Homework 9

(due Wednesday, Nov. 14)

As noted previously, Exam 2 was very important because it is the first time you have had to "put everything together." The majority of the class seems to be getting the basic ideas; however, most of you are also having some difficulty when it comes to the technical details. This homework assignment is designed to fill in any gaps you may have in your understanding.

On the following page you will find an answer key to Exam 2. Do the parts of the exam which you did not get 100% correct on a separate sheet of paper and hand it by Nov. 14. I have made notes of which parts of the problems each of you missed. You have plenty of time to do this assignment; however, you will be getting a homework assignment on angular momentum this coming Friday (Nov. 9), which will be due the following Friday (Nov. 16).

Answer key for Exam 2:

2. (a) $\psi(x,t) = \frac{e^{2\pi i x/L}}{\sqrt{2L}} e^{-iE_1 t/\hbar} + \frac{i e^{4\pi i x/L}}{\sqrt{2L}} e^{-iE_2 t/\hbar}$, $E_n = \frac{\hbar^2 k_n^2}{2m}$

(b) $|\psi(x,t)|^2 = \frac{1}{L}(1 - \sin(kx - \omega t))$, $k = \frac{2\pi}{L}$, $\omega = (E_2 - E_1)/\hbar$

(c) $\tau = \frac{1}{6\pi} \frac{2mL^2}{\hbar}$

(e) Probability $= \frac{1}{2}$

3. (a) $|\psi(0)\rangle = |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) $|\psi(t)\rangle = \begin{pmatrix} i\sin\left(\frac{\gamma B_0}{2} t\right) \\ \cos\left(\frac{\gamma B_0}{2} t\right) \end{pmatrix}$

(c) Prob. $\{S_y = \hbar/2\} = \frac{1}{2}(1 - \sin(\gamma B_0 t))$
Prob. $\{S_y = -\hbar/2\} = \frac{1}{2}(1 + \sin(\gamma B_0 t))$

(d) Prob. $\{S_y = \hbar/2\} = 1$ for $\gamma B_0 t = \frac{3\pi}{2} + 2\pi n$
Prob. $\{S_y = -\hbar/2\} = 1$ for $\gamma B_0 t = \frac{\pi}{2} + 2\pi n$

(e) $\langle S_y \rangle = -\frac{\hbar}{2} \sin(\gamma B_0 t)$

4. (a) $|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-iE_0 t/\hbar} |0\rangle + \frac{i}{\sqrt{2}} e^{-iE_1 t/\hbar} |1\rangle$

(b) $\langle x \rangle(t) = \sqrt{\frac{\hbar}{2m\omega}} \sin(\omega t)$
$\langle p \rangle(t) = \sqrt{\frac{m\hbar\omega}{2}} \cos(\omega t)$

(c) $\langle x^2 \rangle = \frac{\hbar}{m\omega}$, $\langle p^2 \rangle = m\hbar\omega$, $\langle E \rangle = \hbar\omega$

(d) $\Delta x = \sqrt{\frac{\hbar}{m\omega}} \sqrt{1 - \frac{1}{2}\sin^2(\omega t)}$
$\Delta p = \sqrt{m\hbar\omega} \sqrt{1 - \frac{1}{2}\cos^2(\omega t)}$
$\Delta x \Delta p = \hbar \sqrt{\frac{1}{2} + \frac{1}{4}\sin^2(\omega t)\cos^2(\omega t)} \geq \frac{\hbar}{\sqrt{2}} > \frac{\hbar}{2}$