Name:

Exam 1 - PHY 4604 - Fall 2001

September 26, 2001

8:20-10:10PM, NPB 1002

Directions: Please clear your desk of everything except for pencils and pens. The exam is closed book, and you are not allowed calculators or formula sheets. Leave substantial space between you and your neighbor. Show your work on the space provided on the exam. I can provide additional scratch paper if needed.

1. Solving the one dimensional Schrodinger equation

   (a) Write down the time dependent Schrodinger equation.

   (b) What is the meaning of the magnitude squared of the wave function?

   (c) Write down the time independent or stationary state Schrodinger equation. How are they related?

   (d) Consider the following potential V(x) in one-dimension:

$$\begin{aligned} V(x) &= 0 \text{ for } x < x_o \\ V(x) &= V_o \text{ for } x > x_o. \end{aligned}$$

   Suppose $E < V_o$. What is the solution to the time independent Schrodinger equation for $x < x_o$ and for $x > x_o$? Restrict yourself to the physically reasonable solutions.

(e) Write down the boundary conditions at $x = x_o$.

(f) Solve these boundary conditions for the transmission and reflection coefficients for a wave going from left to right. (This is an easy equation to solve; however, if you do not have time to do it just give the definition expressions for the reflection and transmission coefficients.)

2. Short answer section

   (a) What is an orthonormal basis?

   (b) What is completeness?

   (c) What is the adjoint of an operator?

(d) How do you represent an operator, a bra vector, and a ket vector as a matrix?

(e) How do you express a Hermitian operator/matrix in terms of its eigenvalues and eigenvectors?

3. Wave function space

For this problem you may wish to use the following addition theorems

$$\begin{aligned}
\cos(A \pm B) &= \cos(A)\cos(B) \mp \sin(A)\sin(B) \\
\sin(A \pm B) &= \sin(A)\cos(B) \pm \cos(A)\sin(B) \\
\Rightarrow \cos(A)\cos(B) &= \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right), \text{ etc.}
\end{aligned}$$

(a) Show that the following wave functions are orthogonal on the interval $[0, L]$.

$$\psi_n(x) = C_n \cos(\pi n x/L),$$

where $n = 0, 1, 2, \ldots$.

(b) Determine C_n so that the basis is orthonormal. Note that not all the C_n are the same.

(c) Assuming that this basis is complete on the interval $[0, L]$, express the following functions as sums over basis vectors:

$$\begin{aligned}
\phi(x) &= \delta(x - L/2) \\
\phi(x) &= \sin(2\pi x/L) \\
\phi(x) &= 1 \text{ if } x < L/2 \text{ and } 0 \text{ if } x > L/2
\end{aligned}$$

(d) Write down the completeness condition for this basis.

4. Operators and matrices

(a) Determine the eigenvalues and eigenvectors of the following Hermitian matrix.

$$\mathbf{A} = \begin{pmatrix} E & V \\ V & E \end{pmatrix}$$

(b) Verify that these eigenvectors are orthogonal.

(c) Multiply the eigenvectors by a constant so they are orthonormal.

(d) Verify the completeness condition explicitly for the basis composed of these eigenvectors.

(e) Consider the following two other Hermitian operators.

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Is it possible to find simultaneous eigenvectors of $\mathbf{A}$ and $\mathbf{B}$? Is it possible to find simultaneous eigenvectors of $\mathbf{A}$ and $\mathbf{C}$?