Homework 5

(due Monday, November, 7)

We have covered a number of chapters since the last homework assignment. Some of these chapters have more qualitative information. These are important, and you will be tested on them; however, it is tough to make a homework assignment on those chapters. The following are some problems on the more quantitative chapters that are also important for their results.

1. **Boltzmann equation in more depth**
   Chapter 16, Problem 1.

2. **Hartree and Hartree-Fock approximations**

   (a) Do Chpt. 17, Problem 1 for the case when there are only two electrons and two spatial wave functions:

   $$\psi(r_1, s_1, r_2, s_2) = \psi_1(r_1, s_1)\psi_2(r_2, s_2). \tag{1}$$

   (b) Do Chpt. 17, Problem 2 again for the case when there are only two electrons and two spatial wave functions:

   $$\psi(r_1, s_1, r_2, s_2) = \frac{1}{\sqrt{2}}\left(\psi_1(r_1, s_1)\psi_2(r_2, s_2) - \psi_2(r_1, s_1)\psi_1(r_2, s_2)\right). \tag{2}$$

   (c) What is the difference in the energy eigenvalues for the two cases? What is the physical significance of this difference?

3. **Electron-electron scattering**
   We saw in class that the scattering rate for an electron with energy ϵ was proportional to ϵ^2 at low temperatures with the convention that the Fermi energy is taken to be zero. (ϵ is really $E - E_F$). Here we consider a finite temperature.

   Following the argument on pages 346-348, we consider a scattering process in which two electrons with energies ϵ_1 and ϵ_2 are scattered into two states with energies ϵ_3 and ϵ_4. Energy conservation requires that

   $$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4. \tag{3}$$

   We assume that the electron with energy ϵ_1 is at the Fermi energy, $\epsilon_1 = 0$.

   (a) Argue that the electron-electron scattering rate is proportional to

   $$\frac{1}{\tau} \propto \int d\epsilon_2 d\epsilon_3 d\epsilon_3 f(\epsilon_2)(1 - f(\epsilon_3))(1 - f(\epsilon_4))\delta(\epsilon_2 - \epsilon_3 - \epsilon_4)) \tag{4}$$

   (b) Show that the above integral is proportional to $(k_B T)^2$.

   (c) (Bonus) Show that the above integral is equal to $(\pi^2/4)(k_B T)^2$. You may do this either numerically or analytically.