Name:

Exam 2 - PHY 4604 - Fall 2002

Friday, October, 25, 2002

This exam is closed book and notes. You are not allowed (nor will you need) a calculator. Please use the space provided on the exam to do the problems. You may also use the backs of pages if additional space is needed.

1. Short answer section:

   (a) What is the definition of the Hermitian conjugate of an operator A?

   (b) What function corresponds to the state $|x_o\rangle$?

   (c) What differential equations do the expectation values of the position and momentum operators satisfy for the Hamiltonian $H = \frac{p^2}{2m} + V(x)$?

   (d) Express the time dependent Heisenberg operator, $A(t)$, in terms of the time independent Schrodinger operator, A.

   (e) For a discrete set of states what is the completeness condition in Dirac notation?

2. Consider the harmonic oscillator hamiltonian,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

Because the eigenstates of H are complete, an initial wave function at $t = 0$ may be written as

$$|\psi(t=0)\rangle = \sum_{n=0}^{\infty} c_n |n\rangle,$$

where the c_n are complex numbers.

(a) What is the condition on the c_n so that $|\psi(t=0)\rangle$ is normalized?

(b) What is $|\psi(t)\rangle$ expressed in terms of c_n, $|n\rangle$, and ω?

(c) Derive an expression for $\langle x \rangle_t$ for $|\psi(t)\rangle$. The position operator for the harmonic oscillator is $x = \sqrt{\hbar/(2m\omega)}(a^\dagger + a)$.

(d) Find a set of c_n for which $\langle x \rangle_t = 5\sqrt{\hbar/(m\omega)}\sin(\omega t)$. (Take $\psi(t)\rangle$ to be normalized.)

(e) Compute the commutator $[a^\dagger, a^3]$.

3. Consider two electrons described by the Hamiltonian

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V(x_1) + V(x_2),$$

where $V(x) = \infty$ for $x < 0$ and $x > a$; $V(x) = 0$ for $0 < x < a$. Assume that the electrons are in the same spin state.

(a) What is the energy, E, and wave function, $\psi(x_1, x_2)$, of the **second** excited state?

(b) What is the meaning of $|\psi(x_1, x_2)|^2$?

(c) For the state in part (a) what is the probability of finding either one of the electrons at $x = x_o$?

(d) Suppose that the particles are bosons instead of fermions. What is the energy, E, and wave function, $\psi(x_1, x_2)$, of the **second** excited state?