

Study Guide/Practice Exam 2

This exam will cover Chapters 3, 4, and 5 in the book, which is what we have covered since last exam. I was very pleased with the performance of the class on the first exam; however, please do not become overconfident. This exam will be harder because the material is more advanced and builds upon the earlier work. It is also an important exam because we are putting everything together: the formalism, the principles of quantum mechanics, and two physical examples - spin 1/2 and the harmonic oscillator. When you are studying for the exam, try to put all this together. Instead of focusing on how you do a particular kind of problem, try to see what principles are involved and how they fit together. I can change the problem a little to make a memorized procedure not work, but the principles will remain the same.

As for Exam 1, you are not allowed any formula sheets or a calculator (you will not need one). I will also give three points on the exam to anyone who hands in the practice exam next Monday at the beginning of class.

1. Short answer section

I will choose five of the following questions for the exam.

- (a) What are the six “postulates” of quantum mechanics given in the book?
- (b) What is the momentum operator?
- (c) What is the commutator of the position and momentum operators?
- (d) What is the uncertainty principle?
- (e) What differential equations do the expectation values of the position and momentum operators satisfy for the Hamiltonian $H = \frac{p^2}{2m} + V(x)$?
- (f) Express the solution of the time-dependent Schrodinger equation at time t in terms of the wave function at $t = 0$ and the eigenvectors of the hamiltonian.
- (g) What is the time evolution operator for a time independent hamiltonian?
- (h) Explain the difference between the Heisenberg and Schrodinger pictures.
- (i) How does the Stern-Gerlach experiment work?
- (j) For a spin 1/2 particle, what values of the spin in the z-direction can be observed in an experiment which measures S_z ?
- (k) What do the expectation values of the spin do in a magnetic field?
- (l) What are the eigenvalues of the harmonic oscillator hamiltonian?
- (m) What do the raising and lowering operators do when acting on the nth eigenstate of the harmonic oscillator, $|n\rangle$?
- (n) What is the commutation relation between the the raising and lowering operators?

2. Principles of quantum mechanics

The principles of quantum mechanics are used in all these problems. This problem will cover the principles of quantum mechanics in a problem similar to Homework Assignment 5.

- (a) Consider an infinite square well between 0 and L . The energy eigenstates of the hamiltonian are

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi nx}{L}\right).$$

Suppose that the wave function at $t = 0$ is

$$\psi(x, 0) = \frac{1}{\sqrt{L}} \sin\left(\frac{\pi x}{L}\right) + \frac{1}{\sqrt{L}} \sin\left(\frac{2\pi x}{L}\right)$$

What is the wave function at an arbitrary time $t > 0$?

- (b) What is the probability density at time t ?
- (c) Evaluate the expectation values of x and p as a function of time. Do these expectation values satisfy the proper equations of motion? (What are the equations of motion for the expectation values?)
- (d) What is the periodicity in time of the x expectation value?
- (e) Letting this periodicity be τ , sketch the probability density at $t = 0, \tau/4, \tau/2,$ and $3\tau/4$.

3. Spin 1/2

This section will be similar to Homework Assignment 6.

- (a) At time $t = 0$, the spin in the x -direction, S_x , is measured and found to be $\hbar/2$. What is the state vector, $|\psi(0)\rangle$, immediately after the measurement?
- (b) Immediately after this measurement, a uniform magnetic field is applied in the y -direction: $\mathbf{B} = B_o \mathbf{y}$. Determine the wave function at an arbitrary time t : $|\psi(t)\rangle$.
- (c) At this time t , we measure the observables $S_x, S_y,$ and S_z . What values can we find and with what probabilities?
- (d) What relations must exist between B_o and t for the result of one of the measurements to be certain? Give a physical interpretation of this condition.
- (e) What are the expectation values of $S_x, S_y,$ and S_z as a function of time? Give a physical interpretation of these results.

4. Harmonic oscillator

This section will test your knowledge of the solution of the quantum harmonic oscillator. In particular you will need to know the properties of the raising and lowering operators, a^\dagger and a and of the eigenvectors of the harmonic oscillator hamiltonian, $|n\rangle$.

- (a) The operator x can be written in terms of raising and lowering operators as

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger).$$

Determine the matrix elements of x^2 between any two states n and n' : $\langle n'|x^2|n\rangle$.

- (b) Write the operator x^2 as an infinite matrix.
(c) Determine the time dependence of the operator x in the Heisenberg picture.
(d) Compute the expectation value $\langle n|x_H(0)x_H(t)|n\rangle$ and comment on its physical meaning.