Homework 2

(due Friday, September, 7)

In this homework assignment you will be solving several differential equations using the procedure shown in class. Although you are encouraged to refer to my notes and the book, you should do each problem from scratch and show your work in the papers you hand in. There is a fair amount of algebra involved so you are strongly encouraged to start this assignment early.

1. The probability current for the one dimensional Schrodinger equation is

$$j = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right).$$

   Evaluate the probability current for the following two wave functions.

$$\begin{aligned} \psi(x) &= Ae^{ikx} + A'e^{-ikx} \\ \psi(x) &= Be^{\rho x} + B'e^{-\rho x} \end{aligned}$$

   The coefficients, A, A', B, and B', may be complex. Do your results make sense based on how we have defined the transmission and reflection coefficients in class?

2. Consider a potential step of the form: $V(x) = 0$ for $x < 0$ and $V(x) = V_o$ for $x > 0$. Find the solution of the Schrodinger equation for a left moving wave with energy $E > V_o$, which has the form

$$\begin{aligned} \phi(x < 0) &= A_1' e^{-ik_1 x} \\ \phi(x > 0) &= A_2 e^{ik_2 x} + A_2' e^{-ik_2 x}, \end{aligned}$$

   where $k_1 = \sqrt{2mE/\hbar^2}$ and $k_2 = \sqrt{2m(E - V_o)/\hbar^2}$.

   Determine the transmission and reflection coefficients and verify that their sum is one.

$$\begin{aligned} T &= \frac{k_1}{k_2}\frac{|A_1'|^2}{|A_2'|^2} \\ R &= \frac{|A_2|^2}{|A_2'|^2} \end{aligned}$$

3. Consider a potential barrier of the form $V(x) = 0$ for $x < 0$ and $x > L$, and $V(x) = V_o$ for $0 < x < L$. Assume the energy of the electron is below the barrier, $0 < E < V_o$, so the solution to the Schrodinger equation has the form

$$\begin{aligned} \phi(x < 0) &= A_1 e^{ik_1 x} + A_1' e^{-ik_1 x} \\ \phi(0 < x < L) &= B_2 e^{\rho_2 x} + B_2' e^{-\rho_2 x} \\ \phi(L < x) &= A_3 e^{ik_1 x} + A_3' e^{-ik_1 x}, \end{aligned}$$

   where $k_1 = \sqrt{2mE/\hbar^2}$ and $\rho_2 = \sqrt{2m(V_o - E)/\hbar^2}$. Solve the Schrodinger equation for a right moving wave ($A_3' = 0$) and determine the transmission and reflection coefficients. Verify that their sum is one. In the limit of a thick barrier show that the transmission coefficient decays exponentially.