Homework 4
(due Friday, October, 7)

In this homework assignment we cover the semiclassical model of electron dynamics as explained on pages 217 and 218, and the semiclassical theory of conduction in metals. To calculate the properties of metals we use the Boltzmann equation which is implicit in Eq. (13.21) in the book, but not introduced until later (Eq. (16.13)). The goal for this assignment is to get you comfortable solving for the semiclassical dynamics of r and k and solving the Boltzmann equation in some simple cases.

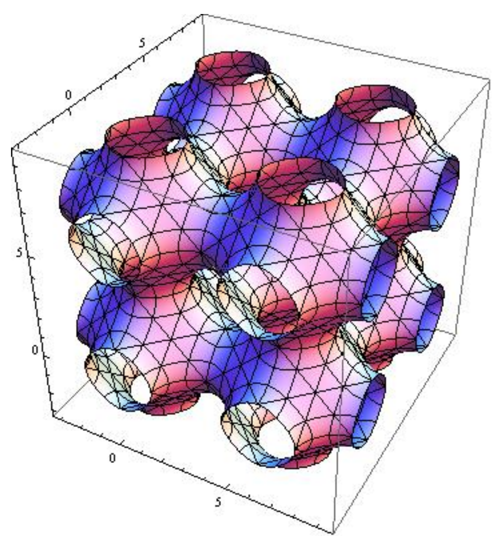
1. **Semiclassical Dynamics**

   (a) For graphene near the corners of the Brillouin zone the dispersion relation is linear, $E = \pm B|k - k_o|$, where $B > 0$ is a constant and k_o is at the corner of the two dimensional Brillouin zone. Solve the equations of the semiclassical dynamics, Eqs. (12.6a) and (12.6b), near k_o in the presence of a magnetic field perpendicular to the graphene sheet. Make sure to consider both the $+B$ and the $-B$ cases. What is the period of the orbits?

   (b) Repeat (a) for the traditional dispersion relation of $E = A|k|^2$ near the top or bottom of a band. Here, A can be positive of negative.

   (c) Compare the results of parts (a) and (b).

   (d) A constant energy surface of a three dimensional simple cubic tight binding model is shown below. Sketch an orbit in a constant magnetic field in the z-direction that is closed. Sketch another orbit that is open.



2. **Boltzmann equation**

   (a) In homework assignment 1, problem 1 you computed the AC conductivity matrix in the presence of a static magnetic field in the z-direction. Compute the conductivity matrix for this same case, except now using the Boltzmann equation.

Assume that the dispersion relation for the states near the Fermi energy has the form $E(k) = E_o + Ak^2$, where A can be either positive or negative. Follow the same steps that we covered in class (see notes for Chapter 13):

i. Write down the Boltzmann equation in the presence of electric and magnetic fields, allowing for time dependence of the electric field. (There will be a $\partial f/\partial t$ term.)

ii. Take the linear response limit by substituting f_{eq} for f in the electric field term, but not in any of the other terms. In particular the term with the magnetic field has f not f_{eq}. We are only doing linear response in the electric field.

iii. Multiply both sides of the Boltzmann equation by $-e\vec{v}$ and integrate $\int d^3k/(4\pi^3)$. Remember that the current density is

$$j = \int \frac{d^3k}{4\pi^3}(-e\vec{v})f. \tag{1}$$

iv. Show that the contribution from the electric field term is

$$\int \frac{d^3k}{4\pi^3}(-e\vec{v})(-e\vec{E}\cdot\frac{1}{\hbar}\vec{\nabla}_k f_{eq}) = -\frac{\sigma_o}{\tau}\vec{E}, \tag{2}$$

where σ_o is the DC conductivity at zero magnetic field.

v. Show that the contribution from the magnetic field term is

$$\int \frac{d^3k}{4\pi^3}(-e\vec{v})(-e\frac{\vec{v}}{c}\times\vec{B}\cdot\frac{1}{\hbar}\vec{\nabla}_k f) = \frac{2Ae}{\hbar^2}(\frac{\vec{j}}{c}\times\vec{B}), \tag{3}$$

where A comes from the dispersion relation.

vi. Compute the conductivity matrix as defined in homework assignment 1.

(b) Specialize to the case of a the DC conductivity (zero frequency). Using your result from part (a) compute the Hall coefficient. What determines whether the Hall coefficient is positive or negative?