Si $E_g \sim 1eV$ $\rho(T)$

Ge $E_g \sim 0.7eV$ impurities

GaAs $E_g \sim 1.4eV$

Direct vs. indirect intrinsic vs. extrinsic

Electrons: $\mathcal{E}(k) \approx \mathcal{E}_c + \hbar^2\left(\frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} + \frac{k_3^2}{2m_3}\right)$

Holes: $\mathcal{E}(k) \approx \mathcal{E}_v - \hbar^2\left(\frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} + \frac{k_3^2}{2m_3}\right)$

Si: conduction band 6 symmetry points
cigar shaped $m_L \approx m_e$, $m_T \approx 0.2m_e$
two deg. valence bands near $k=0$
masses: $\overset{\text{heavy}}{0.49m_e}$, $\overset{\text{light}}{0.16m_e}$
3rd band too

Ge: 8 symmetry related elipsoids <111>
valence band near $k \approx 0$

$$n_c(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$$

$$p_v(T) = \int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \left(1 - \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}\right)$$

$$= \int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$$

$$n_c(T) \approx \int_{\varepsilon_c}^{\infty} d\varepsilon \; g_c(\varepsilon) \, e^{-(\varepsilon-\mu)/k_BT}$$

$$= e^{-(\varepsilon_c-\mu)/k_BT} \overbrace{\int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \, e^{-(\varepsilon-\varepsilon_c)/k_BT}}^{N_c(T)}$$

$$p_v(T) \approx \int_{-\infty}^{\varepsilon_v} d\varepsilon \, g_v(\varepsilon) \, e^{-(\mu-\varepsilon)/k_BT}$$

$$= e^{-(\mu-\varepsilon_v)/k_BT} \overbrace{\int_{-\infty}^{\varepsilon_v} d\varepsilon \; g_v(\varepsilon) \, e^{-(\varepsilon_v-\varepsilon)/k_BT}}^{P_v(T)}$$

$$g_{c,v}(\varepsilon) = \sqrt{2|\varepsilon-\varepsilon_{c,v}|} \, \frac{m_{c,v}^{3/2}}{\hbar^3\pi^2} \longrightarrow N_c(T) = \frac{1}{4}\left(\frac{2m_c k_BT}{\pi\hbar^2}\right)^{3/2}$$

$$P_v(T) = \frac{1}{4}\left(\frac{2m_v k_BT}{\pi\hbar^2}\right)^{3/2}$$

$$n_c \cdot p_v = N_c P_v \, e^{-(\varepsilon_c-\varepsilon_v)/k_BT} = N_c P_v \, e^{-E_g/k_BT}$$

Intrinsic case: $n_c(T) = p_v(T) \equiv n_i(T)$

$$n_i(T) = [N_c P_v]^{1/2} e^{-E_g/2k_BT}$$

$$\ln n_i = \ln N_c - \frac{(\mathcal{E}_c - \mu)}{k_BT} = \ln P_v - \left(\frac{\mu - \mathcal{E}_v}{k_BT}\right)$$

$$= \frac{1}{2}\ln N_c + \frac{1}{2}\ln P_v - \frac{E_g}{2k_BT}$$

$$\rightarrow \quad 0 = \frac{1}{2}\ln\frac{N_c}{P_v} + \frac{\mu - \mathcal{E}_v}{k_BT} - \frac{E_g}{2k_BT}$$

$$\rightarrow \mu = \mathcal{E}_v + \frac{1}{2}E_g + \frac{1}{2}k_BT\ln\left(\frac{P_v}{N_c}\right)$$

$$= \mathcal{E}_v + \frac{1}{2}E_g + \frac{3}{4}k_BT\ln\left(\frac{m_v}{m_c}\right)$$

As $T \rightarrow 0$, μ lies in middle of gap.

