Vector Spaces

size of space:	3 dimensions	N dimensions	countably infinite	continuum infinite
examples:	$v_x\mathbf{x} + v_y\mathbf{y} + v_z\mathbf{z}$	complex N-tuples	complex functions on interval $[0, L]$	complex functions on interval $(-\infty, \infty)$
add/subtract:	add/sub. components	add/sub. components	add/sub. functions	add/sub. functions
scalar multiplication:	multiply each comp.	multiply each comp.	multiply function	multiply function
dot product:	$\mathbf{v} \cdot \mathbf{w} = v_x w_x + v_y w_y + v_z w_z$	$\langle\psi\|\phi\rangle = \sum_{j=1}^{N} \psi^*(j)\phi(j)$	$\langle\psi\|\phi\rangle = \int_0^L dx\, \psi^*(x)\phi(x)$	$\langle\psi\|\phi\rangle = \int_{-\infty}^{\infty} dx\, \psi^*(x)\phi(x)$
basis example:	$\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$	$\psi_n(j) = \delta_{j,n}$ for $n = 1, 2, \ldots, N$	$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right)$ for $n = 1, 2, \ldots, \infty$	$\psi_k(x) = e^{ikx}$ for $-\infty < k < \infty$
orthonormality:	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$	$\langle\psi_n\|\psi_{n'}\rangle = \delta_{n,n'}$	$\langle\psi_n\|\psi_{n'}\rangle = \delta_{n,n'}$	$\langle\psi_k\|\psi_{k'}\rangle = 2\pi\delta(k - k')$
completeness:	$\hat{\mathbf{x}}(\hat{\mathbf{x}} \cdot + \hat{\mathbf{y}}(\hat{\mathbf{y}} \cdot + \hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot = \mathbf{1}$	$\sum_{n=1}^{N} \psi_n^*(j)\psi_n(j') = \delta_{j,j'}$ i.e., $\sum_{n=1}^{N} \|\psi_n\rangle\langle\psi_n\| = \mathbf{1}$	$\sum_{n=1}^{\infty} \psi_n^*(x)\psi_n(x') = \delta(x - x')$ i.e., $\sum_{n=1}^{\infty} \|\psi_n\rangle\langle\psi_n\| = \mathbf{1}$	$\int \frac{dk}{2\pi} \psi_k^*(x)\psi_k(x') = \delta(x - x')$ i.e., $\int \frac{dk}{2\pi} \|\psi_k\rangle\langle\psi_k\| = \mathbf{1}$