Name:

Final Exam - PHY 4604 - Fall 2001

December 3, 2001

3:00-5:00PM, 1101 New Physics Building

Directions: Please clear your desk of everything except for pencils and pens. The exam is closed book, and you are not allowed calculators or formula sheets. Leave substantial space between you and your neighbor. Show your work on the space provided on the exam. I can provide additional scratch paper if needed.

1. **Short answer section**

   (a) Write down the time dependent Schrodinger equation.

   (b) What is the meaning of the magnitude squared of the wave function?

   (c) What is the commutation relation between the raising and lowering operators of the harmonic oscillator?

   (d) What are the commutators of L_x, L_y, L_z and L^2?

   (e) What are the bound state energies of the hydrogen atom?

2. **One-dimensional Schrodinger Equation**

(a) Consider the following potential in one dimension:

$$\begin{aligned} V(x) &= V_o \text{ for } x < 0 \\ &= 0 \text{ for } x > 0. \end{aligned}$$

Assuming that $E > V_o > 0$ find the form of the solutions to the Schrodinger equation for $x > 0$ and for $x < 0$.

(b) Matching boundary conditions at $x = 0$, find the stationary state wave function for a particle moving from left to right. What is the transmission probability?

3. **Wave Function Space & Principles of Quantum Mechanics**

Consider an infinite square well between $x = 0$ and $x = L$. The energy eigenstates of the Hamiltonian are

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right)$$

for $n = 1, 2, \ldots$ These states form a complete orthonormal basis.

(a) Suppose at $t = 0$ the wave function of the system is given by

$$\psi(x, 0) = 1 \text{ if } x < L/2 \text{ and } 0 \text{ if } x > L/2.$$

Express $\psi(x, 0)$ in terms of the $\phi_n(x)$.

(b) Based on your results from part (a), compute the wave function at an arbitrary time, $\psi(x, t)$.

4. **Spin 1/2**

(a) A spin 1/2 particle is in a magnetic field pointing in the z-direction, $B_o\mathbf{z}$. If at $t = 0$, the spin of the particle is measured to point in the $+\mathbf{x}$ direction, what is the wave function as a function of time?

(b) At time, t, the spin of the particle is measured in the $\mathbf{y}$ direction. What are the probabilities for the different possible measured values of the spin?

5. **Harmonic Oscillator**

(a) At $t = 0$ a harmonic oscillator is in the state

$$|\psi(0)\rangle = \frac{2}{5}|0\rangle + \frac{3i}{5}|1\rangle.$$

What is the wave function at an arbitrary time t?

(b) For the harmonic oscillator the position operator may be written as

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a^\dagger + a).$$

Compute the expectation value of the position as a function of time for the above wave function.

6. **Angular Momentum**

(a) Compute the commutator $[J_x^2, J_y]$.

(b) What are the non-zero matrix elements of J_- for $j = 3/2$?

7. **Central Potentials**

Consider the three dimensional harmonic oscillator,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2.$$

(a) What is the radial Schrodinger equation for $u(r)$ with angular momentum l?

(b) Show that $u(r) = C\exp(-Ar^2)$ is a solution to the differential equation for $l = 0$. Is this a physical solution? Explain your answer.