Study Guide/Practice Exam 1

Thus far in the course we have covered an introduction to quantum mechanics, Chapter 1, and the mathematics of quantum mechanics, Chapter 2. This lays the ground work for what we are going to do in the course, although it may not be appearant at this point how it all fits together.

The exam will consist of four parts:

1. Solving the one dimensional Schrodinger equation
2. Short answer section
3. Wave function space
4. Operators and matrices

This will cover everything we have done thus far. For the exam you are not allowed any formula sheets or a calculator (you will not need one). This practice exam/study guide should give you a very good idea of the formulas which you need to memorize for the exam (and hopefully forever).

1. Solving the one dimensional Schrodinger equation

   (a) Write down the time dependent Schrodinger equation.

   (b) What is the meaning of the magnitude squared of the wavefunction?

   (c) Write down the time indpendent or stationary state Schrodinger equation. How are they related?

   (d) How does one make a wavepacket from the plane wave solutions of the Schrodinger equation?

   (e) Suppose in a region of space the potential is a constant, V.

      i. What is the general form of the solution for energies $E > V$?

      ii. What is the general form of the solution for energies $E < V$?

   (f) The exam will ask you to set up a boundary value problem for piecewise constant potentials in one dimension. Here is a simple example for a potential step, which is a slight varient on what we have done thus far.

   A potential step occurs at the position, x_o, with $V = 0$ for $x > x_o$ and $V = V_o$. For an energy $E < V_o$ write down the solution to the (time-independent) Schrodinger equation for $x < x_o$ and $x > x_o$. There will be several unknowns which need to be determined by the boundary conditions. Write down the boundary conditions at $x = x_o$ to get two equations for the unknowns.

   (g) You will also need to know how to compute the transmission and reflection coefficients once you have the solutions to the Schrodinger equation. Here is an example to test your knowledge.

Suppose that the potential, $V(x)$, is equal to V for $x < x_1$ and equal to zero for $x > x_2 > x_1$. The solution to the Schrodinger equation for $x < x_1$ and $x > x_2$ then has the form:

$$\begin{aligned} \psi(x) &= Ae^{ik_1x} + Be^{-ik_1x} \text{ for } x < x_1 \\ \psi(x) &= Ce^{ik_2x} + De^{-ik_2x} \text{ for } x > x_2 \end{aligned}$$

How do you compute the transmission and reflection probabilities for for a wave moving from left to right? How do you compute the transmission and reflection probabilities for for a wave moving from right to left? Make sure to give explicit expressions for T and R in both cases.

2. Short answer section

   This section tests your knowledge of some of the more formal aspects in Chapter 2. In addition to studying these questions, I recommend studying the summary table for vector spaces which I handed out in class.

   (a) What are the three properties of vectors which we are using?

   (b) What is an orthonormal basis? Give examples for different dimensional vector spaces.

   (c) What is completeness? Give examples for different dimensional vector spaces.

   (d) What is a linear operator?

   (e) What is the adjoint of an operator?

   (f) What is a hermitian operator?

   (g) How do you represent an operator, a bra vector, and a ket vector as a matrix?

   (h) How do you express an "observable" operator in terms of its eigenvalues and eigenvectors?

   (i) Under what conditiosn can one guarantee that a basis may be found which is eigenvectors of two different operators, A and B?

3. Wave function space

   This section will be similar to homework assignment 3.

   (a) Show that the following wave functions are orthogonal on the interval $[0, L]$.

   $$\psi_n(x) = C_n e^{ik_n x/L},$$

   where $k_n = 2\pi n$ and n is any integer - positive, negative or zero.

   (b) Determine C_n so that the basis is orthonormal.

(c) Assuming that this basis is complete on the interval $[0, L]$, express the following functions as sums over basis vectors:

$$\begin{aligned}
\phi(x) &= \delta(x - L/2) \\
\phi(x) &= cos(2\pi x/L) \\
\phi(x) &= sin(2\pi x/L) \\
\phi(x) &= 1 \text{ if } x < L/2 \text{ and } 0 \text{ if } x > L/2
\end{aligned}$$

(d) Write down the completeness condition for this basis.

4. Operators and matrices

This section will be similar to homework assignment 4.

(a) Consider the following Hermitian matrix.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

It has the following three eigenvectors:

$$\begin{aligned}
|\psi_1\rangle &= C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
|\psi_2\rangle &= C_2 \begin{pmatrix} 1 \\ e^{2\pi i/3} \\ e^{4\pi i/3} \end{pmatrix} \\
|\psi_3\rangle &= C_3 \begin{pmatrix} 1 \\ e^{4\pi i/3} \\ e^{8\pi i/3} \end{pmatrix}
\end{aligned}$$

Verify that these eignevectors are orthogonal.

(b) Determine C_n so that these vectors are orthonormal.

(c) Determine the eigenvalues for these eigenvectors. (Hint: just operator (multiply) with $\mathbf{A}$ on the eigenvectors.)

(d) Verify the following idendities explicitly:

$$\begin{aligned}
\mathbf{1} &= \sum_{n=1}^{3} |\psi_n\rangle\langle\psi_n| \\
\mathbf{A} &= \sum_{n=1}^{3} \lambda_n |\psi_n\rangle\langle\psi_n|
\end{aligned}$$

(e) Define another operator (matrix) as

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Is it possible to find simultaneous eigenvectors of $\mathbf{A}$ and $\mathbf{B}$?