Name:

Exam 2 - PHY 4604 - Fall 2001

October 30, 2001

6:15-8:05PM, NPB 1002

Directions: Please clear your desk of everything except for pencils and pens. The exam is closed book, and you are not allowed calculators or formula sheets. Leave substantial space between you and your neighbor. Show your work on the space provided on the exam. I can provide additional scratch paper if needed.

1. **Short answer section**

   (a) What is the commutator of the position and momentum operators?

   (b) What differential equations do the expectation values of the position and momentum operators satisfy for the hamiltonian $H = \frac{p^2}{2m} + V(x)$?

   (c) Express the solution of the time-dependent Schrodinger equation at time t in terms of the wave function at $t = 0$ and the eigenvectors of the hamiltonian.

   (d) Explain the difference between the Heisenberg and Schrodinger pictures.

   (e) What is the commutation relation between the raising and lowering operators?

2. **Principles of quantum mechanics**

The wave functions

$$\phi_n(x) = \frac{e^{ik_n x}}{\sqrt{L}},$$

$k_n = 2\pi n/L$ and $n = 0, \pm 1, \pm 2, \ldots$, form a complete orthonormal basis on the interval $[0, L]$. They are also eigenvectors of the hamiltonian $H = \frac{p^2}{2m}$. Since the wave functions are periodic, $\phi_n(0) = \phi_n(L)$, these wave functions may be thought of as being on a ring with circumference L.

(a) Suppose the wave function at $t = 0$ is

$$\psi(x, 0) = \frac{e^{2\pi i x/L}}{\sqrt{2L}} + i\frac{e^{4\pi i x/L}}{\sqrt{2L}}.$$

Determine the wave function, $\psi(x, t)$, at an arbitrary time.

(b) Compute the probability density at time t.

(c) The probability density is periodic in time. What is the period, τ, of the oscillations?

(d) Sketch the probability density at $\tau = 0$, $\tau/4$, $\tau/2$, and $3\tau/4$.

(e) If the position of the particle is measured at $t = \tau/4$, what is the probability that the particle is between 0 and $L/2$, i.e., $0 \leq x \leq L/2$?

3. **Spin 1/2**

(a) At time $t = 0$, the spin in the z-direction, S_z, is measured and found to be $-\hbar/2$. What is the state vector, $|\psi(0)\rangle$, immediately after the measurement?

(b) Immediately after this measurement, a uniform magnetic field is applied in the x-direction: $\mathbf{B} = B_o\mathbf{x}$. Determine the wave function at time t: $|\psi(t)\rangle$.

(c) At this time t, we measure S_y. What values can we find and with what probabilities?

(d) What relations must exist between B_o and t for the result of the measurement to be certain? Give a physical interpretation of this condition.

(e) What is the expectation values of S_y as a function of time? Give a physical interpretation of this result and compare it to the results of (c) and (d) above.

4. **Harmonic oscillator**

(a) At $t = 0$ a harmonic oscillator is in the state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle.$$

What is the wave function at an arbitrary time t?

(b) For the harmonic oscillator the position and momentum operators may be written as

$$\begin{aligned} x &= \sqrt{\frac{\hbar}{2m\omega}}\,(a^\dagger + a) \\ p &= \sqrt{\frac{m\hbar\omega}{2}}\,i\,(a^\dagger - a). \end{aligned}$$

Compute the expectation value of the position and momentum as a function of time for the above wave function. Do the results for $\langle x\rangle$ and $\langle p\rangle$ make sense when compared to the classical equations of motion?

(c) Compute the expectation values of x^2 and p^2 as as a function of time. How are these related to the energy?

(d) Finally, using the results from (b) and (c) above, check the uncertainty principle for this wave function.