}{\hbar} \nabla_k \mathcal{E} = \frac{1}{1+\lambda} v^\circ(k) \\ g(\mathcal{E}_F) &= (1+\lambda) g^\circ(\mathcal{E}_F) \end{aligned}\right\} (6)$$

$$\lambda \lesssim \frac{4\pi e^2}{k_0^2} \int \frac{dS'}{8\pi^3 \hbar v(k')}$$

$$\frac{4\pi e^2}{k_0^2} = \frac{\partial n}{\partial \mu} = \frac{1}{g(\mathcal{E}_F)} = \left[ \int \frac{dS'}{4\pi^3 \hbar v(k')} \right]^{-1} \longrightarrow \lambda \lesssim 1$$

(3) $\mathcal{E}_k - \mathcal{E}_F \gg \hbar\omega_D$

$$\mathcal{E}_k - \mathcal{E}_F = (\mathcal{E}_k^{TF} - \mathcal{E}_F)\left(1 - O\left(\frac{\hbar\omega_D}{\mathcal{E}_k - \mathcal{E}_F}\right)^2\right)$$

## Electron-Phonon Interaction

$$V^{eff}_{k,k'} = \frac{1}{V}\left(\frac{4\pi e^2}{|k-k'|^2+k_0^2}\right)\left(\frac{[\hbar\omega(k-k')]^2}{[\hbar\omega(k-k')]^2-(\mathcal{E}_k-\mathcal{E}_{k'})^2}\right)$$

$$\Delta E = \sum_i \frac{\langle 0|V^{ep}|i\rangle}{E_0-E_i} \qquad k \rightarrow k' + \underset{\substack{\text{emit}\\ \text{phonon}}}{q}$$

no absorp. at $T=0$ since no phonons

$$E_i - E_0 = \mathcal{E}_{k'} + \hbar\omega(k-k') - \mathcal{E}_k$$

$$\Delta E = \sum_{k,k'} n_k(1-n_{k'})\frac{|g_{k,k'}|^2}{\mathcal{E}_k-\mathcal{E}_{k'}-\hbar\omega(k-k')}$$

$$V^{eff}_{k,k'} = \frac{\partial^2 \Delta E}{\partial n_k \partial n_{k'}}$$

$\rightarrow$ $$V^{eff}_{k,k'} = -|g_{k,k'}|^2\left(\frac{1}{\mathcal{E}_k-\mathcal{E}_{k'}-\hbar\omega(k-k')} + \frac{1}{\mathcal{E}_{k'}-\mathcal{E}_k-\hbar\omega(k'-k)}\right)$$

$$= |g_{k,k'}|^2 \frac{2\hbar\omega(k-k')}{[\hbar\omega(k-k')^2-(\mathcal{E}_k-\mathcal{E}_{k'})^2]}$$

$\rightarrow$ $$\boxed{|g_{k,k'}|^2 = \frac{1}{V}\underbrace{\frac{4\pi e^2}{|k-k'|^2+k_0^2}}_{\substack{\text{Free el.'s}\\ |k-k'| \ll k_0}}\frac{1}{2}\hbar\omega_{k-k'}} \quad (7)$$

$$\approx \frac{1}{V}\frac{4\pi e^2}{k_0^2}\frac{1}{2}\hbar\omega_{k-k'} = \frac{1}{V}\frac{2\mathcal{E}_F}{3n_e}\frac{1}{2}\hbar\omega_{k-k'}$$

$$= \frac{\hbar\omega_{k-k'}\cdot\mathcal{E}_F}{3NZ} \quad \text{for } |k-k'| \ll k_0$$

$\rightarrow$ Interaction vanishes linearly w/ phonon q.

## Temperature Dependent Resistivity:

$\varepsilon_k = \varepsilon_{k'} \pm \hbar\omega(k-k')$

high T: $n(q) = \dfrac{1}{e^{\beta\hbar\omega(q)}-1} \approx \dfrac{k_B T}{\hbar\omega(q)}$

$$\rho(T) \sim T \quad \text{for } T \gg \Theta_D$$

low T: $\hbar\omega(q) \lesssim k_B T$

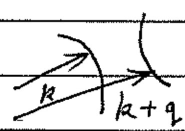
$\hbar c q \lesssim k_B T$

Area $\propto T^2$

Coupling $\propto T$

$\dfrac{1}{\tau_{el-ph}} \propto T^3$

$1 - \cos\theta \approx \dfrac{\theta^2}{2} \Rightarrow \rho \propto T^5$

Still $T^3$ when 

big change in $\vec{v}$

## Phonon drag:

Electron + phonons momentum conserved

$\exp(-\hbar\omega_{min}/kT)$