<u>Extrinsic case:</u> $n_c - p_v = \Delta n \neq 0$

Still: $n_c p_v = N_c P_v e^{-E_g/kT} = n_i^2$

$$\begin{Bmatrix} n_c \\ p_v \end{Bmatrix} = \frac{1}{2}\left[(\Delta n)^2 + 4n_i^2\right]^{1/2} \pm \frac{1}{2}\Delta n$$

$$n_c(T) = N_c(T)\, e^{-(\mathcal{E}_c-\mu)/k_BT}$$
$$n_i(T) = N_c(T)\, e^{-(\mathcal{E}_c-\mu_i)/k_BT}$$
$$\rightarrow \frac{n_c(T)}{n_i(T)} = e^{-(\mu_i-\mu)/k_BT}$$

$$p_v(T) = P_v(T)\, e^{-(\mu-\mathcal{E}_v)/k_BT}$$
$$n_i(T) = P_v(T)\, e^{-(\mu_i-\mathcal{E}_v)/k_BT}$$
$$\frac{p_v(T)}{n_i(T)} = e^{-(\mu-\mu_i)/k_BT}$$

$$\frac{\Delta n}{n_i} = e^{(\mu-\mu_i)/k_BT} - e^{-(\mu-\mu_i)/k_BT} = 2\sinh(\beta(\mu-\mu_i))$$

$|\Delta n| \gg |n_i|$ one kind of carrier dominates:

n-type or p-type

Impurity Levels:

donors - els. to cond. band   higher valence
acceptors - holes to val. band   lower valence

Ge: 4e ion
As: 5e ion
} +e core & weakly bound el.

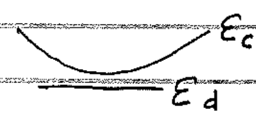
binding 0.013 eV

since ε different (≈16)
(ε ≈ inf. in metal)
also effective mass

$$r_o = \frac{m}{m^*}\,\varepsilon\, a_o$$

$$\mathcal{E} = \frac{m^*}{m}\frac{1}{\varepsilon^2} \times 13.6\,\text{eV}$$

$r_o \sim 100\,\text{Å}$ or more



} close!

Si 1.12 eV $\mathcal{E}_c - \mathcal{E}_d = 0.044\,\text{eV}$ for P donor

Donor level: no els. present

two possible 1 el. states

$$\langle n \rangle = \frac{2\, e^{-\beta(\varepsilon_d - \mu)}}{1 + 2 e^{-\beta(\varepsilon_d - \mu)}} = \frac{1}{\frac{1}{2} e^{\beta(\varepsilon_d - \mu)} + 1}$$

$$n_d = \frac{N_d}{\frac{1}{2} e^{\beta(\varepsilon_d - \mu)} + 1}$$

Acceptor: 0-hole 2 els.

1-hole 1 els.

$$p_a = \frac{N_a}{\frac{1}{2} e^{\beta(\mu - \varepsilon_a)} + 1}$$

$$n_c + n_d - p_v - p_a = N_d - N_a$$ (charge neutrality)

Suppose $\varepsilon_d - \mu \gg k_B T$

$\mu - \varepsilon_a \gg k_B T$

$\rightarrow n_d \ll N_d, \; p_a \ll N_a.$

$$\rightarrow \Delta n = n_c - p_v = N_d - N_a$$

From p. 4,

$$\left\{ \begin{matrix} n_c \\ p_v \end{matrix} \right\} = \frac{1}{2}\left[(N_d - N_a)^2 + 4n_i^2\right]^{1/2} \pm \frac{1}{2}\left[N_d - N_a\right]$$

$$\frac{N_d - N_a}{n_i} = 2 \sinh \beta(\mu - \mu_i)$$

For the strongly doped case, $|N_d - N_a| \gg n_i$,

$$\left.\begin{aligned} n_c &\approx N_d - N_a \\ p_v &\approx \frac{n_i^2}{N_d - N_a} \end{aligned}\right\} N_d > N_a$$

$$\left.\begin{aligned} n_c &\approx \frac{n_i^2}{N_a - N_d} \\ p_v &\approx N_a - N_d \end{aligned}\right\} N_a > N_d$$