Name:

Exam 2 Part 2 - Solid State Physics - Fall 2015

December 7, 2015

<u>Directions:</u> Please clear your desk of everything except for pencils and pens. The exam is closed book, and you are not allowed a formula sheet, but you may use a calculator. Leave substantial space between you and your neighbor. Show your work in the space provided on the exam. I can provide additional scratch paper if needed.

The entire exam is out of 100 points. Each subquestion, (a), (b), (c), ... is worth 5 points. This part of the exam is out of 50 points.

1. **Lattice Vibrations**

In your homework assignment on the classical harmonic crystal we derived the following expression for the matrix, $D(\vec{R})$, for a potential energy, $\phi(\vec{R})$, that only depends on the distance between atoms $R = |\vec{R}|$.

$$D_{m,n}(\vec{R} \neq 0) = -\left(\phi''(R) - \frac{\phi'(R)}{R}\right) \hat{R}_m \hat{R}_n - \frac{\phi'(R)}{R}\delta_{m,n} \tag{1}$$

Here, the indices m and n correspond to the x, y, and z directions, and $\hat{R}$ is a the unit vector in the direction of $\vec{R}$.

In this problem we consider a **simple cubic crystal** with lattice spacing a and atoms of mass M.

(a) Express the Fourier transform $D(k)$ in terms of the constants

$$A = 2\phi'(a)/a \tag{2}$$

$$B = 2\left(\phi''(a) - \frac{\phi'(a)}{a}\right) \tag{3}$$

for this simple cubic crystal case.

(b) Solve for the angular frequency of the normal modes, $\omega_s(k)$, where s indicates the three possible polarizations.

(c) Take the limit as $k \to 0$ and determine the speed of sound in terms of A and B. Does the speed of sound depend on the polarization or the direction of k in this case?

(d) Sketch $\omega_s(k)$ for k along the x-axis. Indicate on your plot the Brillouin zone boundaries.

(e) How do you expect the lattice contribution to the specific heat for the quantum harmonic crystal to depend on temperature in the low temperature limit and in the high temperature limit?

2. **Transport and Semiconductors**

(a) Write down the Boltzmann equation in the relaxation time approximation for electrons in a uniform time dependent electric field.

(b) In a spatial region a semiconductor has N_d donors and N_a acceptors per unit volume. In the same region the density of electrons in the conduction band is n, and the density of holes in the valence band is p. What is the charge density of the semiconductor in this region?

(c) Assume that the density of states in the conduction and valence bands has the form

$$g_c(E > E_c) = G_c\sqrt{E - E_c} \tag{4}$$

$$g_v(E < E_v) = G_v\sqrt{E_v - E}. \tag{5}$$

Assuming that the gap, $E_c - E_v$, is much larger than the the temperature and that the chemical potential is in between E_c and E_v, derive expressions for the density of electrons in the conduction band, n, and the density of holes in the valence band, p. Your expressions should include G_c, G_v, k_BT, and μ (chemical potential).

You may wish to use the integral

$$\int_0^\infty \sqrt{\epsilon}e^{-\beta\epsilon}d\epsilon = \frac{\sqrt{\pi}}{2}(k_BT)^{3/2}. \tag{6}$$

(d) Using the result from the previous part show that the product of n and p is independent of the chemical potential, μ. It only depends on temperature.

(e) Take the semiconductor to be neutral (uncharged) and $N_d \gg N_a$. Assume that all the donors and acceptors are ionized. Using the results form parts (b) and (d), give approximate non-vanishing expressions for the density of electrons and holes. (Non-vanishing means that zero is not an acceptable answer.)