Properties of Matter:

For a given $n = 1, 2, 3, \ldots$ the allowed l values are $l = 0, 1, 2, \cdots, n-1$. Thus, for $n = 1$, $l = 0$. For $n = 2$, $l = 0, 1$. Remember, for a given l, the are many m values: $m = -l, \cdots, l$. Finally, an electron may be in either the $|+\rangle$ or $|-\rangle$ spinstate.

All of the above imply that the levels in the hydrogen atom are degenerate:

$n = 1$, $E = -\frac{E_I}{1^2}$ $\quad l = 0 \quad$ degeneracy $= \overset{\text{for spin}}{2} \cdot (2 \cdot 0 + 1) = 2$

$n = 2$, $E = -\frac{E_I}{2^2}$ $\quad l = 0 \quad$ degeneracy $= 2 \cdot (2 \cdot 0 + 1) = 2$

$\quad l = 1 \quad$ degeneracy $= 2 \cdot (2 \cdot 1 + 1) = \underline{6}$

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$n = 3$, $E = -\frac{E_I}{3^2}$ $\quad l = 0 \quad$ degeneracy $= 2 \cdot (2 \cdot 0 + 1) = 2$

$\quad l = 1 \quad$ degeneracy $= 2 \cdot (2 \cdot 1 + 1) = 6$

$\quad l = 2 \quad$ degeneracy $= 2 \cdot (2 \cdot 2 + 1) = \underline{10}$

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$\vdots$

general n, $E = -\frac{E_I}{n^2}$ $\quad l = 0, 1, \cdots, n-1 \quad$ total degeneracy $= 2n^2$

<u>Pauli exclusion principle:</u> (covered next semester)

When you have more than 1 particle, say 2 particles, the wave function depends on each particles position: $\psi(r_1, r_2)$.

$$|\psi(r_1,r_2)|^2 = \text{prob. density of finding particle 1 at } r_1 \text{ and particle 2 at } r_2.$$

If the particles are <u>identical</u>, e.g. 2 electrons, then the hamiltonian is invariant under interchanging the 2 particles: $r_1 \leftrightarrow r_2$.

$$P\psi(r_1, r_2) = \psi(r_2, r_1)$$

$[P, H] = 0 \rightarrow$ We can take the eigenstates of H to satisfy $P\psi = \lambda\psi$.

Since $P^2\psi(r_1,r_2) = P\psi(r_2,r_1) = \psi(r_1,r_2)$
$= P\lambda\psi(r_1,r_2) = \lambda^2\psi(r_1,r_2)$,

$$\lambda^2 = 1 \Rightarrow \underbrace{\lambda = +1}_{\text{bosons}} \text{ or } \underbrace{\lambda = -1}_{\text{fermions}}.$$

Electrons are fermions.

One implication of $P\psi(r_1,r_2) = -\psi(r_1,r_2) = \psi(r_2,r_1)$ for fermions like electrons is that

$$\psi(r,r) = 0.$$

In otherwords, two particles (identical fermions) can not be at the same place at the same time.

If particle 1 is in state φ_a, and particle 2 is in state φ_b, then the fermion wave function

<u>can not</u> be $\psi(r_1,r_2) = \varphi_a(r_1)\varphi_b(r_2)$, but

must be antisymmetric like

$$\psi(r_1,r_2) = \frac{1}{\sqrt{2}}\left(\varphi_a(r_1)\varphi_b(r_2) - \varphi_b(r_1)\varphi_a(r_2)\right).$$

Clearly, if $a=b$, then $\psi = 0$. $\Rightarrow$ Two fermions can not be in the same state. //

Periodic table:

For atoms with more than 1 electron, one fills the states starting from the lowest energy ones.

H: 1 electron in 1s orbital

He: 1 electron in 1s orbital for each spin

Le: 1 electron in 1s orbital for each spin
and 1 electron in 2s orbital

⋮

Note: The chemistry notation is $\ell = 0$ s level
$\ell = 1$ p level
$\ell = 2$ d level.

Note: These wave functions are only approximate solutions to the true many body state, e.g.

$H\psi = E\psi$ with

$$H = \sum_{n=1}^{N} \left(\frac{p_n^2}{2m} + V(r_n) \right) + \frac{1}{2} \sum_{n \neq m} \frac{e^2}{4\pi\varepsilon_0 |r_n - r_m|} .$$

Solids:

The stability of matter, e.g. why atoms formsolids, depends on three physical laws:

1. Electrostatics
2. Schrodinger wave equation
3. Pauli exclusion principle

A simple example: Take 2 hydrogen atoms that are very far apart and bring them closer together.

Potential (electrostatic) energy increases because electrons repel.

Kinetic energy decreases because the electrons can spread out between the two atoms.

For not-too-small distances the decrease in kinetic energy out weighs the increase in potential energy and the 2 atoms bind to form H_2.