Homework 8
(due Friday, Oct. 26)

1. Harmonic oscillator wavefunctions:

   (a) Starting from the ground state wavefunction,

   $$\phi_0(x) = \langle x|0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{1}{2}\frac{m\omega}{\hbar}x^2\right),$$

   and the formula for a general eigenstate,

   $$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle,$$

   derive expressions for the first three excited states, $\phi_1(x)$, $\phi_2(x)$, and $\phi_3(x)$.

   (b) Let $\xi = \sqrt{\hbar/m\omega}$. Plot the wavefunctions $\sqrt{\xi}\phi_n(x)$ versus x/ξ for $n = 0, 1, 2, 3$.

   (c) Plot the probability density $\xi|\phi_n(x)|^2$ versus x/ξ for $n = 0, 1, 2, 3$.

   (d) The parity operator, P, maps a wavefunction $\phi(x)$ to $\phi(-x)$. What are the eigenvalues of the parity operator for the states $\phi_n(x)$ for $n = 0, 1, 2, 3$?

2. Time dependence of the harmonic oscillator:

   For each of the $t = 0$ wavefunctions shown below determine

   (a) the wavefunction at a general time, $|\psi(t)\rangle$,

   (b) the expectation value of position as a function of time, $\langle\psi(t)|x|\psi(t)\rangle$,

   (c) the expectation value of momentum as a function of time, $\langle\psi(t)|p|\psi(t)\rangle$,

   (d) the amplitude and period of the oscillations.

$$\begin{aligned}
|\psi(t=0)\rangle &= \cos(\theta)|0\rangle + \sin(\theta)|1\rangle \\
|\psi(t=0)\rangle &= \cos(\theta)|1\rangle + \sin(\theta)|2\rangle \\
|\psi(t=0)\rangle &= \cos(\theta)|0\rangle + \sin(\theta)|2\rangle
\end{aligned}$$