Homework 3

(due Friday, September, 14)

This homework assignment will give you some explicit experience working in wave function space. You will develop some intuition for how wave functions add, and you will get some experience in using a computer to plot functions. I am not going to specify which computer program to use. You should use whatever you are comfortable with. Some of you will use a spread sheet, some will use plotting packages, some will use mathematical programs like Maple, and some will actually write a computer code. Whatever package you use, make sure to label your axes and to include enough points to make your plots look smooth.

1. In class we introduced the complete orthonormal set of functions on the interval $[0, L]$,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right).$$

Using these functions, any function, $\phi(x)$, on the interval $[0, L]$ can be written as

$$\begin{aligned} \phi(x) &= \sum_{n=1}^{\infty} \psi_n(x) \langle \psi_n | \phi \rangle, \text{where} \\ \langle \psi_n | \phi \rangle &= \int_0^L dx \psi_n(x) \phi(x). \end{aligned}$$

In the following set $L = 1$. Evaluate analytically the coefficients $\langle \psi_n | \phi \rangle$ for the following four ϕ's.

$$\begin{aligned} \phi(x) &= 1 \text{ for } x > 0.5 \text{ and } = 0 \text{ for } x < 0.5 \\ \phi(x) &= x \\ \phi(x) &= \cos(2\pi x) \\ \phi(x) &= \delta(x - 0.25) \end{aligned}$$

2. Using the coefficients in part 1, make four plots, one for each of the different $\phi(x)$. In each plot evaluate the finite sum

$$\phi(x) = \sum_{n=1}^{N} \psi_n(x) \langle \psi_n | \phi \rangle$$

for $N = 5$, $N = 10$, and $N = 20$. Also, plot the exact function, $\phi(x)$. Thus, there should be four curves in each plot. Include along with the plot any programs or other documentation to show your work in making the plots.

From the plots, does it appear that the sum will converge to the function as $N \to \infty$?