Probability current:

Continuity equation: $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$

$$P(t) = \int d^3r\, \rho(r,t)$$

$$\frac{dP}{dt} = \int d^3r\, \frac{\partial \rho}{\partial t} = \int d^3r\, (-\vec{\nabla}\cdot\vec{j})$$

$$= -\int dS\, \vec{j}\cdot\hat{n}$$

$\frac{dP}{dt} > 0 \Rightarrow$ current entering $\int dS\, \vec{j}\cdot\hat{n} < 0$

$\frac{dP}{dt} < 0 \Rightarrow$ current leaving $\int dS\, \vec{j}\cdot\hat{n} > 0$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi \qquad \times \psi^*$$

$$-i\hbar \frac{\partial \psi^*}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi^* + V\psi^* \qquad \times \psi$$

subtract: $i\hbar\left(\psi^*\frac{\partial\psi}{\partial t} + \frac{\partial\psi^*}{\partial t}\psi\right) = -\frac{\hbar^2}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$

$$\rightarrow \frac{\partial|\psi|^2}{\partial t} = \frac{-\hbar}{2mi}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$$

$$= -\vec{\nabla}\cdot\frac{\hbar}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

$$\rightarrow \vec{j} = \frac{\hbar}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

One-dimensional case:

$|\psi(x)|^2 dx$ = prob. finding between x & $x+dx$

$$|\psi(x,t+dt)|^2 dx - |\psi(x,t)|^2 dx =$$

$$= j(x,t)\,dt - j(x+dx,t)\,dt$$

$$\Rightarrow \frac{|\psi(x,t+dt)|^2 - |\psi(x,t)|^2}{dt} = \frac{j(x,t) - j(x+dx,t)}{dx}$$

$$\Rightarrow \frac{\partial |\psi|^2}{\partial t^2} = -\frac{\partial j}{\partial x}$$, j = probability current

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \qquad < \times \psi^*$$

$$-i\hbar \frac{\partial \psi^*}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + V\psi^* \qquad < \times \psi$$

SUBTRACT:

$$i\hbar\left(\psi^* \frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi^*}{\partial t}\right) = \frac{-\hbar^2}{2m}\psi^* \frac{\partial^2 \psi}{\partial x^2} + \frac{\hbar^2}{2m}\psi \frac{\partial^2 \psi^*}{\partial x^2}$$

$$\Rightarrow \frac{\partial |\psi|^2}{\partial t} = \frac{-\hbar}{2mi}\left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 \psi^*}{\partial x^2}\right)$$

$$= \frac{-\hbar}{2mi}\frac{\partial}{\partial x}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$

$$\Rightarrow j = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$