

Name:

Exam 3 - PHY 4604 - Fall 2002

Monday, December 9, 2002

This exam is closed book and notes. You are not allowed (nor will you need) a calculator. Please use the space provided on the exam to do the problems. You may also use the backs of pages if additional space is needed.

1. Short answer section

- (a) What is the orthogonality condition for the spherical harmonics?

- (b) What is the form of the solution to the Schrodinger equation for a central potential?

- (c) What is the radial Schrodinger equation for $u(r)$? How is $u(r)$ related to $R(r)$? What is the normalization condition for $u(r)$?

- (d) What is the Bohr radius (numerically)?

- (e) What are the bound state energies of the hydrogen atom?

2. Angular momentum

- (a) Compute the commutator $[L_x^2, L_y^2]$. (The result should be a sum of products of three operators.)

(b) For $l = 2$ compute the matrix representation of the operator L_y .

3. Central potentials

Consider the the potential in three dimensions: $V(r) = 0$ for $r < a$ and $V(r) = \infty$ for $r \geq a$.

(a) Using the spherical Bessel functions what is the general form of the radial wave function, $R(r)$, for angular momentum l and energy $E > 0$?

(b) What is the boundary condition at $r = a$? What is the behavior of $R(r)$ as $r \rightarrow 0$?

(c) What is the solution for $R(r)$ for $l = 0$ consistent with (b) and (c) above? Specify $R(r)$ explicitly up to an overall normalization constant.

(d) Plot the radial wave function, $R(r)$.

(e) Determine the allowed energies.

4. Spin

- (a) Consider a system of spin $1/2$. What are the eigenvalues and eigenvectors of the operator $(S_x + S_z)/\sqrt{2}$? This operator is the spin operator in the $(\mathbf{x} + \mathbf{z})/\sqrt{2}$ direction.

(b) Suppose a measurement of this operator is made, and the system is found to be in the state corresponding to the smaller eigenvalue. What is the probability that a subsequent measurement of S_z yields $-\hbar/2$?

(c) How would the results of part (b) change if we measured S_x instead of S_z ?