

Homework 7

(due Friday, December, 2)

This is the last homework assignment! I am sorry to give it out right before Thanksgiving, but it is due the Friday after Thanksgiving. This assignment contains one problem from each of the chapters that we have covered since the last assignment. As usual these are intended primarily to have you go back and study the material covered in class so that you can do modest modifications to that material.

1. Measuring Phonons

Chapter 24, Problem 2.

2. Anharmonic Effects

Chapter 25, Problem 3.

3. Phonons in Metals

Consider the process when an electron with wave vector k scatters to one with wave vector k' by either emitting or absorbing a phonon. The following is a more mathematical formulation of the discussion on p. 525 for the electron-phonon scattering rate.

(a) Argue that the scattering rate for absorption is proportional to

$$\int \frac{d^3k'}{(2\pi)^3} |g_{k,k'}|^2 \frac{1}{e^{\beta\hbar\omega_{|k-k'|}} - 1} 2\pi\delta(\epsilon_k - \epsilon_{k'} + \hbar\omega_{|k-k'|}), \quad (1)$$

where we assume that $\omega_{|k-k'|} = c|k - k'|$. The incident electron state (k) also needs to be occupied and the scattered electron state (k') empty, but this is the part that gives one T^3 for absorption.

(b) Evaluate this integral in the low temperature limit by reducing the k' integral to an energy integral, $\int N(\epsilon_{k'})d\epsilon_{k'}$ and an angular integral $\int \Omega_{k'}/4\pi$. The density of states at the Fermi energy can taken to be constant, $N(E_F)$, in the integral. $|k - k'|$ depends on the angle between the two vectors:

$$|k - k'| \approx \sqrt{2k_F^2 - 2k_F^2 \cos(\theta)} = 2k_F \sin(\theta/2) \approx k_F \theta. \quad (2)$$

Using Eq. (26.42) show that the scattering rate is in Eq. (1) proportional to $(k_B T)^3$.

4. Dielectric Properties of Insulators

(a) Chapter 27, Problem 5.

(b) For the case of $n = 2$ in the previous problem,

$$\epsilon(\omega) = A + \frac{B_1}{\omega^2 - \omega_1^2} + \frac{B_2}{\omega^2 - \omega_2^2}, \quad (3)$$

plot $\epsilon(\omega)$ vs. ω as in Fig. 27.5 and plot ω vs. k as in Fig. 27.6.