Study Guide/Practice Exam 3

This exam will cover Chapters 6 and 7 in the book, which is what we have covered since last exam. Even if you did well on the first two exams, it is encouraged that you study for this exam because the final will be cumulative. If you have done well on all three exams, the studying for the final should be easy.

As for the previous exams, you are not allowed any formula sheets or a calculator (you will not need one). I will also give three points on the exam to anyone who hands in the practice exam at the beginning of class on Friday Nov. 30.

Exam 3: Monday, Dec. 3 from 6:15-8:10PM in Room 310 Larsen Hall
Final : Wednesday, Dec. 12 from 3:00-5:00PM in our regular classroom

1. **Short answer section**

   I will choose five of the following questions for the exam.

   (a) What are the commutators of L_x, L_y, L_z, and L^2?

   (b) Define L_+ and L_-. Express L_x and L_y in terms of L_+ and L_-.

   (c) What are the eigenvalues of the J^2 and J_z operators?

   (d) What is J_+ acting on $|j, m\rangle$? What is J_- acting on $|j, m\rangle$? What is J_z acting on $|j, m\rangle$?

   (e) What are the allowed values of L^2 and L_z for the spherical harmonics? Why?

   (f) What is the orthogonality condition for the spherical harmonics?

   (g) What is the completeness condition for the spherical harmonics?

   (h) What is a symmetry in quantum mechanics? Give some examples of symmetries we have studied.

   (i) How is momentum related to the translation operator?

   (j) How is angular momentum related to the rotation operator?

   (k) How are the commutation relations for the angular momentum operators related to rotations in three dimensions?

   (l) What happens if you rotate a state $|j, m\rangle$ by 2π about the z-axis? How does your result depend on the value of m (and j)?

   (m) What is the form of the solution to the Schrodinger equation for a central potential?

   (n) What is the radial Schrodinger equation for $R(r)$? What is the normalization condition for $R(r)$?

   (o) What is the radial Schrodinger equation for $u(r)$? How is $u(r)$ related to $R(r)$? What is the normalization condition for $u(r)$?

   (p) What is the behavior of $u(r)$ as $r \to 0$ for angular momentum l and a not too singular potential?

(q) What is the Bohr radius (numerically)?

(r) What is a Rydberg (numerically)?

(s) What is the asymptotic behavior of the eigenstates of the hydrogen atom for energies less than zero?

(t) What are the bound state energies of the hydrogen atom?

(u) What are the degeneracies of the bound states of the hydrogen atom?

2. **Angular momentum**

   (a) Compute the commutator $[[L_+, L_z], L_-]$ and compare it to $[[L_+, L_-], L_z]$.

   (b) Expand the function, $f(\theta, \phi) = 1 + \sin(\theta)\sin(\phi + \frac{\pi}{4})$ in terms of the spherical harmonics. You may look in your book for the spherical harmonics. On the exam I would give you the spherical harmonics on a formula sheet. Do not leave your final answer in terms of integrals.

   (c) For $l = 1$ compute the matrix elements of the operator $L_z L_+$ and express it as a matrix.

3. **Central potentials**

   In this section you will solve a problem with a central potential. The following is one of the standard examples.

   A three dimensional harmonic oscillator has a Hamiltonian of

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2.$$

   (a) What is the radial Schrodinger equation for $u(r)$ for this Hamiltonian and angular momentum l?

   (b) For large r what is the asymptotic form of the wave function? (Hint: At large r look for a solution of the radial Schrodinger equation of the form $\exp(f(r))$, where $f(r)$ is some function to be determined.)

   (c) What is the behavior of $u(r)$ as $r \to 0$ for a given l?

   (d) Show that the asymptotic solution is part (b) is an exact solution for $l = 0$. What is the energy of this state?

   (e) Show that the wave function $\psi(x, y, z) = X(x)Y(y)Z(z)$ is an eigenstate of this Hamiltonian when when $X(x)$, $Y(y)$, and $Z(z)$ are eigenstates of the *one* dimensional harmonic oscillator. From what you know about the harmonic oscillator, prove that the eigenstate in part (d) is the ground state, and it is non-degenerate.

4. **Symmetry, Rotations, and the Hydrogen Atom**

   This section is a collection of other things which we have covered in Chapters 6 and 7. There will likely be two shorter problems in this section.

   (a) Take a solid object like a book and perform the following sequence of rotations. (i) Rotate about the x-axis by 90 degrees. (ii) Rotate about the y-axis by 90 degrees. (iii) Rotate about the x-axis by -90 degrees. The cumulative effect at the end of all three of these operations can be described by a single rotation. What is that rotation?

   (b) Perform the same operation using the rotation operators for spin 1/2 particles: $\exp(-i(\theta/\hbar)L_n)$, where L_n is the angular momentum operator in the direction of rotation and θ is the angle of rotation. Compare your results to those in part (a).

   (c) Plot the magnitude of both the radial wave functions, $|R_n(r)|$, and the angular wave functions $|Y_l^m(\theta, \phi)|$ for the $n = 1$, 2, and 3 states. You may look in your book to find these functions. On the exam I would give you these functions on a formula sheet.